Contact mechanics and elements of tribology

Lecture 3.
Surface Roughness

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MINES PARIS | PSL | Centre des Matériaux | CNRS

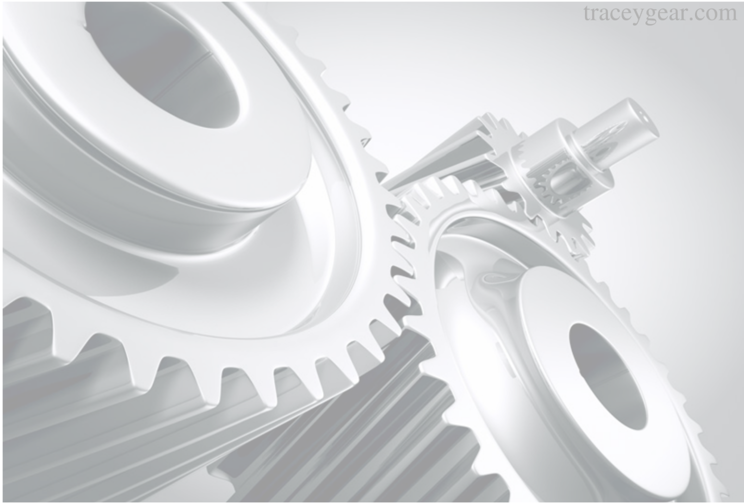
@ Centre des Matériaux (& virtually)
February 25, 2026

Outline

- Introduction
- Measurement techniques
- Classifications
- Main characteristics
- PDF and PSD
- Random process model of roughness
- Computational roughness models
- Reading

Introduction

Contact under microscope



Contact under microscope

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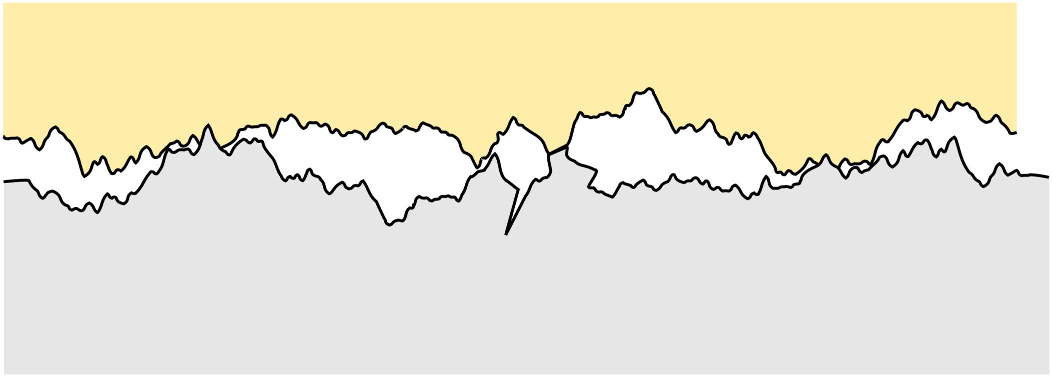

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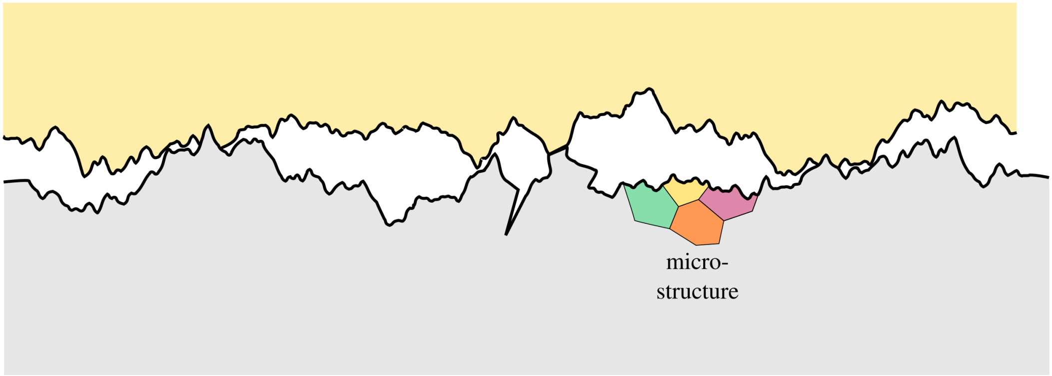

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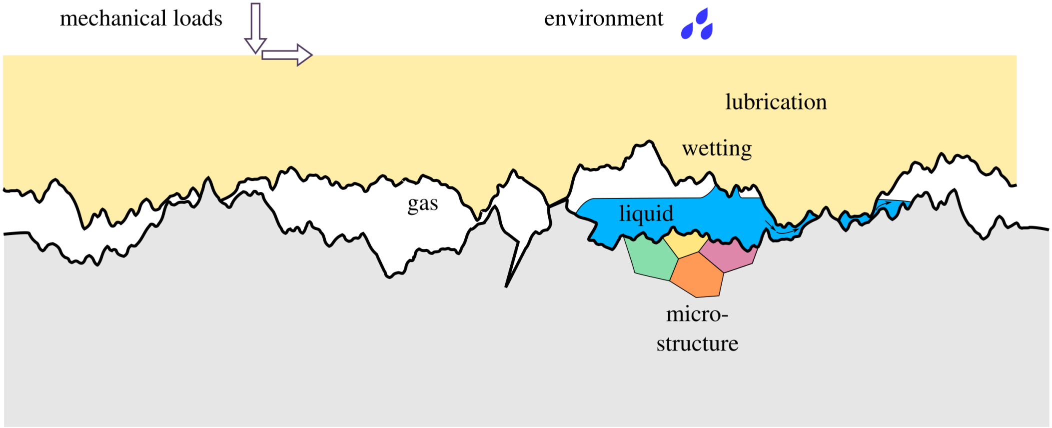

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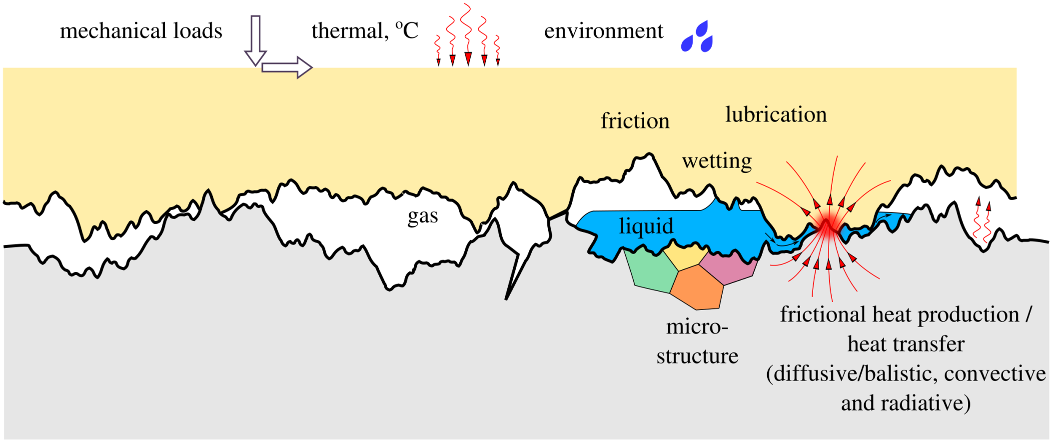

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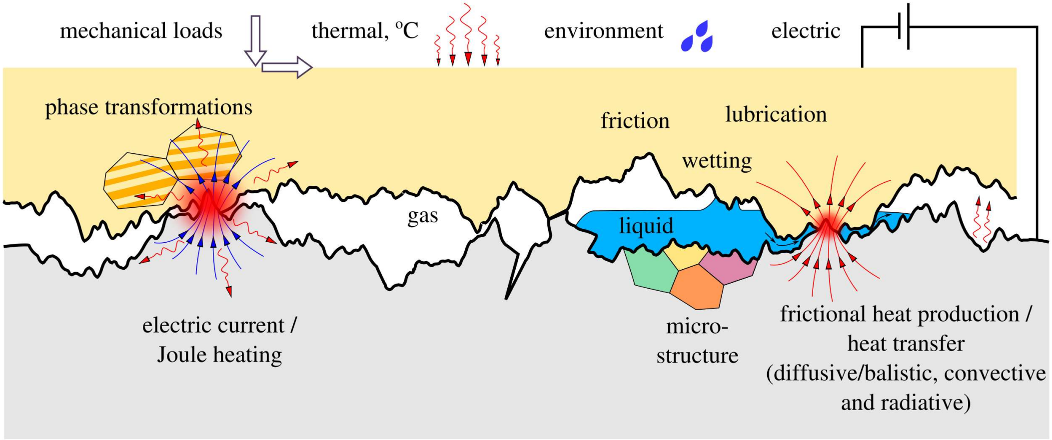

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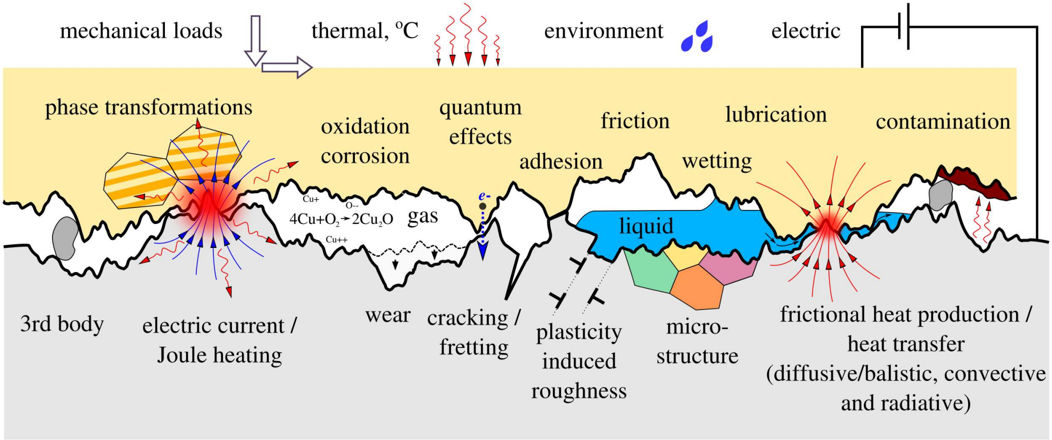

Roughness

- **Natural and industrial surfaces are *rough*:**
  - processing
  - polishing
  - coating
  - microstructure
  - surface energy
  - deformation
  - aging
  - environment

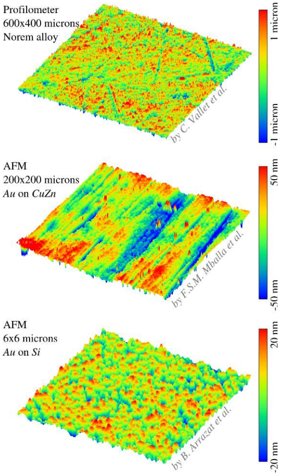

Fig. Examples of rough surfaces

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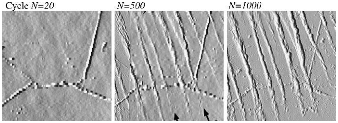

Fig. Persistent slip marks [1]

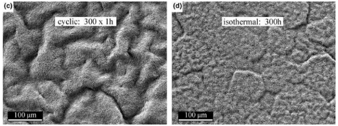

Fig. Rumpling (thermal cycling induced roughness in air)[2]

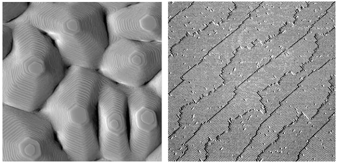

Fig. Epitaxial surface growth [3,4]

[1] J.Polák, J. Man & K. Orbtlík, Int J Fatigue 25 (2003)
[2] V.K. Tolpygo, D.R. Clarke, Acta Mat 52 (2004)
[3] M. Einax, W. Dieterich, P. Maass, Rev Mod Phys 85 (2013)
[4] J.R. Arthur, Surf Sci 500 (2002)

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- **Roughness affects:**
  - stress-strain state
  - dry friction
  - wear
  - adhesion
  - fluid flow
  - sealing
  - energy transfer

