

Contact mechanics and elements of tribology

Lecture 4.

Micromechanical contact: roughness

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@ Centre des Matériaux (& virtually)
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- Introduction
- Measurement techniques
- Classifications
- Main characteristics
- PDF and PSD
- Random process model of roughness
- Computational roughness models
- Reading

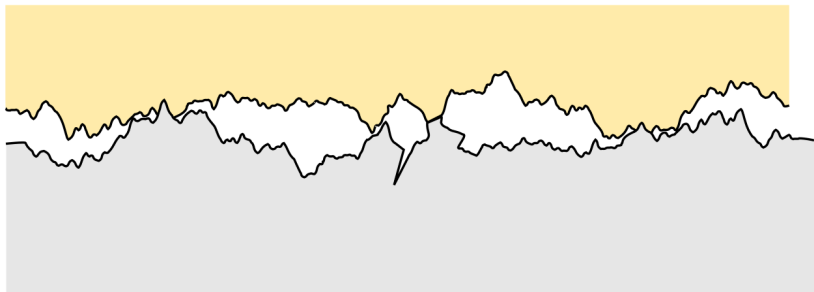
Contact under microscope



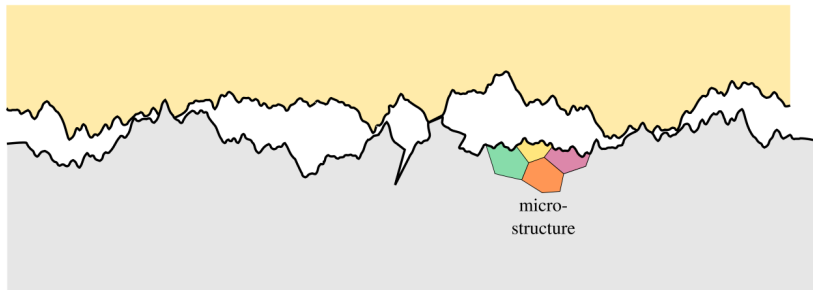
Contact under microscope



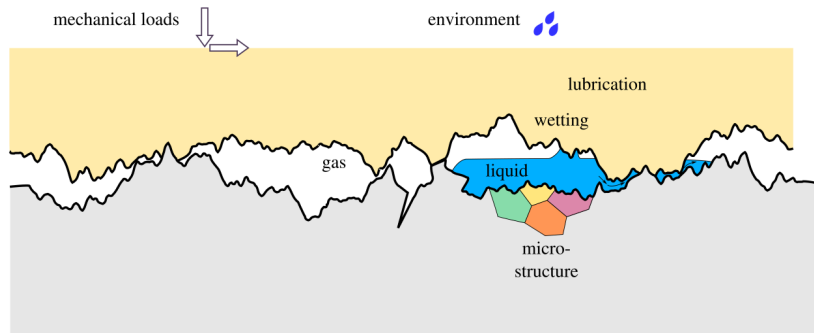
Contact under microscope



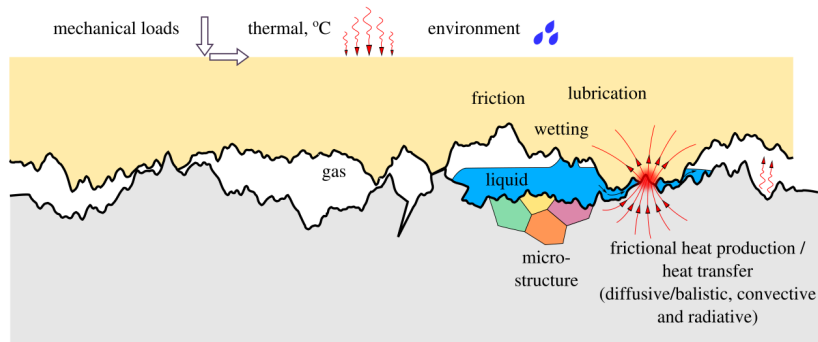
Contact under microscope



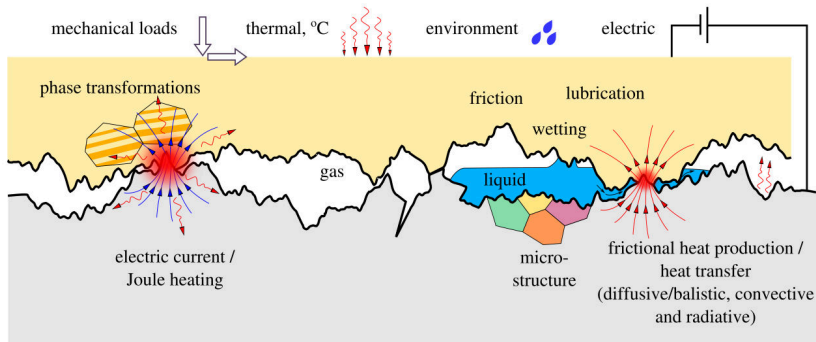
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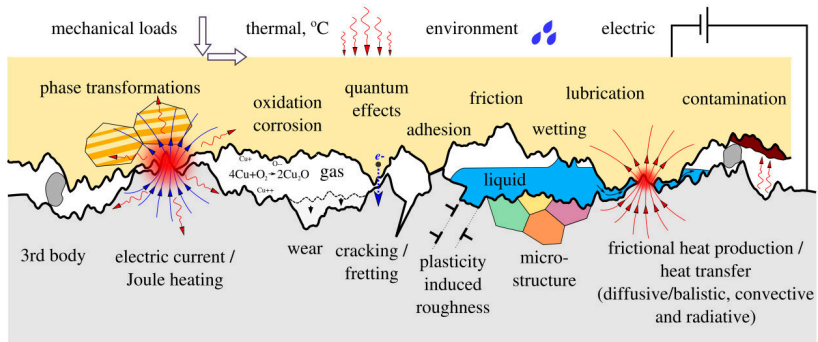
Contact under microscope



Contact under microscope



Contact under microscope



■ Natural and industrial surfaces are *rough*:

- processing
- polishing
- coating
- microstructure
- surface energy
- deformation
- aging
- environment

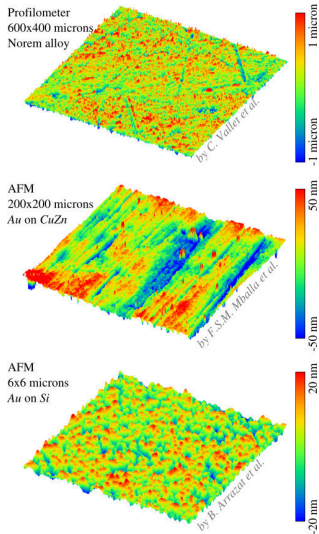


Fig. Examples of rough surfaces

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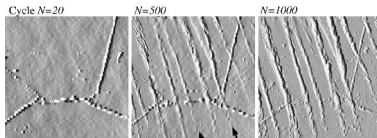