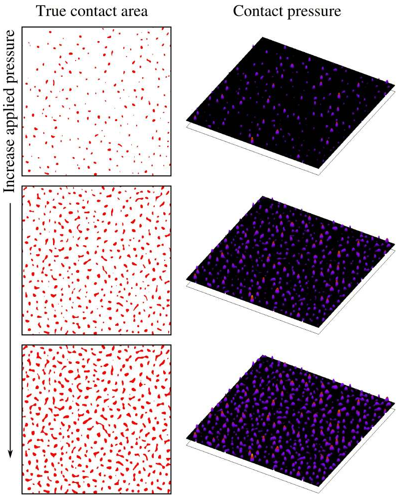

Fig. True contact area and stress fluctuations

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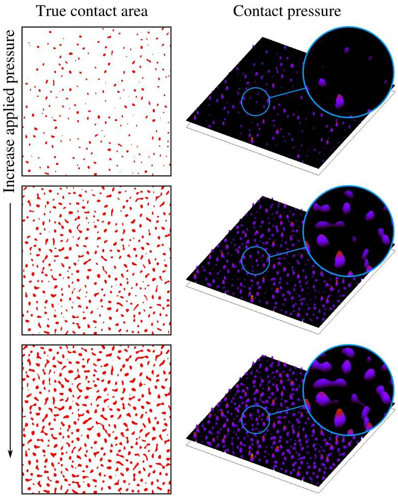

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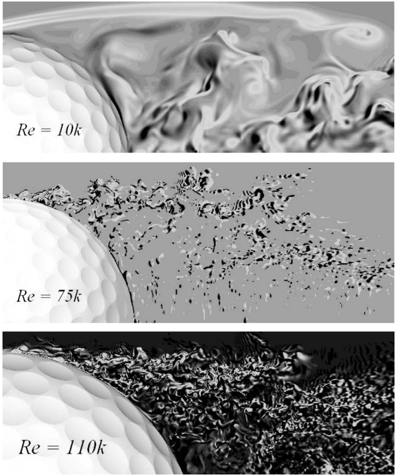

Fig. Numerical simulation of airflow around a (dimpled) golf ball [5]

[5] C.E. Smith, PhD thesis (2011)

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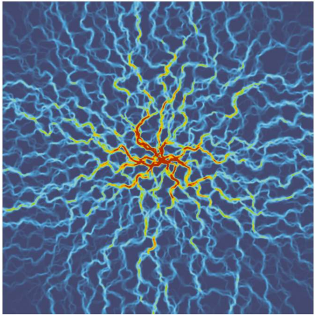


Fig. Fluid passage through free volume between rough surfaces

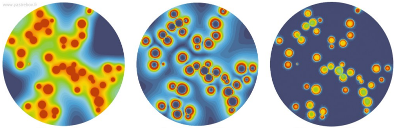


Fig. Heat transfer between rough surfaces (asperity-based model)

Roughness

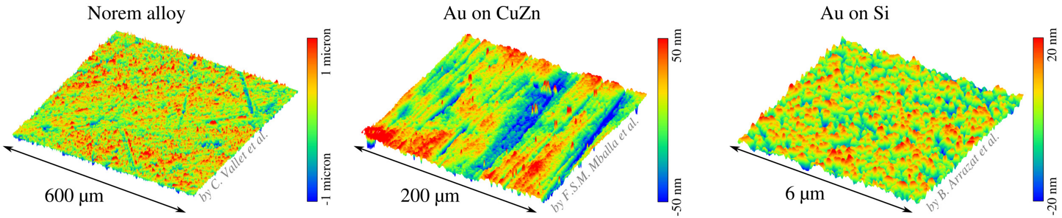
Visible to the naked eye

Granite

Asphalt concrete

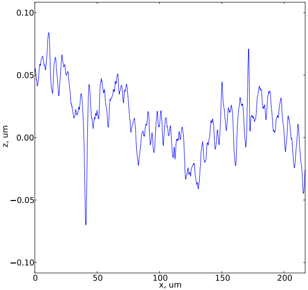
Cracked rock

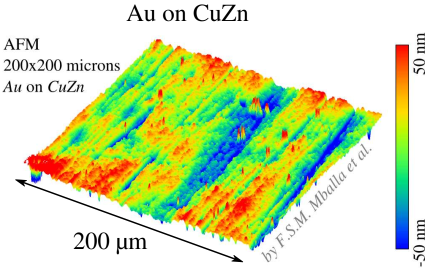
Seemingly smooth



Roughness: in plane and out of plane scales

$$\frac{\text{in-plane factor}}{\text{out-of-plane factor}} = 1000$$





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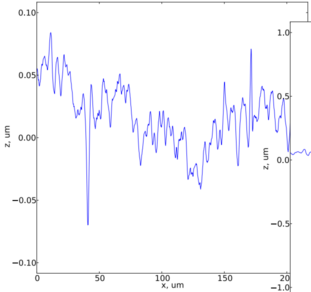
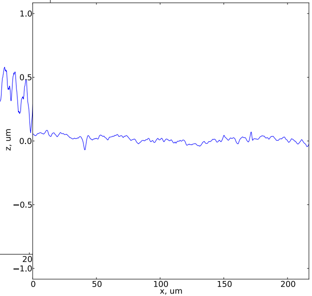
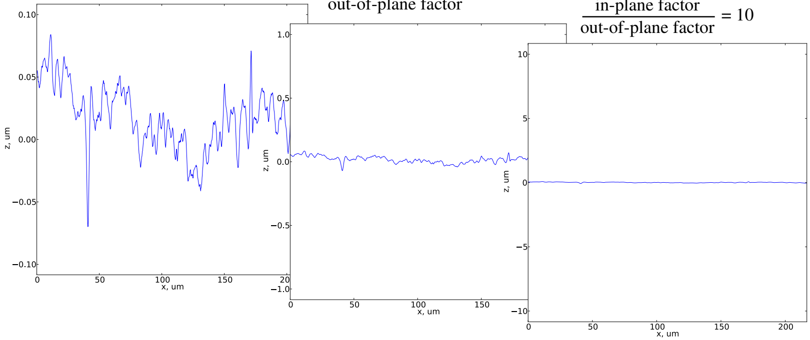


$$\frac{\text{in-plane factor}}{\text{out-of-plane factor}} = 100$$



Roughness: in plane and out of plane scales

$$\frac{\text{in-plane factor}}{\text{out-of-plane factor}} = 1000 \qquad \frac{\text{in-plane factor}}{\text{out-of-plane factor}} = 100 \qquad \frac{\text{in-plane factor}}{\text{out-of-plane factor}} = 10$$



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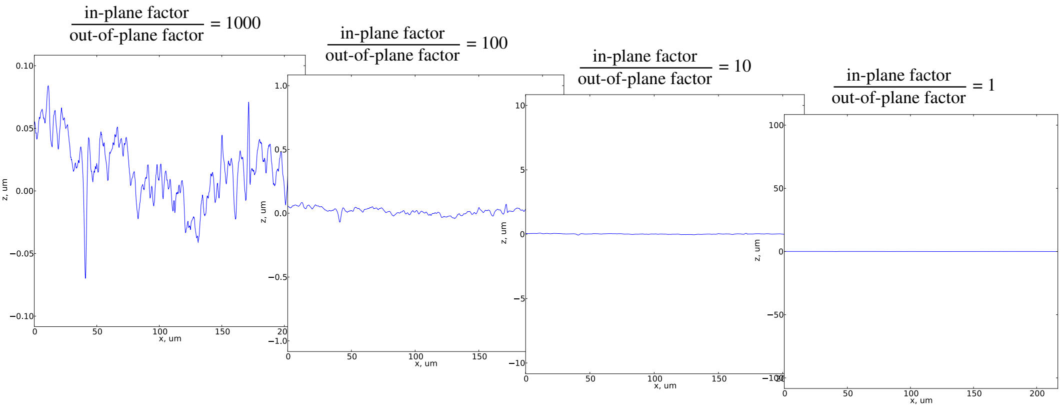


Surface metrology

Surface metrology techniques

Stylus measurements

- Mechanical contact of a tip with surface
- Force $\geq 3\mu$g, tip radius $\geq$ 50 nm
- Mainly for profile measurements $y(x)$

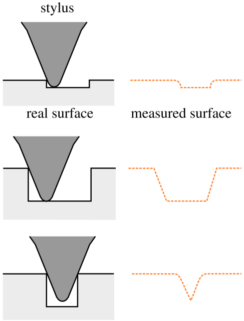
Optical measurements

- Confocal (laser scanning) microscopy
  - *highest lateral resolution*
- Interferometry (WLI):
  - *highest vertical resolution*
  - *10 to 100 times faster than CM*
- Scanning Electronic Microscopy (SEM):
  - *in secondary electron emission*
  - *electrons penetrate in the matter → roughness smoothing*
  - *conducting materials*

Nano-contact measurements

- Atomic Force Microscopy (AFM)
  roughness + adhesive and elastic properties
- Scanning Tunneling Microscope (STM)

Stylus profilometer



Valley measurement error

Stylus measurements
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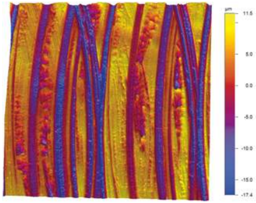
Stylus profilometer

Modern stylus profilometer
www.bruker.com



Roughness measurements ($\Delta z \approx 30$ μm)
www.icryst.com

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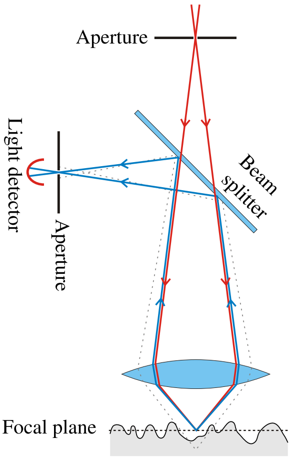
Confocal microscopy



Principle of confocal microscopy
adapted from www.wikipedia.org

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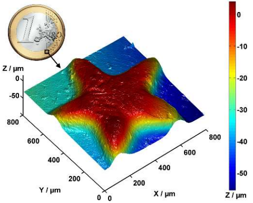
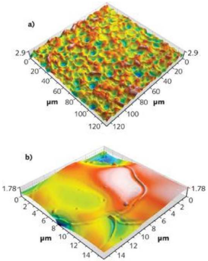
Confocal microscopy



1 euro surface www.wikipedia.org



Stainless steel machined with micro-electric discharge
www.laserfocusworld.org

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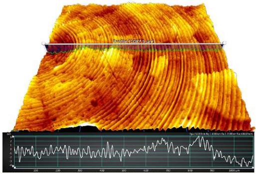
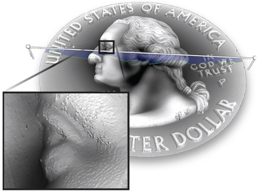
White Light Interferometry



Diamond-turned optics www.zygo.com



US quarter surface www.zygo.com

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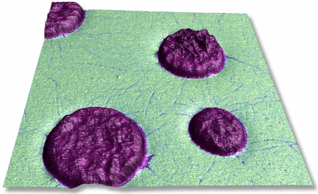
AFM

Modern AFM
www.bruker.com



Roughness and elastic moduli (color) of polymer blend
www.bruker.com

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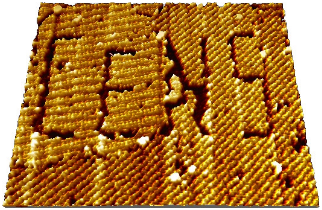
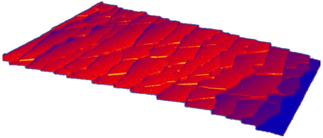


Fig. Center for NanoScience logo imprinted at atomic scale
www.cens.de



Atomic steps on platinum surface (500×500 nm)
www.icryst.com

Roughness Characterization

Roughness: classification

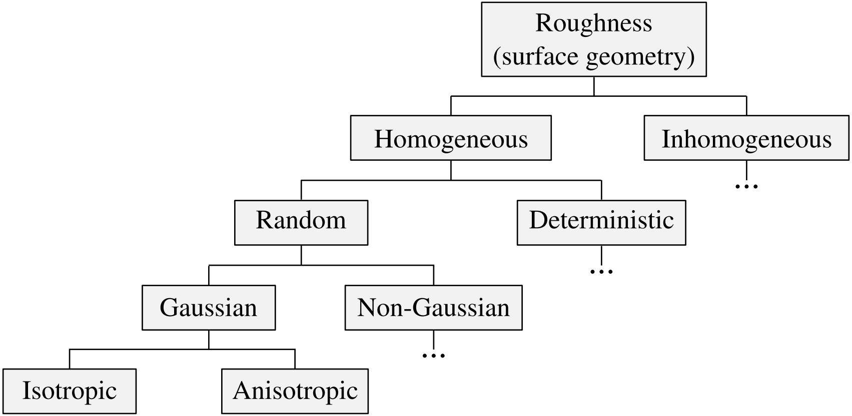
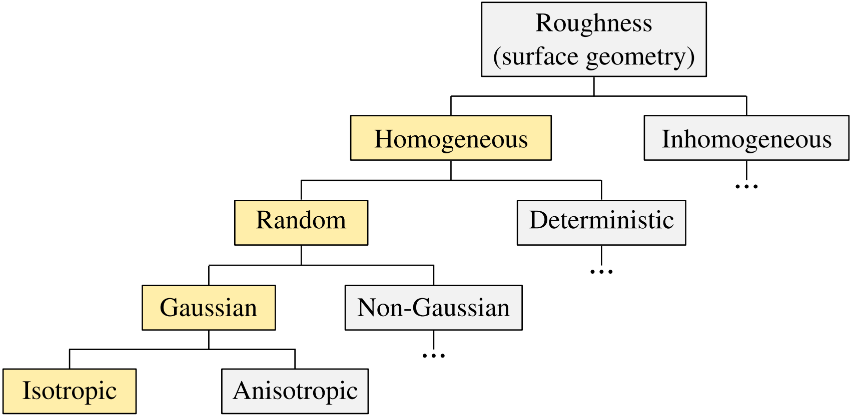


Fig. Roughness classification according to Nayak[1]

[1] Nayak, J. Lub. Tech. (ASME) 93:398 (1971)

Roughness: classification



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Roughness and geometry/form

- How to separate roughness from surface geometry?
- Most metrology software enable *simple* shape removal

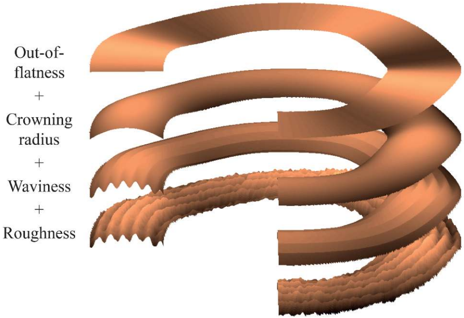

Fig. Circular metalic seal with turned copper surface[1]

[1] F.P. Rafòls, Licentiate Thesis, LTU 2016.

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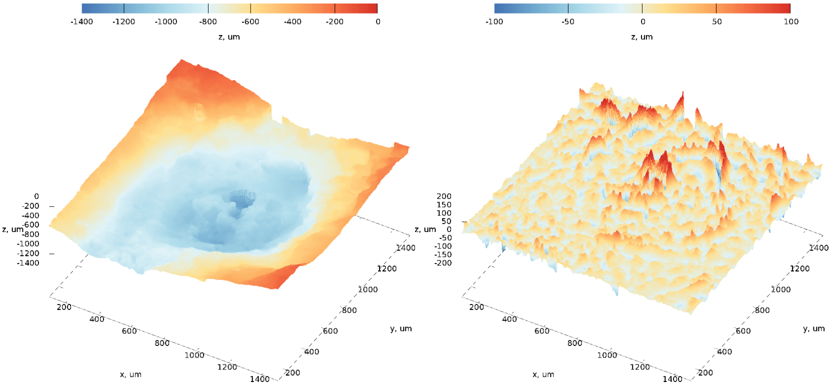

Fig. (left) impact crater, (right) shape is filtered out

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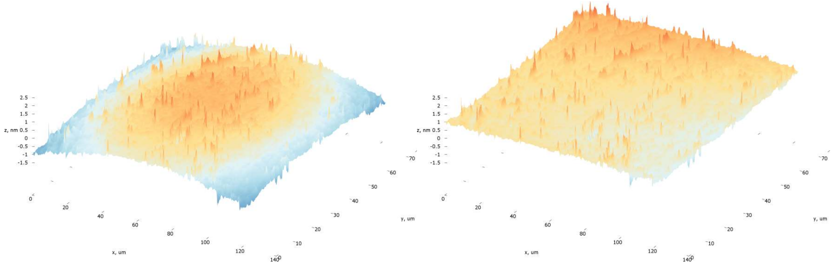
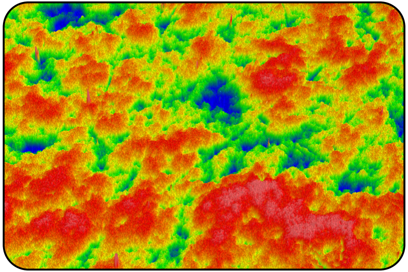


Fig. (left) spherical indenter, (right) $z = a(x^2 + y^2)$ shape is substracted

Surface characterization

- Let $z(x,y) : \mathbb{R}^2 \mapsto \mathbb{R}$ is a single-valued function
- For the average operator $\langle \bullet \rangle = \frac{1}{A} \int_A \bullet \, dA$
- Centered $\langle z \rangle = 0$
- No tilt $\langle \nabla z \rangle = 0$
- No curvature $\langle \nabla\nabla z \rangle = 0$



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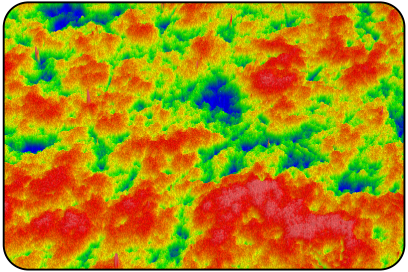


1. Height probability density (PDF) of z:

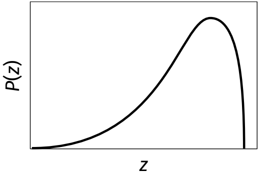
$$P(z)$$

2. Fourier Transform of z:

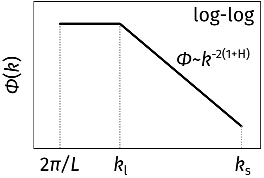
$$\mathrm{FT}(z(x,y)) = \hat{z}(k_x, k_y)$$

PDF and PSD

PDF
probability density

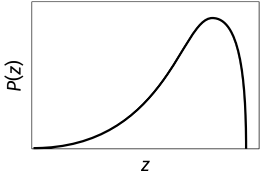


PSD
spectral density



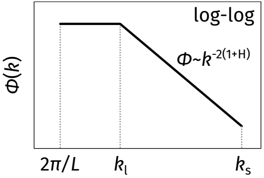
PDF and PSD

PDF
probability density



- Function
  $P(z) = \langle \delta(z - z'(x,y)) \rangle$
- Probability moments
  $s_p = \int_{z\in\mathbb{R}} z^p P(z)\,dz$

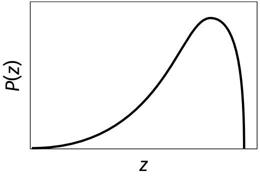
PSD
spectral density



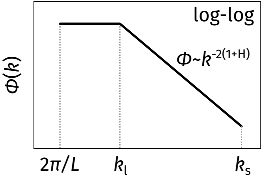
- Function
  $\Phi(k_x, k_y) = \frac{1}{4\pi^2 A} |\mathrm{FT}(z(x,y))|^2$
- Spectral moments
  $m_{pq} = \int_{k_x,k_y\in\mathbb{R}} k_x^p k_y^q \Phi(k_x,k_y)\,dk_x\,dk_y$

PDF and PSD

PDF
probability density



PSD
spectral density



- Function
  $P(z) = \langle \delta(z - z'(x,y)) \rangle$
- Probability moments
  $s_p = \int_{z\in\mathbb{R}} z^p P(z)\,dz$

- Function
  $\Phi(k_x, k_y) = \frac{1}{4\pi^2 A} |\mathrm{FT}(z(x,y))|^2$
- Spectral moments
  $m_{pq} = \int_{k_x,k_y\in\mathbb{R}} k_x^p k_y^q \Phi(k_x,k_y)\,dk_x\,dk_y$

Link between the PDF and the PSD: $\boxed{\langle z^2 \rangle = s_2 = m_{00}}$

Properties

Often rough surfaces are

- Gaussian

$$P_{\text{ens.}}(z) = \mathcal{N}(0,\sigma^2)$$

- Isotropic (rotational invariance of the spectrum)

$$\Phi_{\text{ens.}}(k_x, k_y) = \Phi_{\text{ens.}}(|\underline{k}|)$$

- Self-affine

$$\Phi_{\text{ens.}}(|\underline{k}|) \sim |\underline{k}|^{-2(1+H)}, \quad H \in (0,1)$$

- Ergodic

$$P_A(z) = \langle \delta(z - z'(x,y)) \rangle_A \xrightarrow[A\to\infty]{} P_{\text{ens.}}(z)$$

Parametrize roughness

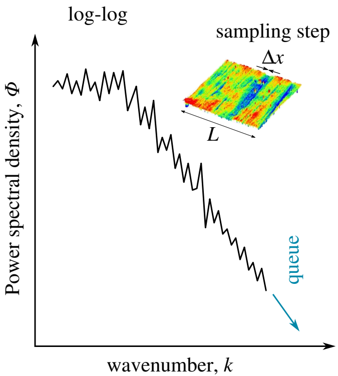
Sampling-step independent characteristics:

- Root mean squared roughness*

$$S_q = \sqrt{\langle z^2 \rangle} = \sqrt{s_2} = \sqrt{m_0}$$

- Hurst exponent $0 < H < 1$ of the PSD decay

$$\Phi \sim k^{-2(1+H)}$$



***Warning!** S_q can be scale-dependent[1,2], i.e. $S_q = S_q(L)$

[1] R.S. Sayles and T.R. Thomas, Nature 271:431-434 (1978).
[2] A. Pradha, M.H. Müser, L. Pastewka, T. Jacobs *and many others*. "The Surface Topography Challenge...", Tribology Letters (2025)

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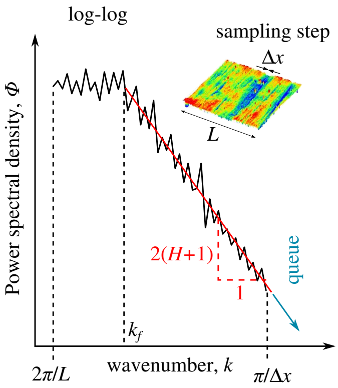

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Parametrize roughness

Sampling-step dependent characteristics:[3]

- Root mean squared gradient

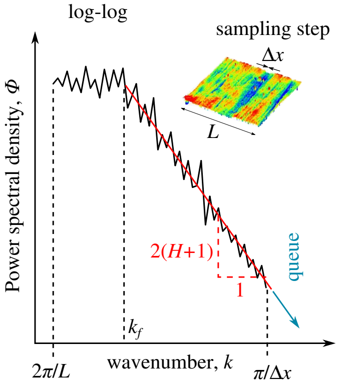
$$S_{dq} = \sqrt{\langle|\nabla z|^2\rangle} = \sqrt{2m_2}$$

- Root mean squared curvature

$$S_{ddq} = \sqrt{\langle\|\nabla\nabla z\|_F^2\rangle} = \sqrt{m_4}$$

- Nayak parameter

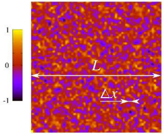
$$\alpha = \frac{m_0 m_4}{m_2^2}$$



[3] P.R. Nayak, Random Process Model of Rough Surfaces, ASME J Lubric Techn, 1971

Roughness modelling

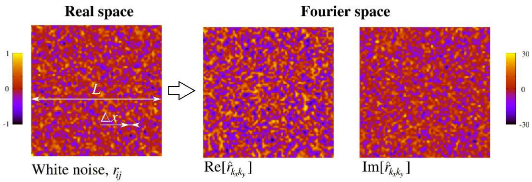
Real space **Fourier space**



White noise, r_{ij}

[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)
[2] Roughness generator `https://github.com/vyastreb/rfgen`

Roughness modelling



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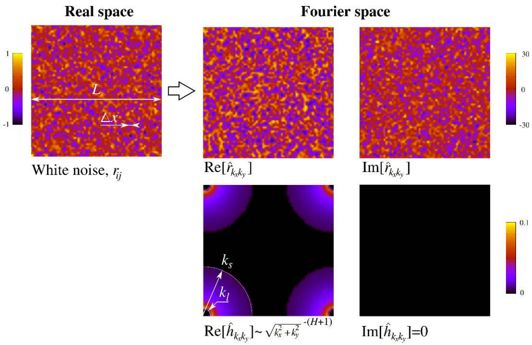
Roughness modelling



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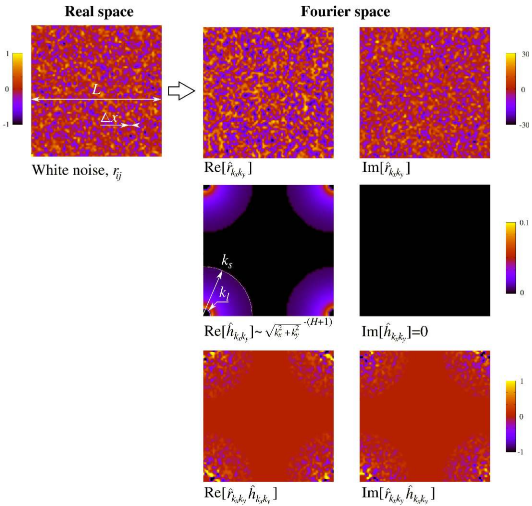