Fig. Persistent slip marks [1]

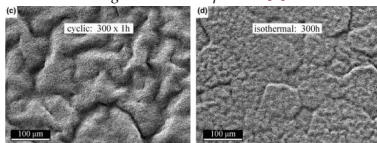


Fig. Rumpling (thermal cycling induced roughness in air)[2]

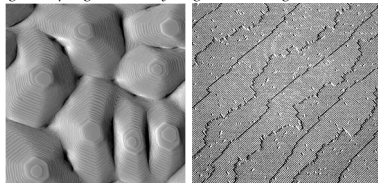


Fig. Epitaxial surface growth [3,4]

[1] J. Polák, J. Man & K. Orbtlík, *Int J Fatigue* 25 (2003)

[2] V.K. Tolpygo, D.R. Clarke, *Acta Mat* 52 (2004)

[3] M. Einax, W. Dieterich, P. Maass, *Rev Mod Phys* 85 (2013)

[4] J.R. Arthur, *Surf Sci* 500 (2002)

■ Natural and industrial surfaces are *rough*:

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■ Roughness affects:

- stress-strain state
- dry friction
- wear
- adhesion
- fluid flow
- sealing
- energy transfer

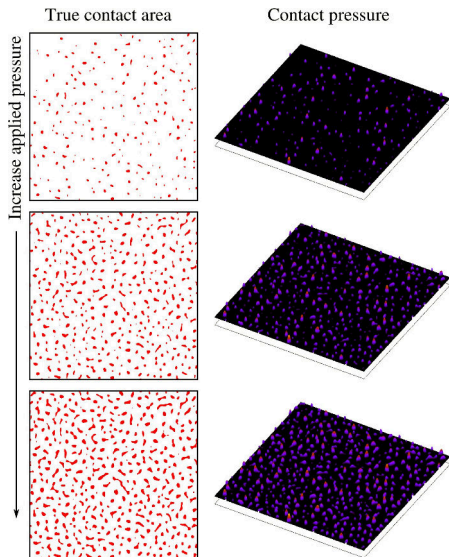


Fig. True contact area and stress fluctuations

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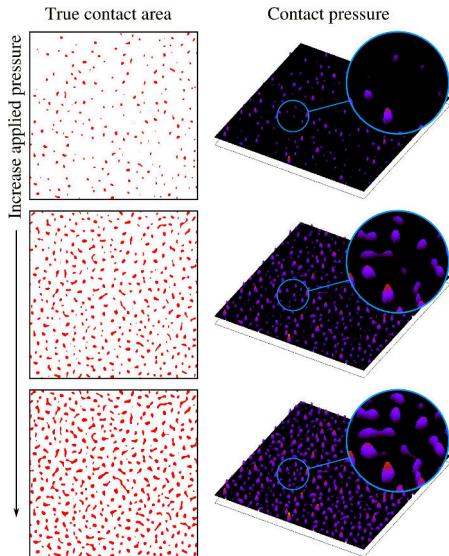


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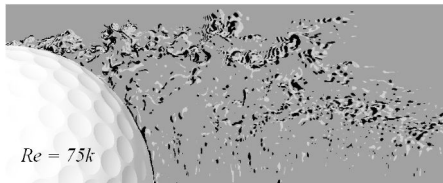
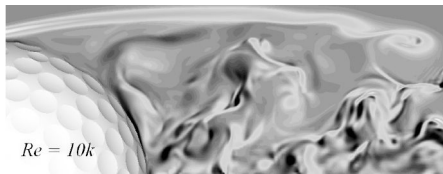


Fig. Numerical simulation of airflow around a (dimpled) golf ball [5]

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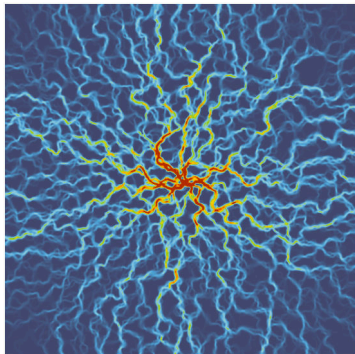


Fig. Fluid passage through free volume between rough surfaces

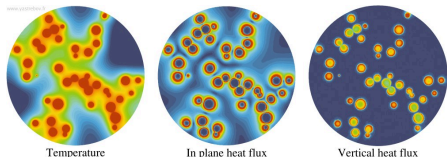
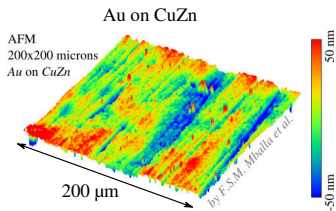
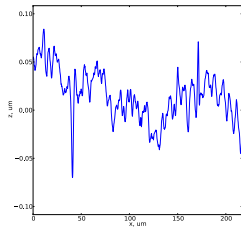


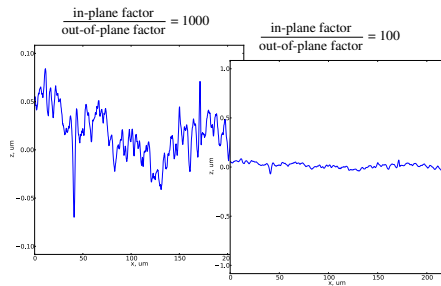
Fig. Heat transfer between rough surfaces (asperity-based model)

Roughness: in plane and out of plane scales

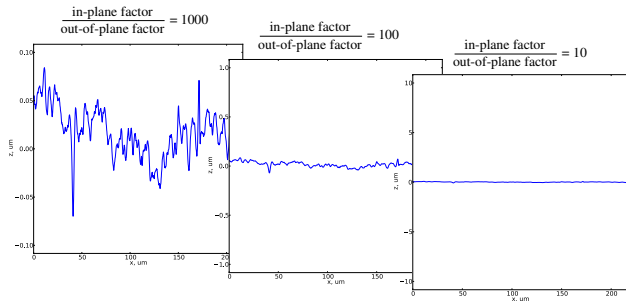
$$\frac{\text{in-plane factor}}{\text{out-of-plane factor}} = 1000$$



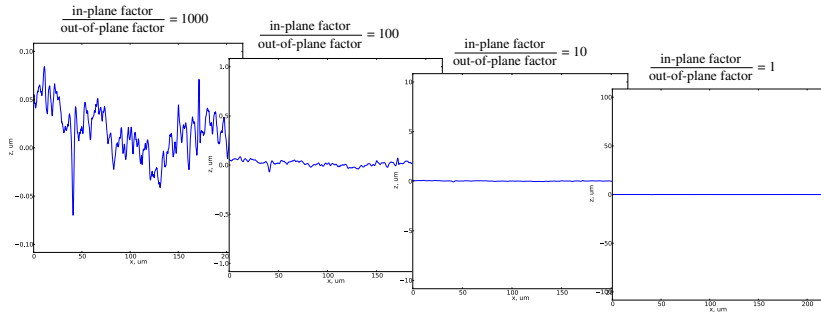
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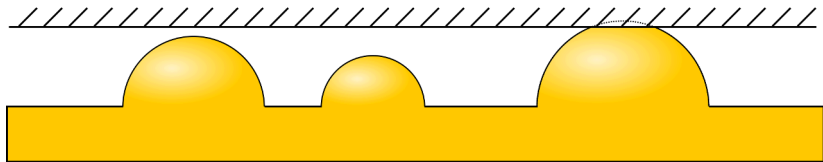


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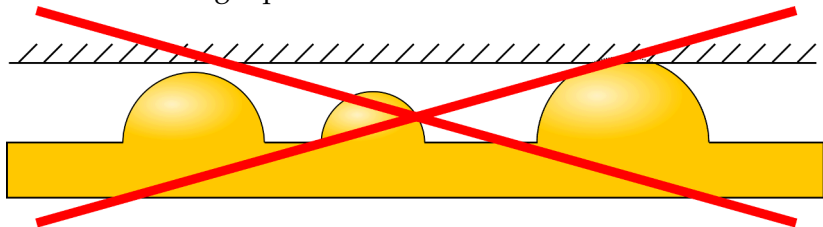
Confusing representation that should be avoided:



Still Hertzian solution remains valid at asperity tips

Roughness: in plane and out of plane scales

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Still Hertzian solution remains valid at asperity tips

■ Stylus measurements

- Mechanical contact of a tip with surface
- Force $\geq 3\mu\text{g}$, tip radius $\geq 50\text{ nm}$
- Mainly for profile measurements $y(x)$

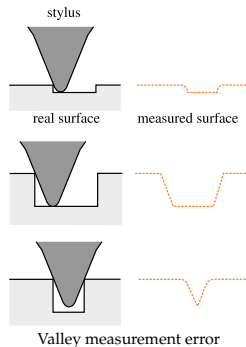
■ Optical measurements

- Confocal (laser scanning) microscopy
 - *highest lateral resolution*
- Interferometry (WLI):
 - *highest vertical resolution*
 - *10 to 100 times faster than CM*
- Scanning Electronic Microscopy (SEM):
 - *in secondary electron emission*
 - *electrons penetrate in the matter* \rightarrow *roughness smoothing*
 - *conducting materials*

■ Nano-contact measurements

- Atomic Force Microscopy (AFM)
 - roughness + adhesive and elastic properties*
- Scanning Tunneling Microscope (STM)

Stylus profilometer



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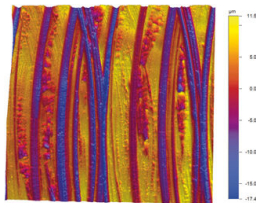
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Stylus profilometer



Modern stylus profilometer
www.bruker.com



Roughness measurements ($\Delta z \approx 30\ \mu\text{m}$)
www.icryst.com

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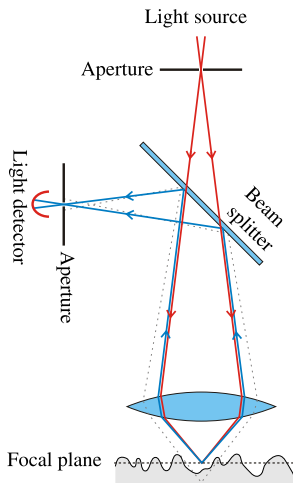
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Confocal microscopy



Principle of confocal microscopy
adapted from www.wikipedia.org

Surface metrology techniques

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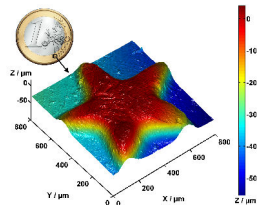
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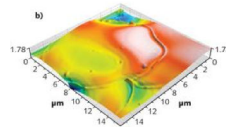
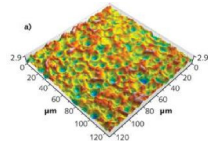
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Confocal microscopy



1 euro surface www.wikipedia.org



Stainless steel machined with micro-electric discharge www.laserfocusworld.org

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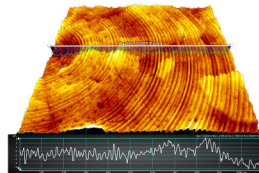
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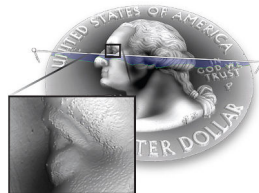
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White Light Interferometry



Diamond-turned optics www.zygo.com



US quarter surface www.zygo.com

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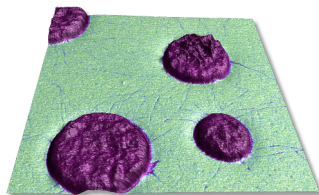
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AFM



Modern AFM
www.bruker.com



Roughness and elastic moduli (color) of
polymer blend
www.bruker.com

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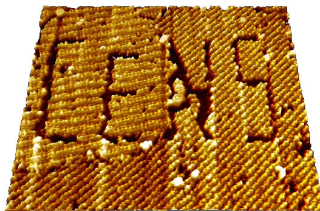
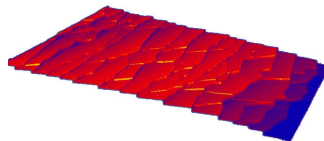


Fig. Center for NanoScience logo imprinted at atomic scale

www.cens.de



Atomic steps on platinum surface
(500×500 nm)

www.icryst.com

Roughness: classification

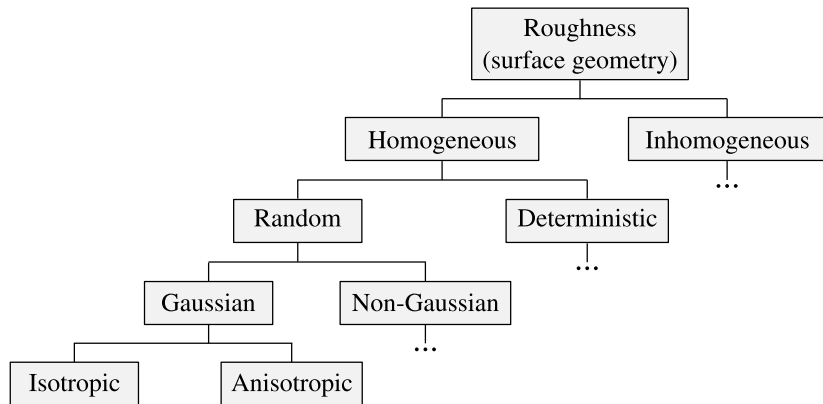


Fig. Roughness classification according to Nayak^[1]

[1] Nayak, J. Lub. Tech. (ASME) 93:398 (1971)