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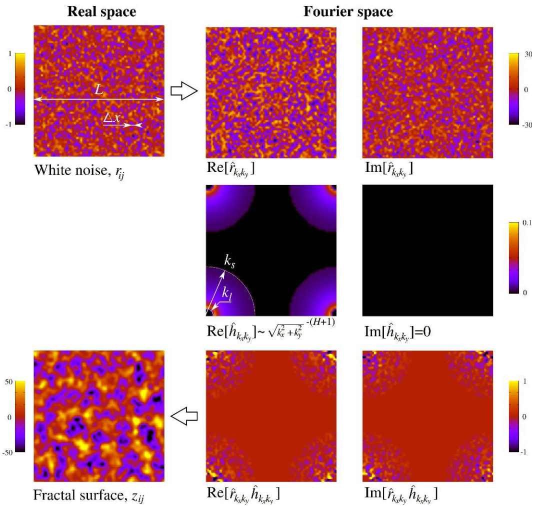

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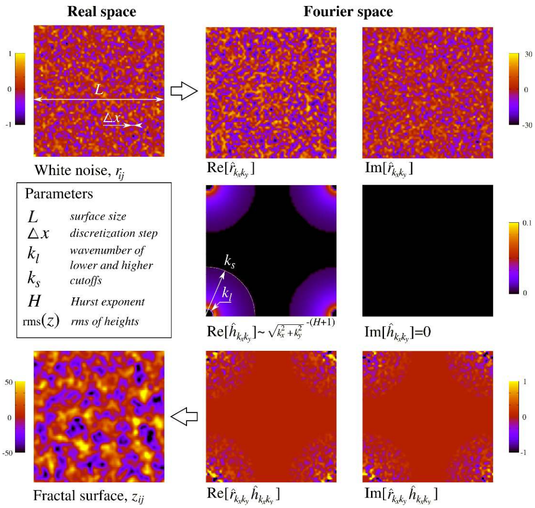
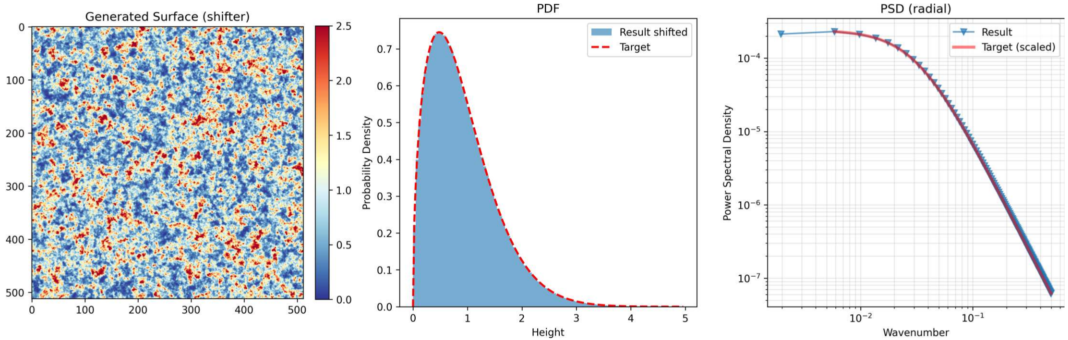


[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)
[2] Roughness generator https://github.com/vyastreb/rfgen

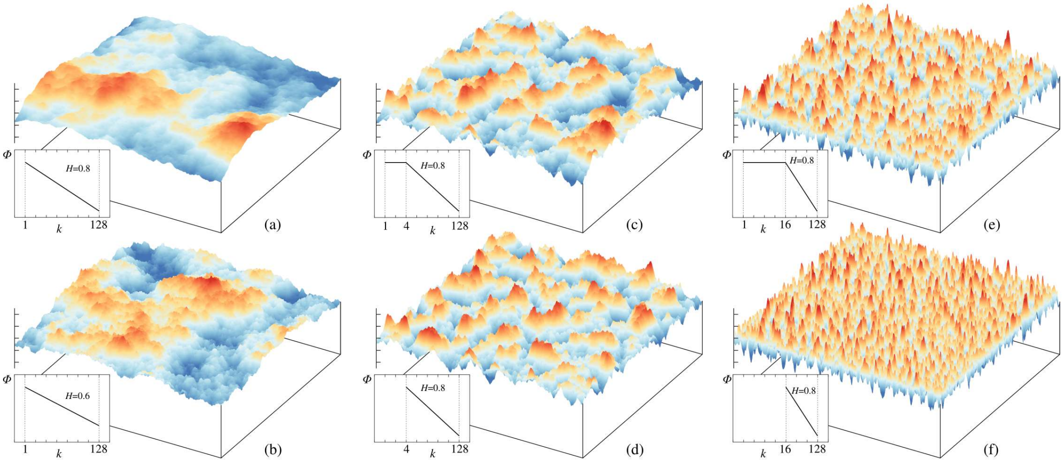
Non-Gaussian roughness modelling

- Example: Weibull PDF and Matérn PSD



[1] Schreiber, T. and Schmitz, A. (1996). Improved surrogate data for nonlinearity tests. Phys. Rev. Lett. 77.
[2] Pérez-Ràfols, F. and Almqvist, A. (2019). Generating randomly rough surfaces with given height probability distribution and power spectrum. Tribol. Int. 131
[3] Roughness generator `https://github.com/vyastreb/rfgen`

Model roughness: isotropic, Gaussian, self-affine



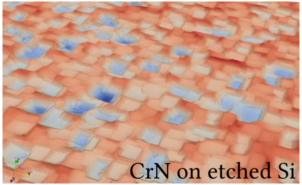
[2] VY, G. Anciaux, J.F. Molinari, Tribol. Int. 114:161-171 (2017)

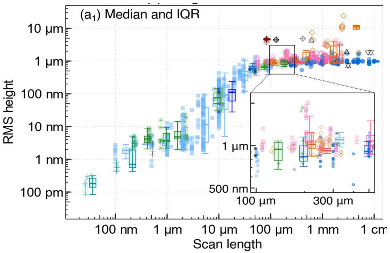
The Surface-Topography Challenge

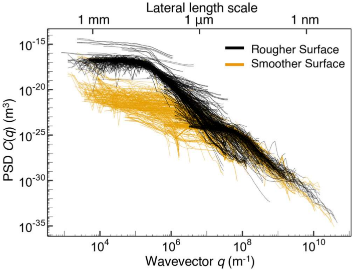
A. Pradhan, M.H. Müser, L. Pastewka, T. Jacobs *et al.*, Tribology Letters 73, 2025

Data

- 20 countries
- 64 research groups
- 153 scientists and engineers
- 2088 measurements (FAIR data)

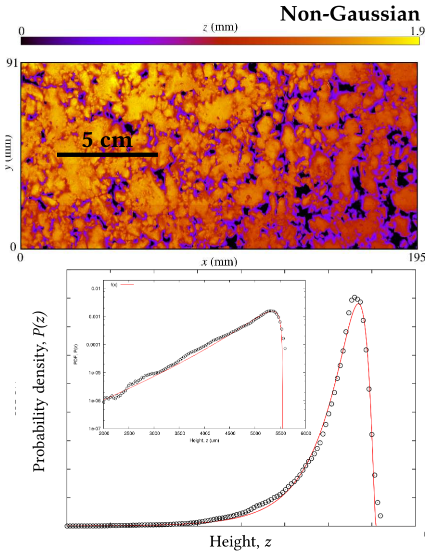




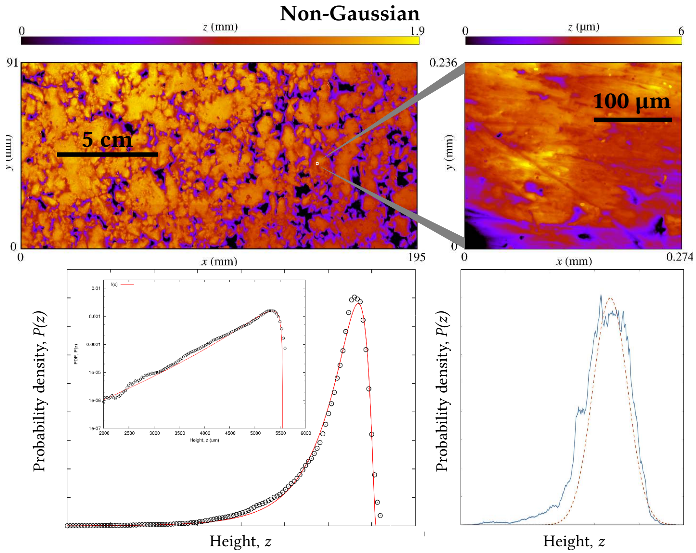


Control the short $\lambda_s = 2\pi/k_s$ and the long $\lambda_l = 2\pi/k_l$ cutoff wavelengths and H exponent

Real-life example: Asphalt roughness

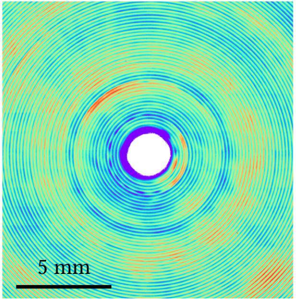


Real-life example: Asphalt roughness



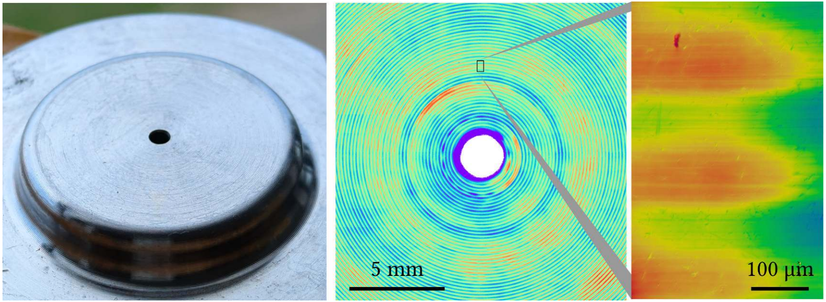
Real-life example: Turned surface

Anisotropic



Real-life example: Turned surface

Anisotropic



Real-life example: Turned surface

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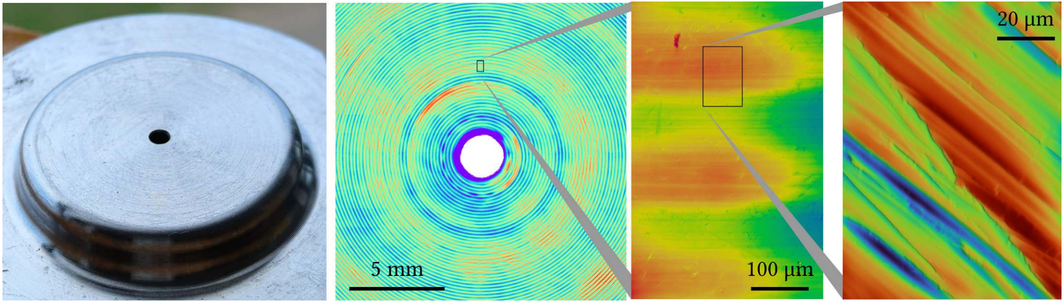
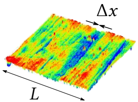


Take-home messages

1. Scale- and sampling-sensitive characteristics



RMS roughness $S_q(L)$

RMS gradient $S_{dq}(\Delta x)$

Take-home messages

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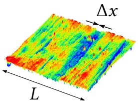
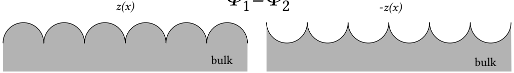


RMS roughness $S_q(L)$

RMS gradient $S_{dq}(\Delta x)$

2. Average characteristics $\langle|\nabla z|^2\rangle$ should be used with caution

$\Phi_1 = \Phi_2$



Take-home messages

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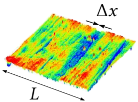

RMS roughness $S_q(L)$

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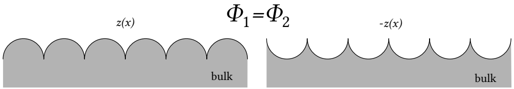
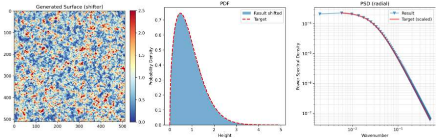


3 PSD and PDF are independent characteristics

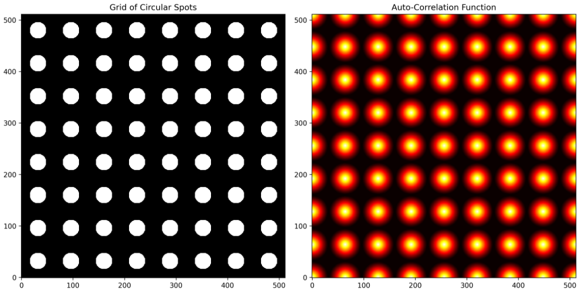


More about PSD

Autocorrelation function

- Continuous autocorrelation function

$$R(\Delta x, \Delta y) = \lim_{L\to\infty} \frac{1}{L^2} \int_0^L \int_0^L z(x+\Delta x, y+\Delta y) z(x,y)\, dxdy$$



Autocorrelation function

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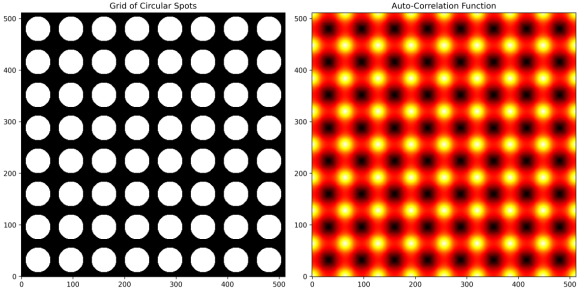

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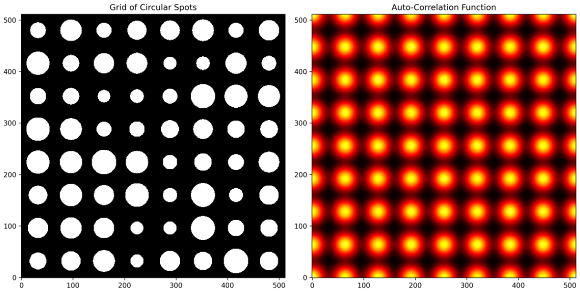

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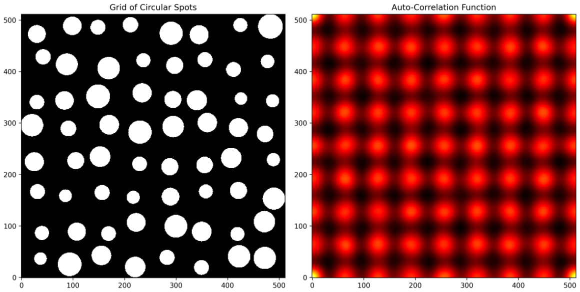

Power spectral density (PSD)

- Fourier Transform: $\hat{f}(k) = \dfrac{1}{2\pi}\int\limits_{-\infty}^{\infty} f(x)\exp(-ikx)\,dx$ where x is the spatial coordinate, $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength.
- PSD is the Fourier Transform of R

$$\Phi(k_x, k_y) \equiv \hat{R}(k_x, k_y) = \mathrm{FT}\left[z(x + \Delta x, y + \Delta y) * z(x, y)\right]$$

- Using convolution theorem
  $\Phi(k_x, k_y) = \hat{z}(k_x, k_y)\,\hat{z}^*(k_x, k_y) = \hat{z}^2(k_x, k_y)$
- Interpretation: energy distribution by frequencies
- Usage: signal analysis, seismology, microstructure characterization, roughness.

Spectral moments

- Spectral moment m_{pq}, $p,q \in \mathbb{N}$:

$$m_{pq} = \iint\limits_{-\infty}^{\infty} k_x^p k_y^q \Phi(k_x, k_y)\, dk_x dk_y$$

- Generalized spectral moment m_{pq}, $p,q \in \mathbb{R}^+$
- For isotropic surface: $m_2 = m_{20} = m_{02}, \quad m_4 = 3m_{22} = m_{40} = m_{04}$
- Averaging:

$$m_2 = \frac{m_{20} + m_{02}}{2}, \quad m_4 = \frac{m_{40} + 3m_{22} + m_{04}}{3}$$

- Physical meaning:

Height variance[1]: $m_0 = \left\langle (z - \langle z \rangle)^2 \right\rangle$

Gradient variance: $2m_2 = \left\langle (\nabla z - \langle \nabla z \rangle)^2 \right\rangle$

Curvature variance: $m_4 = \left\langle (\nabla \cdot \nabla z - \langle \nabla \cdot \nabla z \rangle)^2 \right\rangle$

[1] Longuet-Higgins, M. S. (1957). The statistical analysis of a random, moving surface. Phil. Trans. Royal Society of London. Series A 249(966):321-387.
[2] Nayak, P. R. (1971). Random Process Model of Rough Surfaces. Journal of Lubrication Technology, 93(3):398-407

[1] Variance is the square of the standard deviation

Summary

- Fractal (self-affine) roughness
- Power spectral density (PSD)

$$\Phi(k) \sim k^{-2(H+1)}$$

  k is a wavenumber,
  H is the Hurst exponent.
- **Isotropic**/anisotropic surfaces
- **Gaussian**/non-Gaussian height distribution $P(h)$

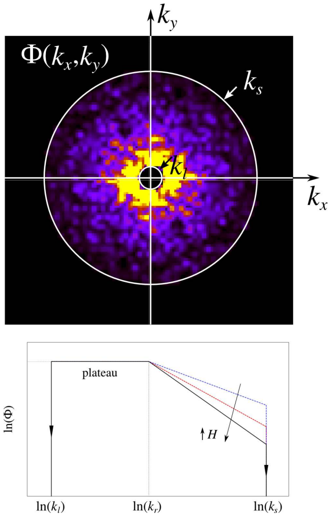

Fig. 3D and radial power spectral densities

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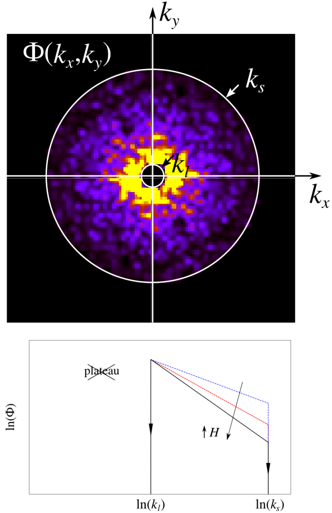

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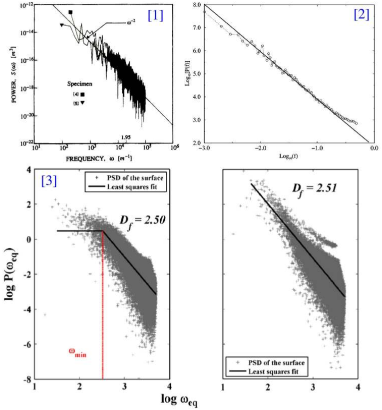

Fig. Power spectral density, measurements

[1] Majumdar, Tien, Wear 136 (1990)
[2] Schmittbuhl, Jørgen Måløy, Phys. Rev. Lett. 78 (1997)
[3] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. 42 (2009)

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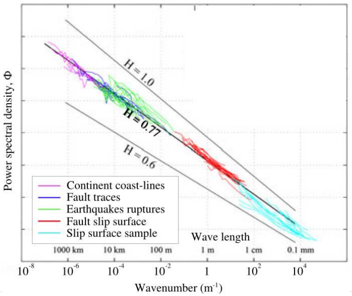

Fig. Power spectral density, geological scales

Adapted from
[4] Renard, Candela, Bouchaud, Geophys. Res. Lett. 40 (2013)

Summary

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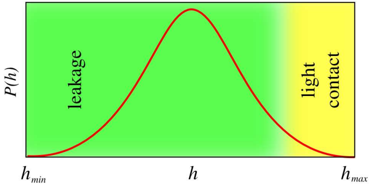


Fig. Height distribution $P(h)$

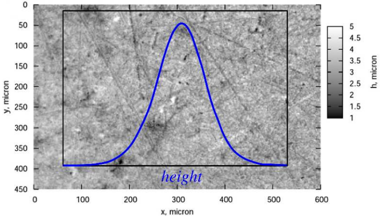


Fig. Height distribution of a polished metal surface

Summary

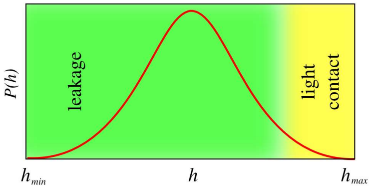
- Fractal (self-affine) roughness
- Power spectral density (PSD)

$$\Phi(k) \sim k^{-2(H+1)}$$

  k is a wavenumber,
  H is the Hurst exponent.

- **Isotropic**/anisotropic surfaces
- **Gaussian**/non-Gaussian height distribution $P(h)$
- **Characteristics:**
  - $\sqrt{\langle z^2 \rangle}$ - *rms heights*
  - $\sqrt{\langle |\nabla z|^2 \rangle}$ - *rms slope (surface gradient)*
  - $\alpha = m_{00} m_{40} / m_{20}^2$ - *breadth of the spectrum (Nayak's parameter*[B]*),*
- **Random process theory**

[A] Longuet-Higgins, Philos. Trans. R. Soc. A 250:157 (1957)
[B] Nayak, J. Lub. Tech. (ASME) 93:398 (1973)
[C] Greenwood, Wear 261: 191 (2006)
[D] Borri, Paggi, J. Phys. D Appl Phys 48:045301 (2015)



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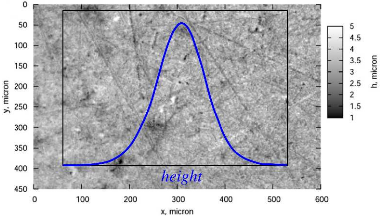


Fig. Height distribution of a polished metal surface

Fractals

Fractals

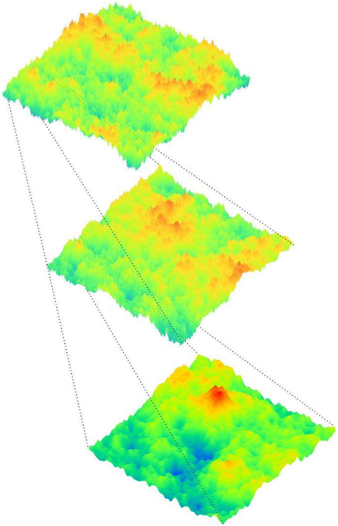


Fig. Example of a rough surface for $H = 0.3$

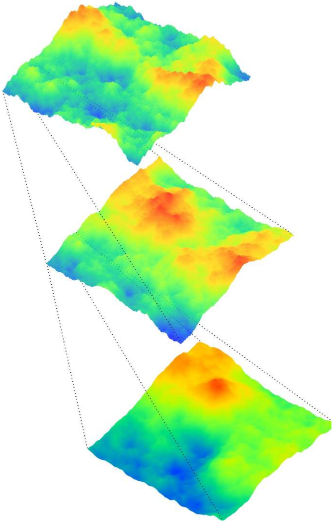
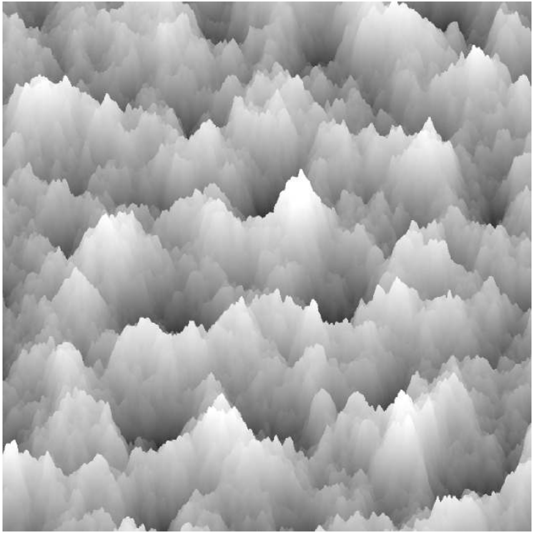


Fig. Example of a rough surface for $H = 0.8$

NB: the Hurst exponent H and the fractal dimension D in 2D space are interconnected via $D = 3 - H$