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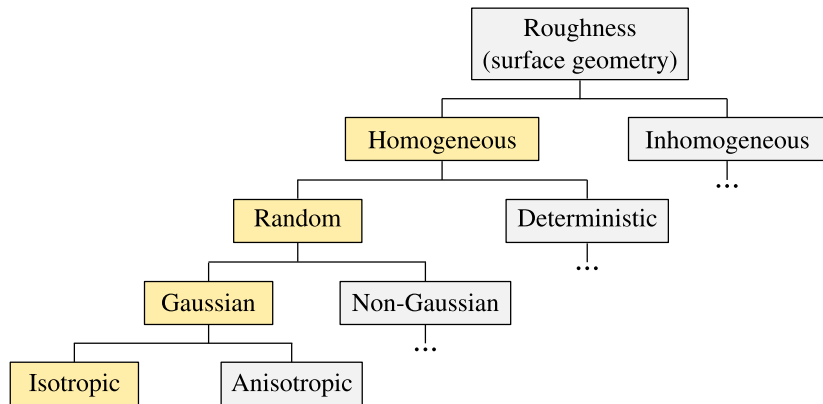


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Roughness and geometry/form

- Roughness vs geometry of surfaces
- Sometimes macroscopic geometry is subtracted (filtered out):
form→error of form→wavyness→roughness
- Non-trivial to remove macroscopic shape
- Most roughness measurement tools enable shape removal

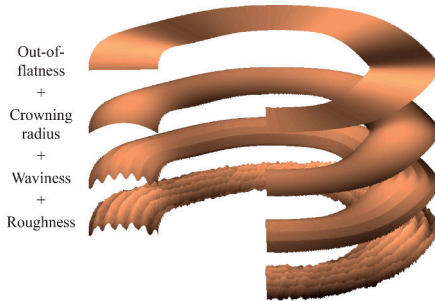


Fig. Circular metallic seal with turned copper surface^[1]

[1] F.P. Rafòls, Licentiate Thesis, LTU 2016.

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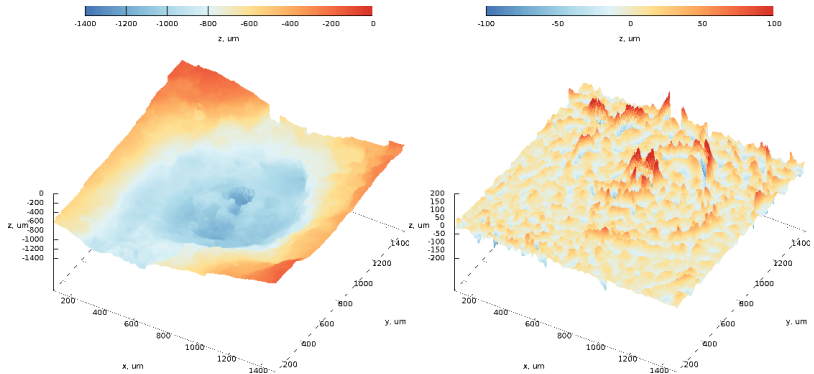


Fig. (left) impact crater, (right) shape is filtered out

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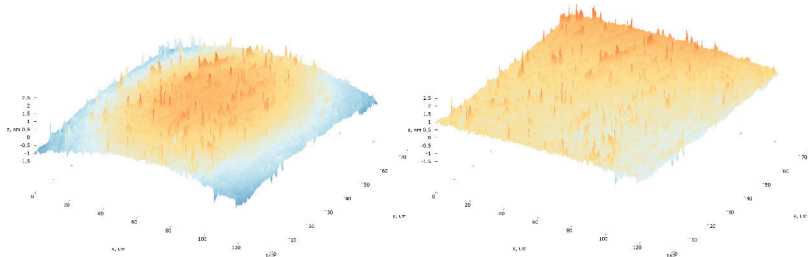


Fig. (left) spherical indenter, (right) $z = a(x^2 + y^2)$ shape is subtracted

Integral quantities

- Average of absolute values [l.u.] (profile - R_a , surface - S_a)

$$S_a = \frac{1}{A} \int_A |z(x, y) - \bar{z}| dA, \quad S_a = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N |z_{ij} - \bar{z}|$$

- Standard deviation of height [l.u.] (σ or R_q for profile, S_q for surface)

$$\sigma = \sqrt{\frac{1}{A} \int_A (z(x, y) - \bar{z})^2 dA}, \quad \sigma = \frac{1}{N} \sqrt{\sum_{i=1}^N \sum_{j=1}^N (z_{ij} - \bar{z})^2}$$

- Maximal valley depth R_v, S_v , maximal peak height R_p, S_p [l.u.]
very sensitive to sample area
- Skewness [adim] (γ_1 or R_{sk}, S_{sk})

$$\gamma_1 = \frac{1}{A\sigma^3} \int_A (z(x, y) - \bar{z})^3 dA, \quad \gamma_1 = \frac{1}{N^2\sigma^3} \sum_{i=1}^N \sum_{j=1}^N (z_{ij} - \bar{z})^3$$

- Kurtosis [adim] (κ or R_{ku}, S_{ku})

$$\kappa = \frac{1}{A\sigma^4} \int_A (z(x, y) - \bar{z})^4 dA, \quad \kappa = \frac{1}{N^2\sigma^4} \sum_{i=1}^N \sum_{j=1}^N (z_{ij} - \bar{z})^4$$

Integral quantities II

- Average of absolute value of gradient (slope) [adim] (profile - R_{da} , surface - S_{da})

$$S_{da} = \langle |\nabla z(x, y) - \bar{\nabla}z| \rangle = \frac{1}{A} \int_A |\nabla z(x, y) - \bar{\nabla}z| dA$$

$$S_{da} = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \left| \frac{z_{i+1,j} - z_{i,j} - \bar{\Delta}z_x}{\Delta x} \right| + \left| \frac{z_{i,j+1} - z_{i,j} - \bar{\Delta}z_y}{\Delta y} \right|$$

- Standard deviation of gradient (slope) [adim] (profile - R_{dq} , surface - S_{dq})

$$S_{dq} = \sqrt{\langle |\nabla z(x, y) - \bar{\nabla}z|^2 \rangle} = \sqrt{\frac{1}{A} \int_A |\nabla z(x, y) - \bar{\nabla}z|^2 dA}$$

$$S_{dq} = \sqrt{\frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \left| \frac{z_{i+1,j} - z_{i,j} - \bar{\Delta}z_x}{\Delta x} \right|^2 + \left| \frac{z_{i,j+1} - z_{i,j} - \bar{\Delta}z_y}{\Delta y} \right|^2}$$

- Often in integrated slope measurements a smoothing filter is used, for example, according to ASME B46.1 standard