Fractals



Flight over a rough surface

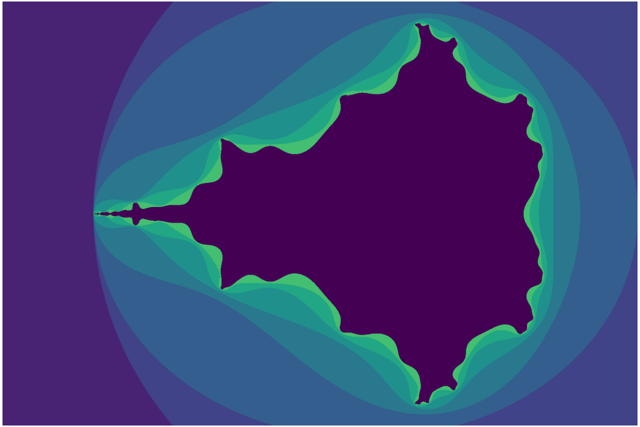
Fractals

Romanesco broccoli

Fractals

- Mandelbrot set (not a fractal)
- Recursive function

$$z_{i+1} = z_i^2 + z, \quad z \in \mathbb{C}$$



Fractals

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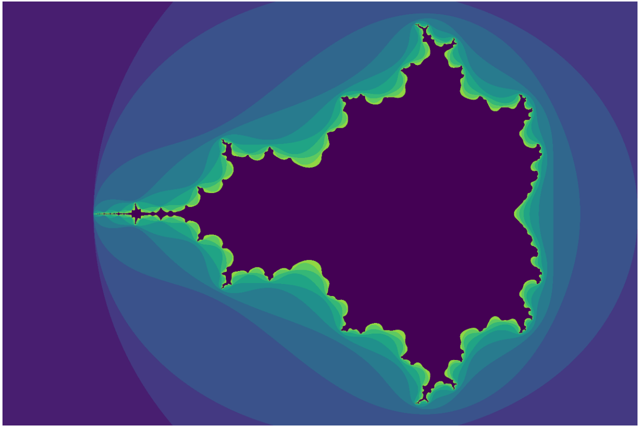

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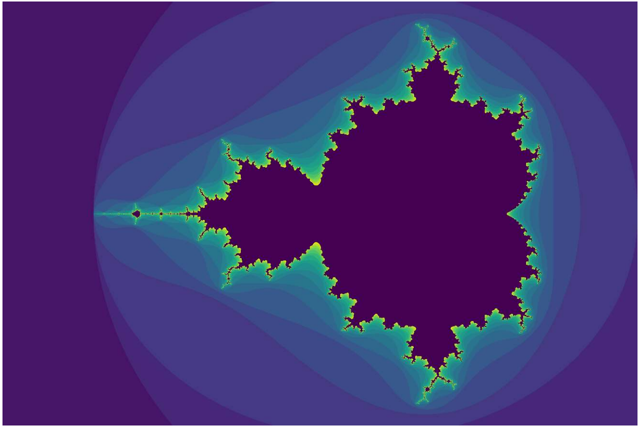

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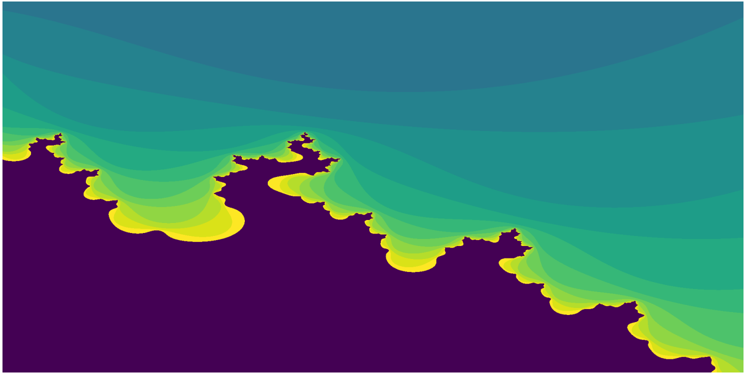

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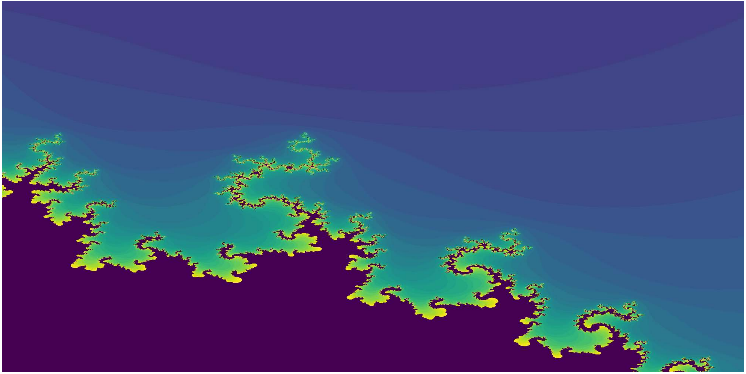

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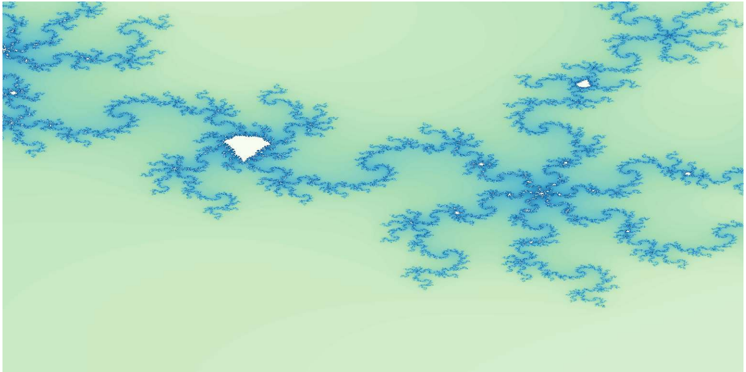

Fractals

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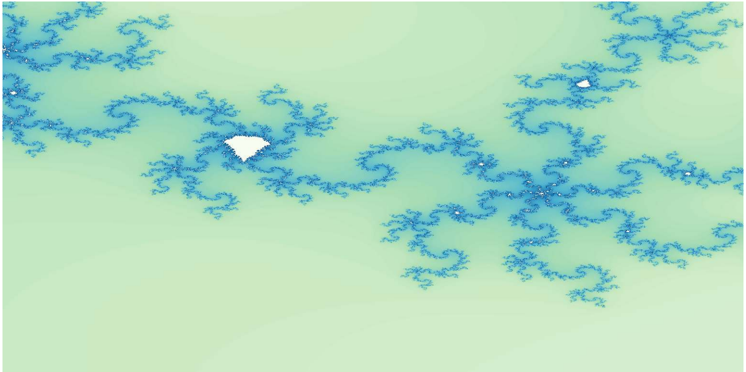
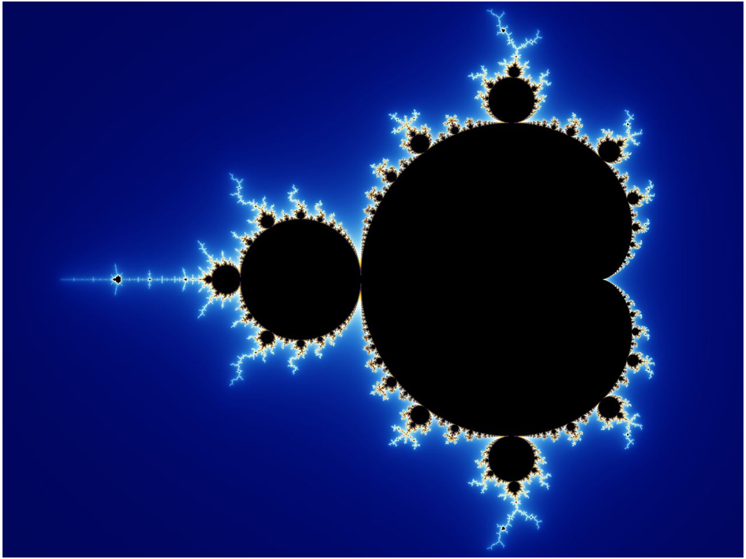
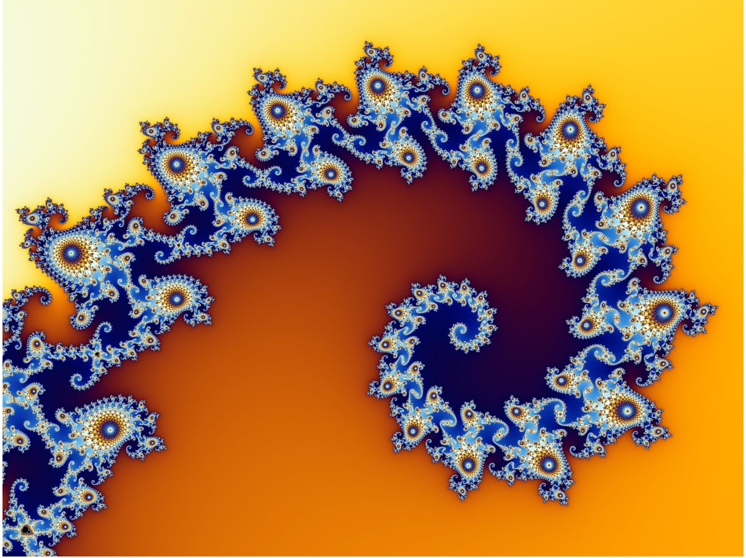


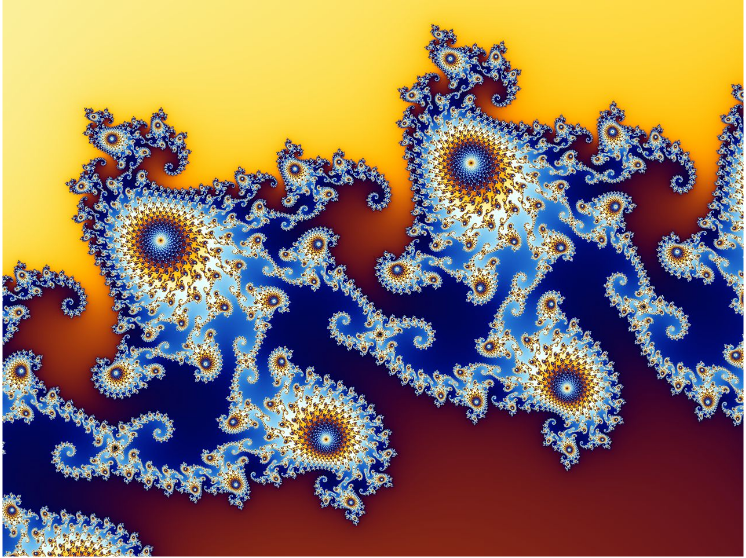


Mandelbrot set (Wikipedia)

Fractals

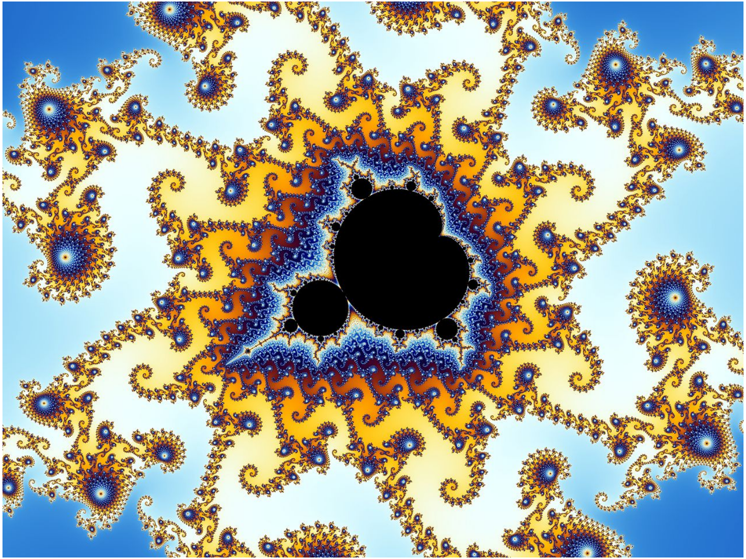


Element of Mandelbrot set (Wikipedia)



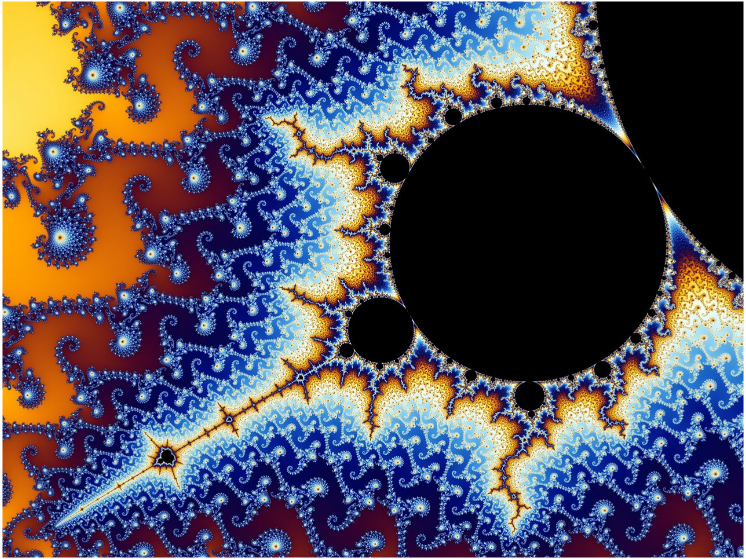
Element of Mandelbrot set (Wikipedia)