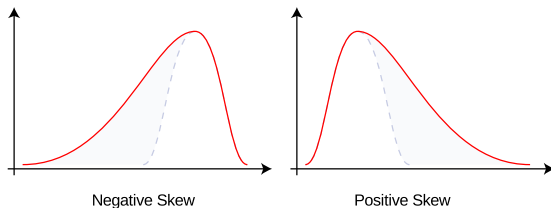
$$\frac{\partial z}{\partial x} \approx \frac{1}{60\Delta x} (z_{i+3,j} - 9z_{i+2,j} + 45z_{i+1,j} - 45z_{i-1,j} + 9z_{i-2,j} - z_{i-3,j})$$

Main characteristics: probability density

- Probability density of heights $P(z)$
- Properties and moments

$$1 = \int_{-\infty}^{\infty} P(z) dz, \quad \bar{z} = \int_{-\infty}^{\infty} zP(z) dz, \quad \sigma = \sqrt{\int_{-\infty}^{\infty} (z - \bar{z})^2 P(z) dz}$$
$$\mu_q = \int_{-\infty}^{\infty} z^q P(z) dz \quad \text{then} \quad \mu_0 = 1, \mu_1 = \bar{z}, \mu_2 = \sigma^2 + \bar{z}^2$$

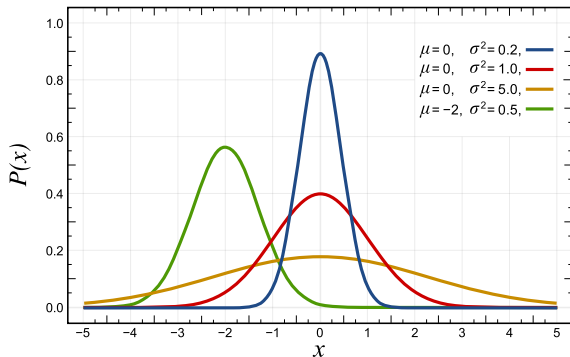
- Link to skewness



Main characteristics: probability density II

Distribution examples

- Normal (Gaussian): $P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu_1)^2}{2\sigma^2}\right], x \in \mathbb{R}$

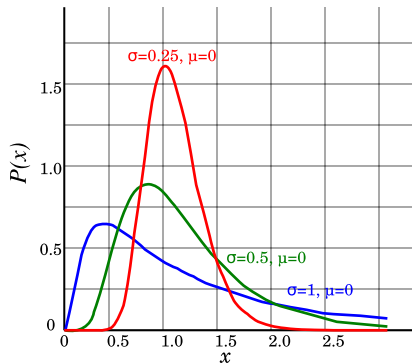


adapted from wikipedia.org

Main characteristics: probability density II

Distribution examples

- Lognormal: $P(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\log(x) - \mu_1)^2}{2\sigma^2}\right] x \in \mathbb{R}^+$

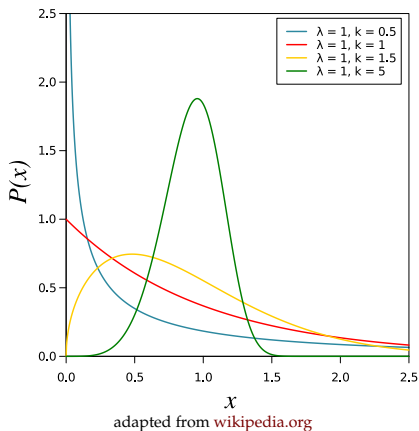


adapted from wikipedia.org

Main characteristics: probability density II

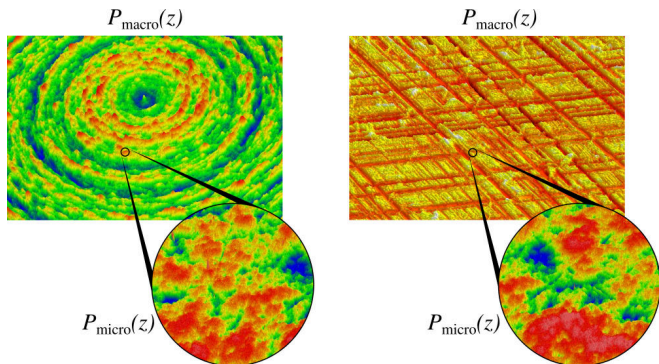
Distribution examples

- Weibull: $P(x) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} \exp -(x/\lambda)^k \quad x \in \mathbb{R}^+$



Real rough surfaces

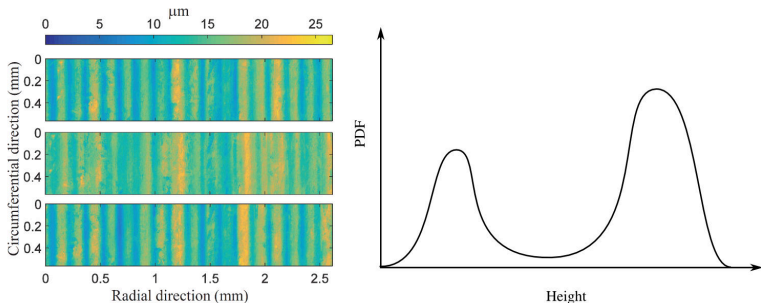
- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
- Wear, polishing, flattening results in removal of the right distribution tail $P(z > z_0) \rightarrow 0$: negative skewness



Macro- and microscopic roughness (left: diamond-turned surface, right: cross-hatched surface)
Images from www.zygo.com used

Real rough surfaces

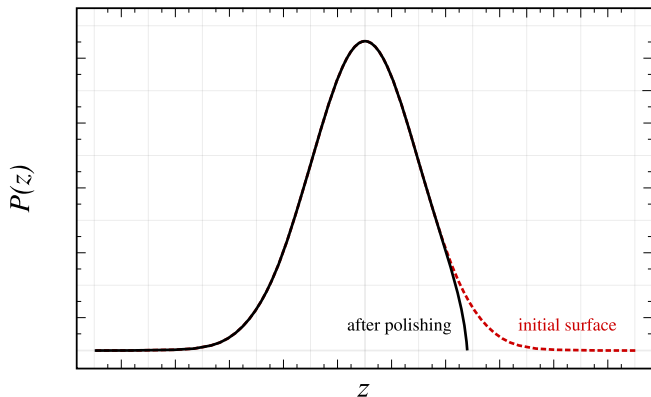
- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
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Turned surface topography and a sketch of height PDF
Pérez-Ràfols, Larsson, Almqvist, Tribol Int 94 (2016)

Real rough surfaces

- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
- Wear, polishing, flattening results in removal of the right distribution tail $P(z > z_0) \rightarrow 0$: negative skewness



Initial surface with Gaussian PDF and surface after polishing

Real rough surfaces

- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
- Wear, polishing, flattening results in removal of the right distribution tail $P(z > z_0) \rightarrow 0$: negative skewness



Very fresh asphalt concrete



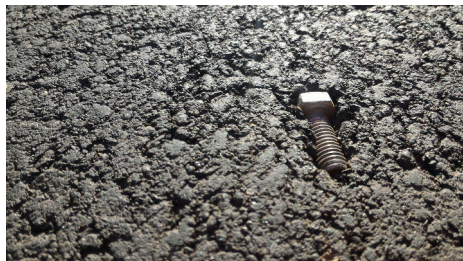
Normal asphalt concrete

Real rough surfaces

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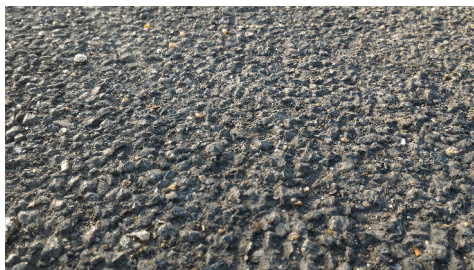
Very fresh asphalt concrete



Normal asphalt concrete . . . with a bolt ☺

Real rough surfaces

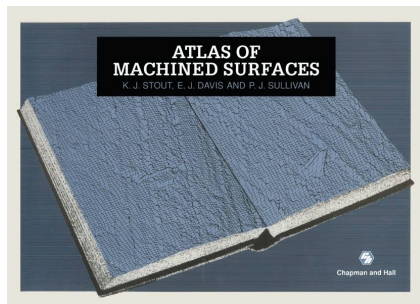
- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
- Wear, polishing, flattening results in removal of the right distribution tail $P(z > z_0) \rightarrow 0$: negative skewness



Old asphalt concrete with worn out bitumen

Real rough surfaces

- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
- Wear, polishing, flattening results in removal of the right distribution tail $P(z > z_0) \rightarrow 0$: negative skewness



Atlas of machined surfaces (with height distributions)

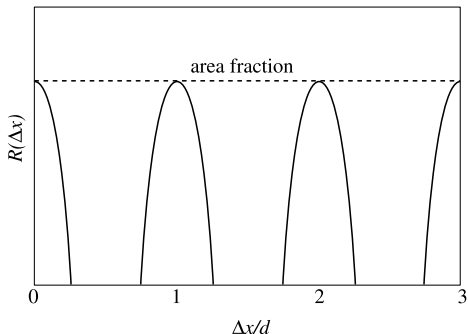
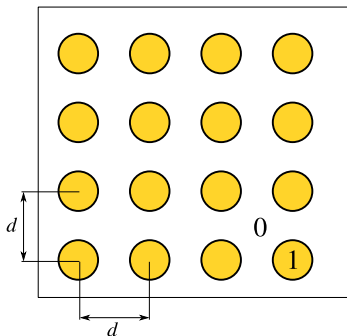
Autocorrelation function

- Continuous autocorrelation function

$$R(\Delta x, \Delta y) = \lim_{L \rightarrow \infty} \frac{1}{L^2} \int_0^L \int_0^L z(x + \Delta x, y + \Delta y) z(x, y) dx dy$$

- Discrete autocorrelation function for a surface $N \times N$

$$R(\Delta x, \Delta y) = \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} z(x + \Delta x, y + \Delta y) z(x, y)$$



Power spectral density (PSD)

- Recall: Fourier Transform: $\hat{f}(k) = \int_{-\infty}^{\infty} f(x) \exp(-2\pi i k x) dx$
- Recall: Discrete Fourier Transform: $\hat{f}_k = \sum_{n=0}^{N-1} x_n \exp(-2\pi i k n / N)$
- where x is the spatial coordinate, $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength.
- PSD is the Fourier Transform of R
 $\Phi(k_x, k_y) \equiv \hat{R}(k_x, k_y) = \text{FFT} [z(x + \Delta x, y + \Delta y) * z(x, y)]$
- Using convolution theorem
 $\Phi(k_x, k_y) = \hat{z}(k_x, k_y) \hat{z}^*(k_x, k_y) = \hat{z}^2(k_x, k_y)$
- Interpretation: energy distribution by frequencies
- Usage: signal analysis, seismology, microstructure characterization, roughness.

Spectral moments

- Spectral moment m_{pq} , $p, q \in \mathbb{N}$:

$$m_{pq} = \iint_{-\infty}^{\infty} k_x^p k_y^q \Phi(k_x, k_y) dk_x dk_y$$

$$m_{pq} = \left[\frac{2\pi}{L} \right]^{p+q} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} i^p j^q \Phi(2\pi i/L, 2\pi j/L)$$

- Generalized spectral moment m_{pq} , $p, q \in \mathbb{R}^+$
- For isotropic surface: $m_2 = m_{20} = m_{02}$, $m_4 = 3m_{22} = m_{40} = m_{04}$
- Averaging:

$$m_2 = \frac{m_{20} + m_{02}}{2}, \quad m_4 = \frac{m_{40} + 3m_{22} + m_{04}}{3}$$

- Physical meaning:

$$\text{Height variance}^1: m_0 = \langle (z - \langle z \rangle)^2 \rangle$$

$$\text{Gradient variance: } 2m_2 = \langle (\nabla z - \langle \nabla z \rangle)^2 \rangle$$

$$\text{Curvature variance: } m_4 = \langle (\nabla \cdot \nabla z - \langle \nabla \cdot \nabla z \rangle)^2 \rangle$$

¹Variance is a squared standard deviation

Summary

- Fractal (self-affine) roughness
- Power spectral density (PSD)
$$\Phi(k) \sim k^{-2(H+1)}$$

k is a wavenumber,
 H is the Hurst exponent.
- **Isotropic**/anisotropic surfaces
- **Gaussian**/non-Gaussian height distribution $P(h)$

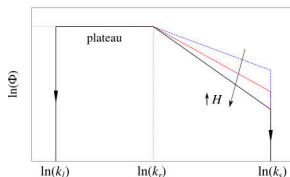
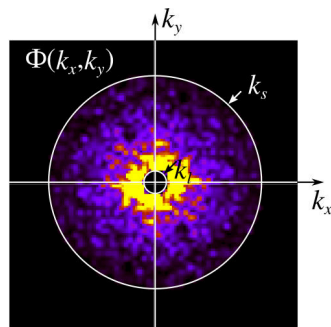


Fig. 3D and radial power spectral densities

Summary

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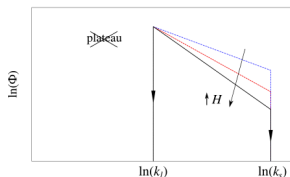
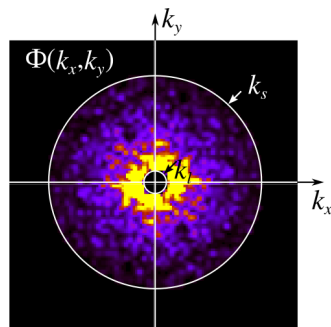