Fractals



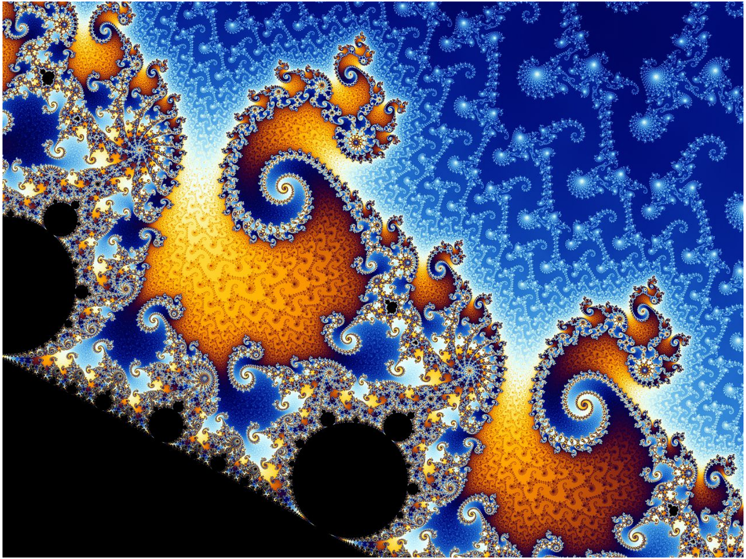
Element of Mandelbrot set (Wikipedia)

Fractals

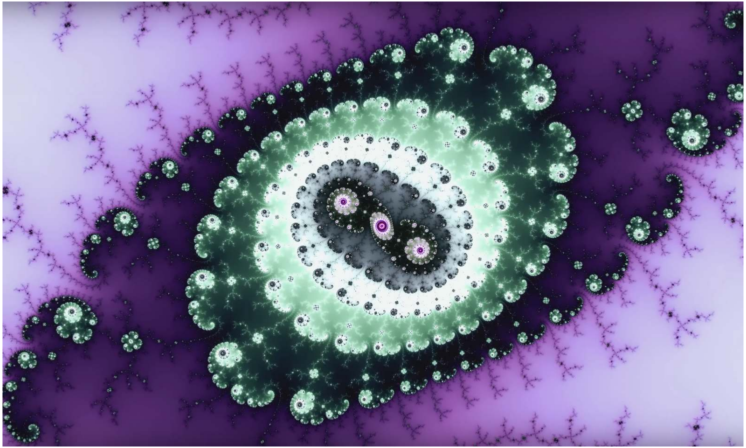


Element of Mandelbrot set (Wikipedia)

Fractals



Element of Mandelbrot set (Wikipedia)

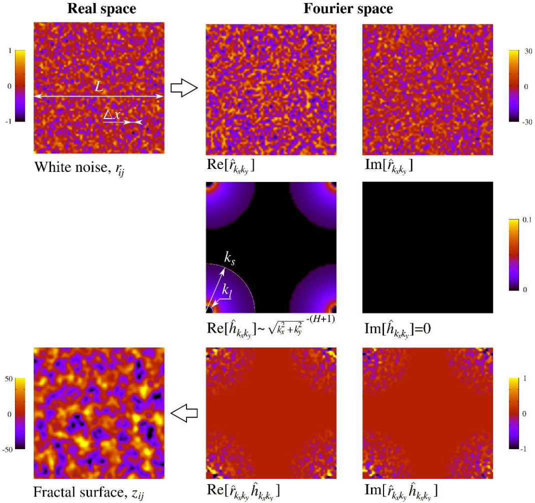
See animation 

https://youtu.be/pCpLWbHVNhk

70 minutes of 4K "Eye of the Universe - Mandelbrot Fractal Zoom" e^{1091}

Effect of Model Parameters

Synthesized rough surfaces



[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s

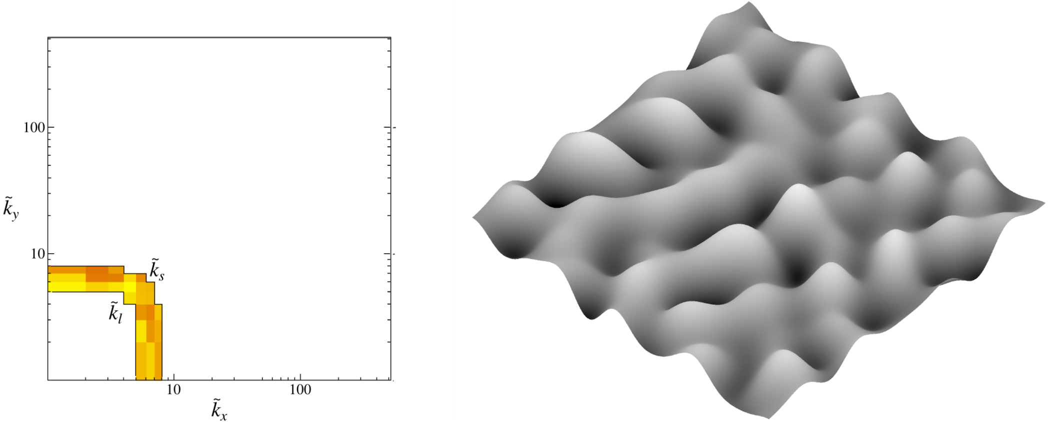
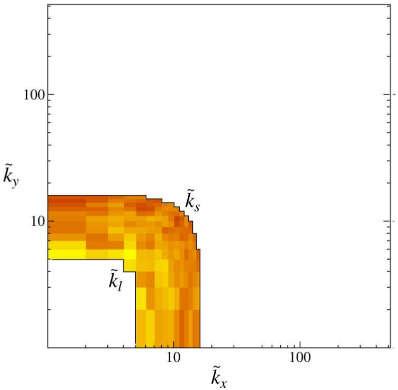


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 8$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



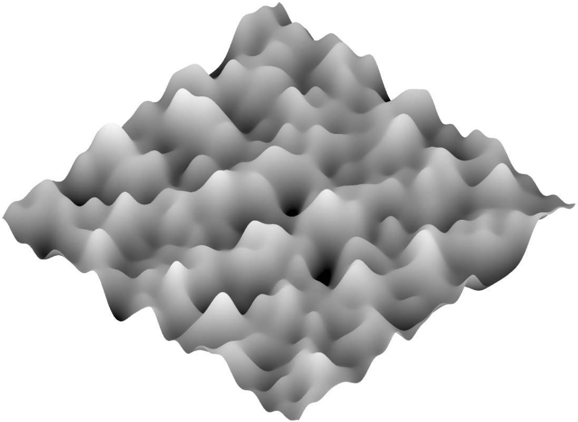


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 16$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s

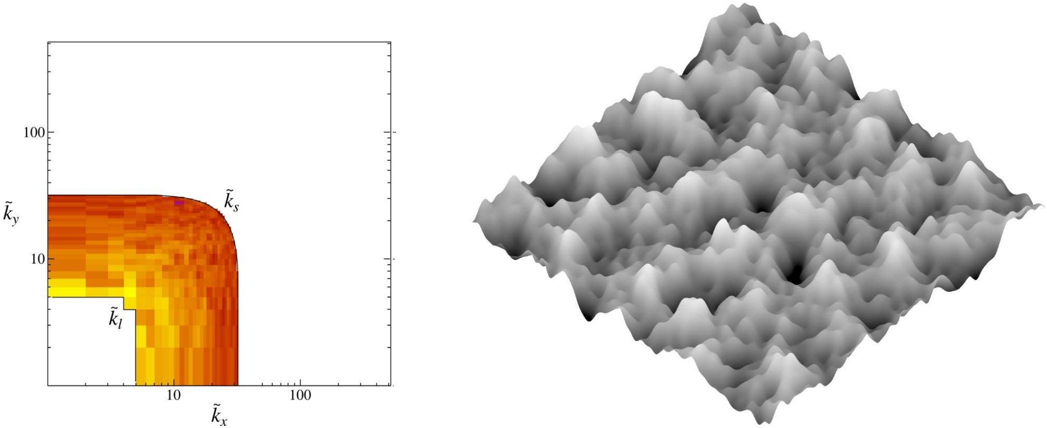
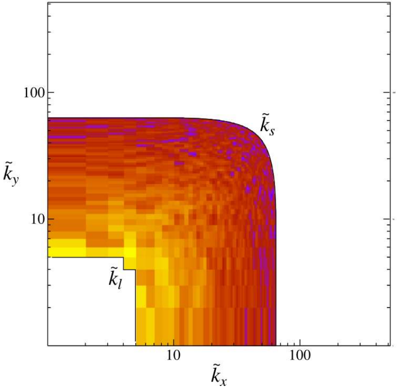


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 32$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



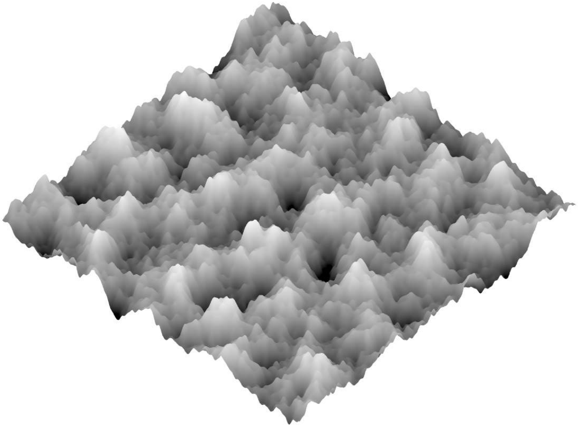


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 64$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s

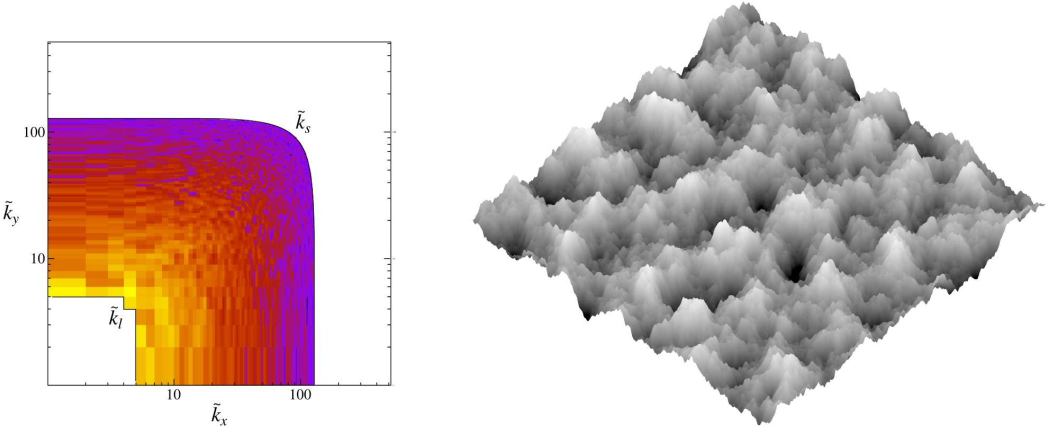
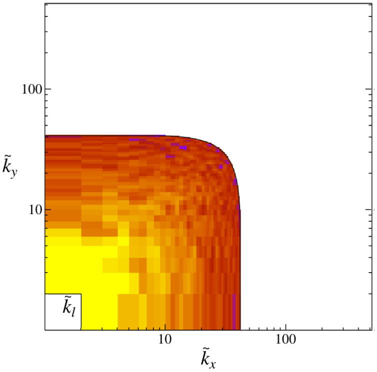


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 128$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$



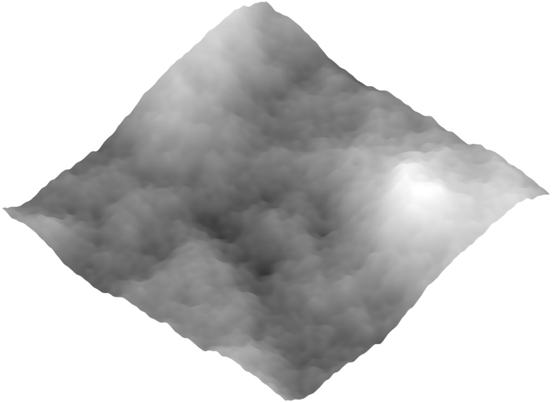


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 1, \quad k_s = 43$

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$

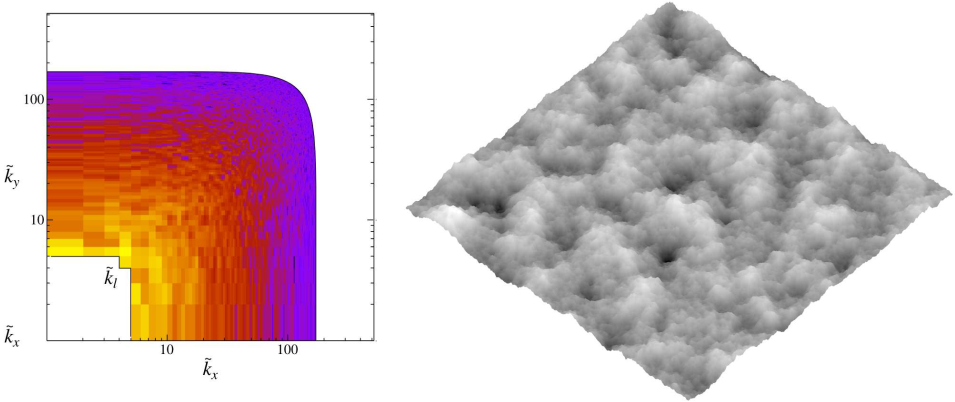


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 171$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$