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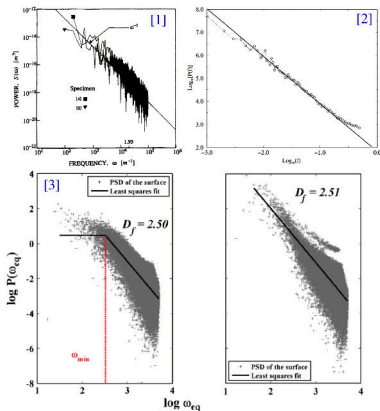


Fig. Power spectral density, measurements

- [1] Majumdar, Tien, *Wear* 136 (1990)
- [2] Schmittbuhl, Jørgen Måløy, *Phys. Rev. Lett.* 78 (1997)
- [3] Vallet, Lasseux, Sainsot, Zahouani, *Tribol. Int.* 42 (2009)

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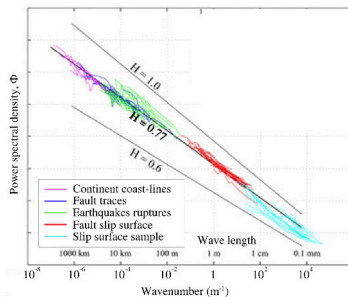


Fig. Power spectral density, geological scales

Adapted from

[4] Renard, Candela, Bouchaud, *Geophys. Res. Lett.* 40 (2013)

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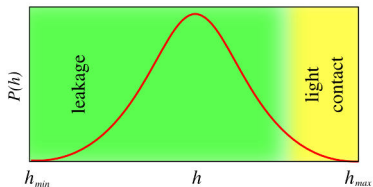


Fig. Height distribution $P(h)$

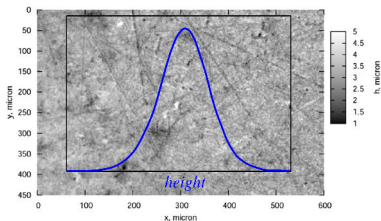


Fig. Height distribution of a polished metal surface

Summary

- Fractal (self-affine) roughness
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- **Isotropic**/anisotropic surfaces
- **Gaussian**/non-Gaussian height distribution $P(h)$

- **Characteristics:**

- $\sqrt{\langle z^2 \rangle}$ - rms heights
- $\sqrt{\langle |\nabla z|^2 \rangle}$ - rms slope (surface gradient)
- $\alpha = m_{00}m_{40}/m_{20}^2$ - breadth of the spectrum (Nayak's parameter^[B]),

$$\text{spectral moments } m_{pq} = \iint_{-\infty}^{\infty} k_x^p k_y^q \Phi(k_x, k_y) dk_x dk_y$$

- **Random process theory**

[A] Longuet-Higgins, Philos. Trans. R. Soc. A 250:157 (1957)

[B] Nayak, J. Lub. Tech. (ASME) 93:398 (1973)

[C] Greenwood, Wear 261: 191 (2006)

[D] Borri, Paggi, J. Phys. D Appl Phys 48:045301 (2015)

V.A. Yastrebov

Lecture 4

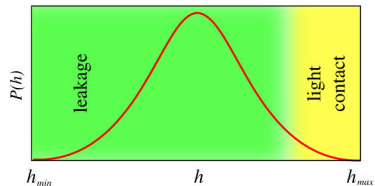


Fig. Height distribution $P(h)$

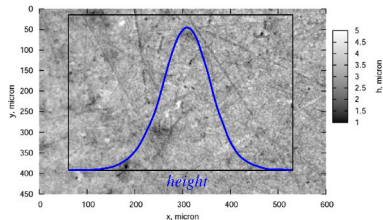


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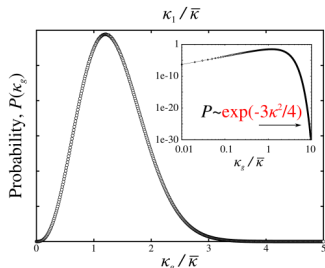
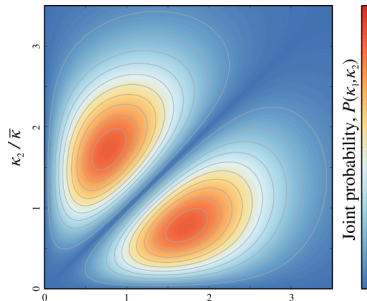
[B] Nayak, J. Lub. Tech. (ASME) 93:398 (1973)

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V.A. Yastrebov

Lecture 4



$$P \sim \kappa^2 \exp(-3\kappa^2/4) \operatorname{erf}(3\kappa/2)$$

Distribution of asperity curvatures

Summary

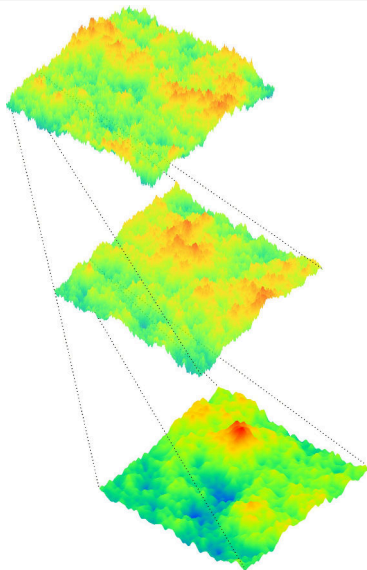


Fig. Example of a rough surface for $H = 0.3$

Recall: the Hurst exponent H and the fractal dimension D in 2D space are interconnected via $D = 3 - H$

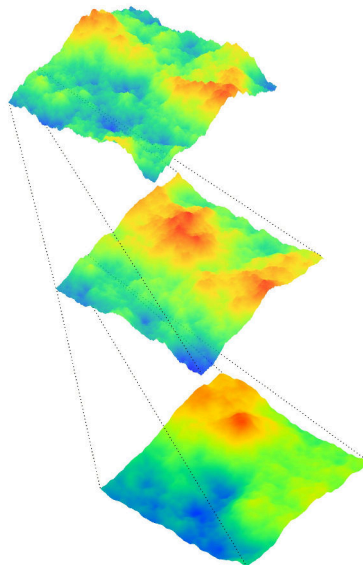
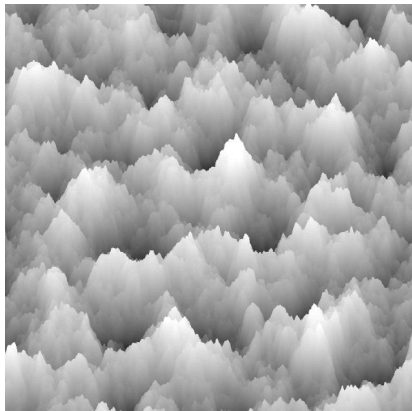