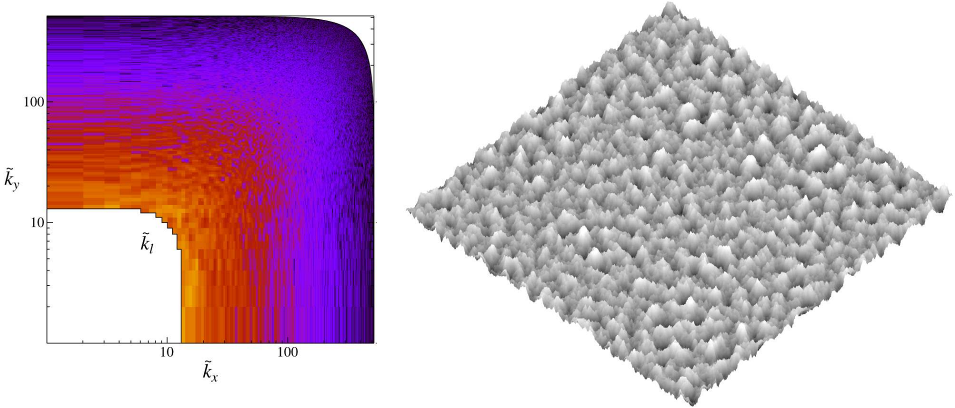


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 12, \quad k_s = 512$

Effect of parameters: illustration

- Effect of the ratio of the higher cutoff to the discretization k_s/N

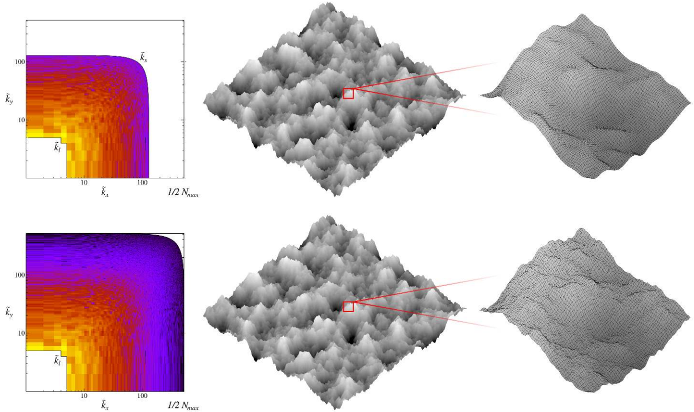


Fig. Power spectral densities (Fourier space) and corresponding rough surfaces (real space) for
$k_l = 12, \quad k_s/N = 1/8$ **VS** $k_l = 12, \quad k_s/N = 1/2$

Effect of parameters: illustration

- Effect of the ratio of the higher cutoff to the discretization k_s/N

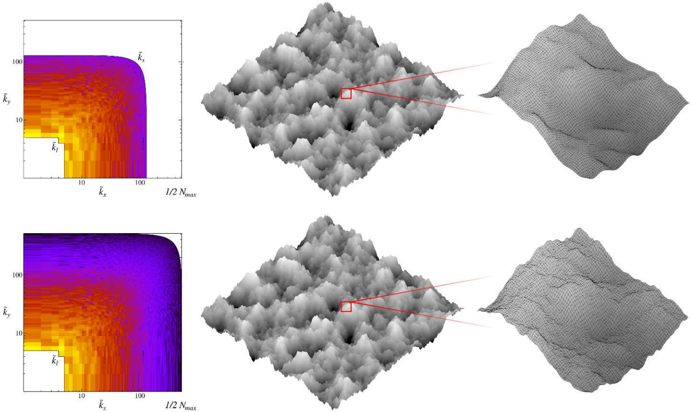


Fig. Power spectral densities (Fourier space)
and corresponding rough surfaces (real space) for
$k_l = 12, \quad k_s/N = 1/8$ **(fine)** **VS** $k_l = 12, \quad k_s/N = 1/2$ **(too coarse)**
for mechanical simulations

Effect of parameters: illustration

- Data interpolation (Shanon, bi-cubic Bézier surfaces)

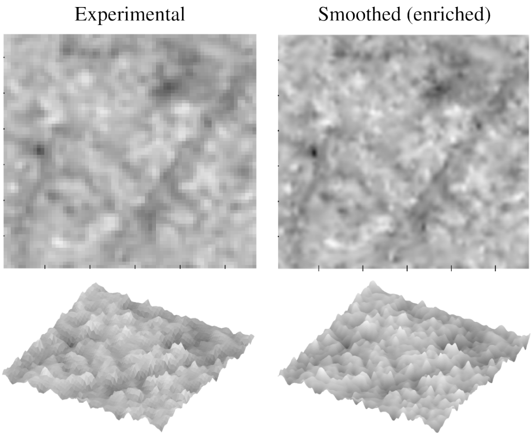


Fig. Bi-cubic Bézier interpolation of an experimental rough surface

[1] Hyun, Robbins, Tribol. Int. (2007)
[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)

Asperity analysis

- Detect summits (z_{ij} higher than neighbor points) and evaluate second derivatives

$$\frac{\partial^2 z}{\partial x^2} = \frac{z_{i+1j} + z_{i-1j} - 2z_{ij}}{2\Delta x^2} \qquad \frac{\partial^2 z}{\partial y^2} = \frac{z_{i+1j} + z_{i-1j} - 2z_{ij}}{2\Delta x^2} \qquad \frac{\partial^2 z}{\partial x \partial y} = \frac{z_{i+1j+1} + z_{i+1j+1} - z_{i+1j-1} - z_{i-1j+1}}{4\Delta x^2}$$

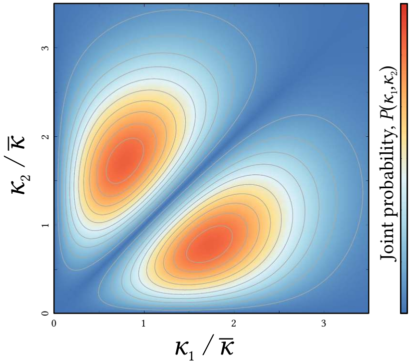
- Principal curvatures $\kappa_{1,2}$:

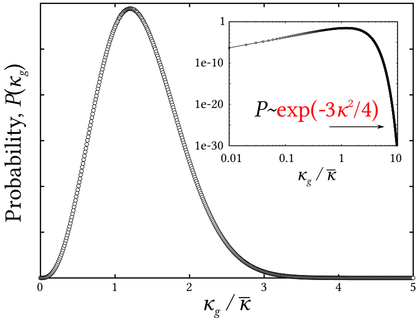
$$\kappa_{1,2} = \frac{1}{2}\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) \pm \sqrt{\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 + \frac{1}{4}\left(\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2}\right)^2}$$

- Saddle point $\kappa_1\kappa_2 < 0$, extrema $\kappa_1\kappa_2 > 0$
- Mean geom. curvature which can be safely used in Hertz theory:

$$\bar{\kappa} = \sqrt{\kappa_1\kappa_2}$$

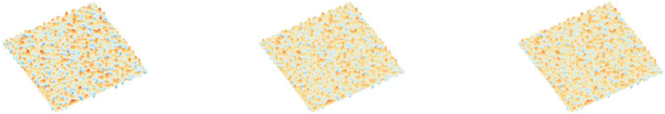
Asperity analysis





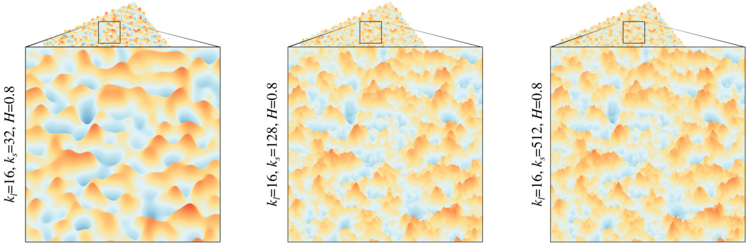
$P \sim \kappa^2 \exp(-3\kappa^2/4)\ \mathrm{erf}(3\kappa/2)$

Statistics of asperity curvature



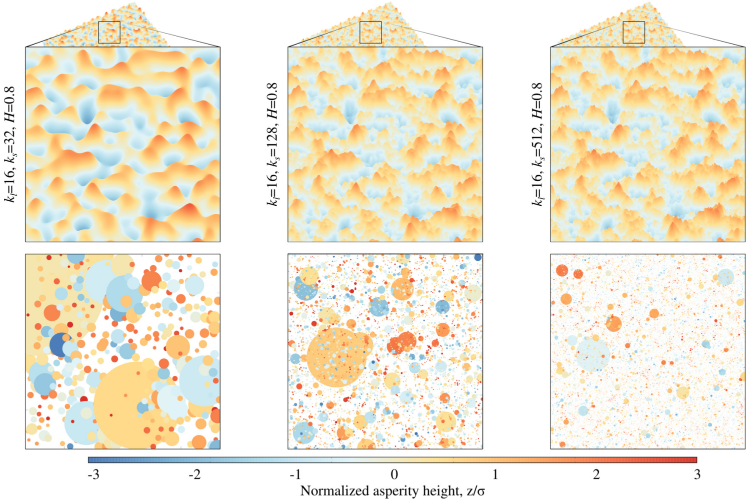
Rough surfaces and associated asperities

Asperity analysis



Rough surfaces and associated asperities

Asperity analysis



Rough surfaces and associated asperities

Summary

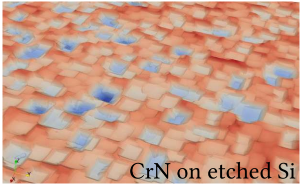
- Surface roughness metrology:
  stylus, confocal, WLI, AFM, STM
- Roughness characteristics:
  PDF, PSD, Hurst exp., spectral moments, Nayak parameter...
- How to model roughness
- Fractal aspect & Mandelbrot set
- Asperity characteristics

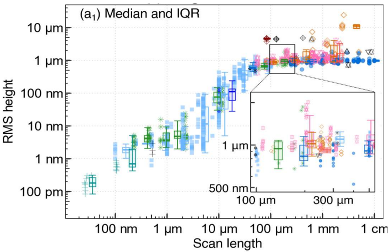
The Surface-Topography Challenge

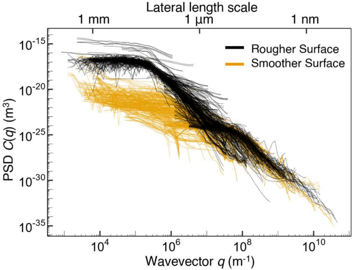
A. Pradhan, M.H. Müser, L. Pastewka, T. Jacobs *et al.*, Tribology Letters 73, 2025

Data

- 20 countries
- 64 research groups
- 153 scientists and engineers
- 2088 measurements (FAIR data)







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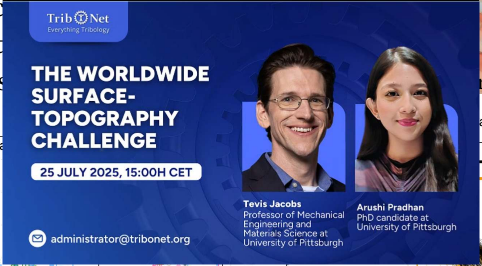
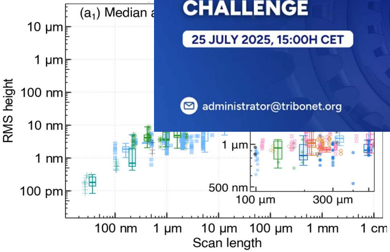
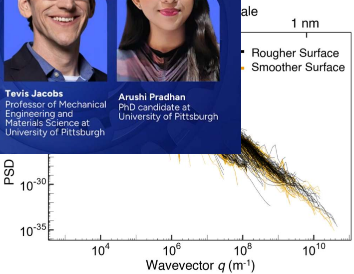
video presentation







☺ Thank you for your attention!