Fig. Example of a rough surface for $H = 0.8$



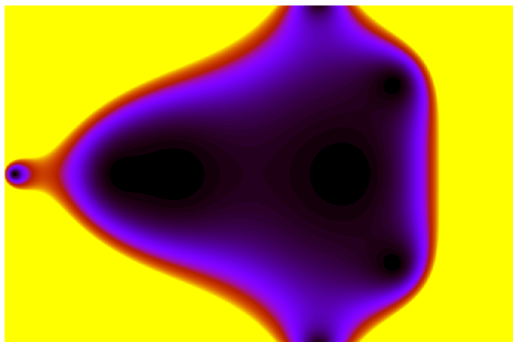
Flight over a rough surface



Romanesco broccoli

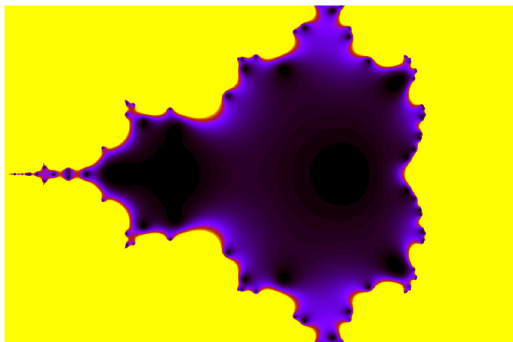
- Mandelbrot set (not a fractal)
- Recursive function

$$z_{i+1} = z_i^2 + z, \quad z \in \mathbb{C}$$



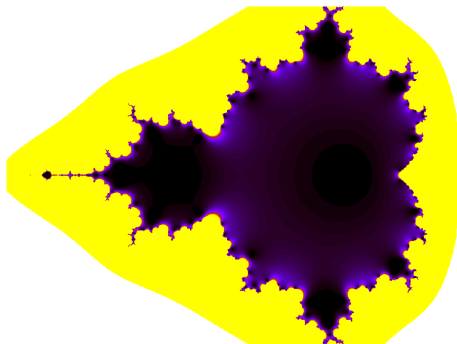
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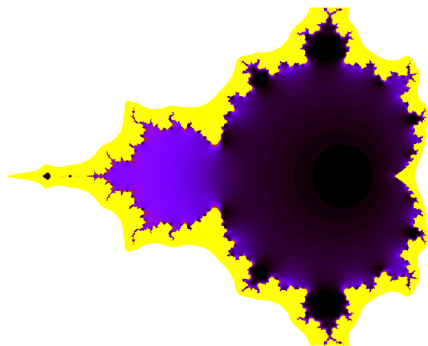
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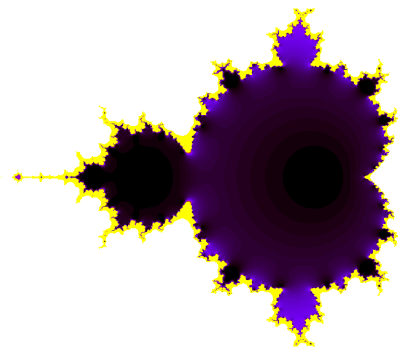
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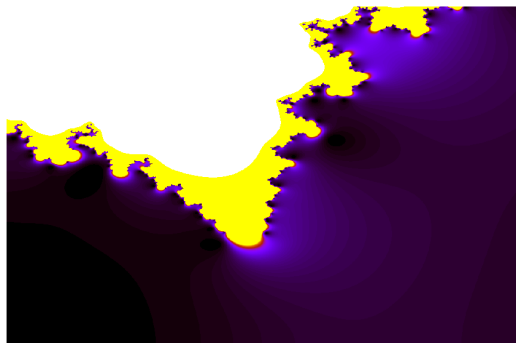
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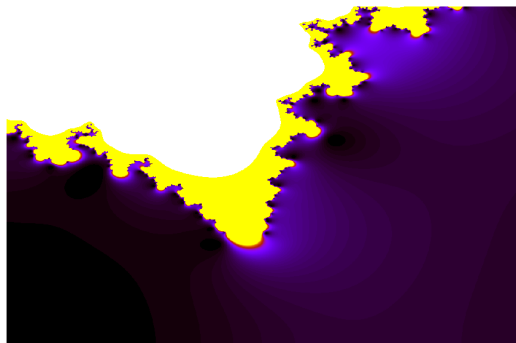
- Mandelbrot set (not a fractal)
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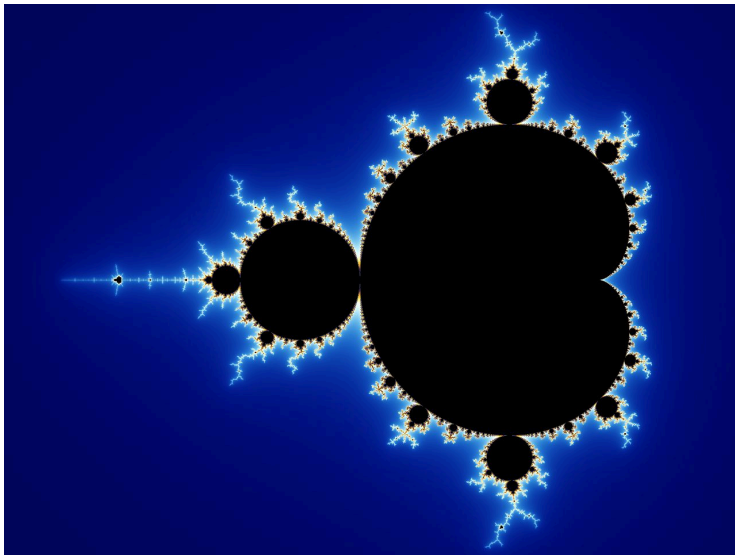
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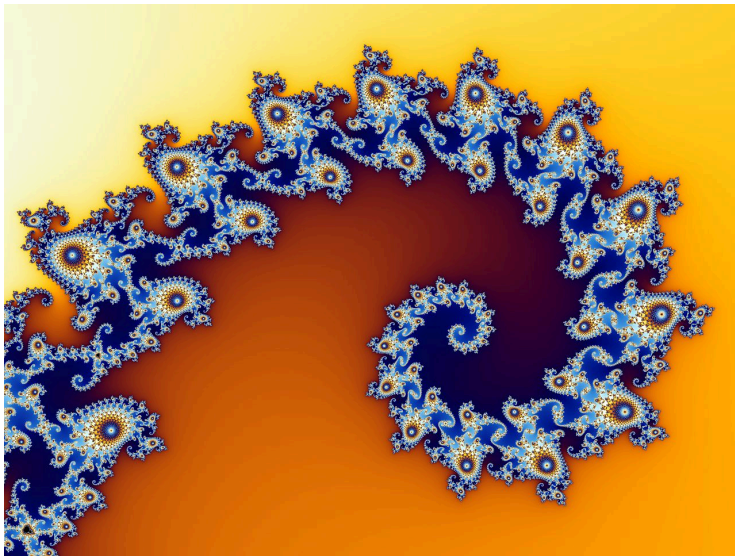
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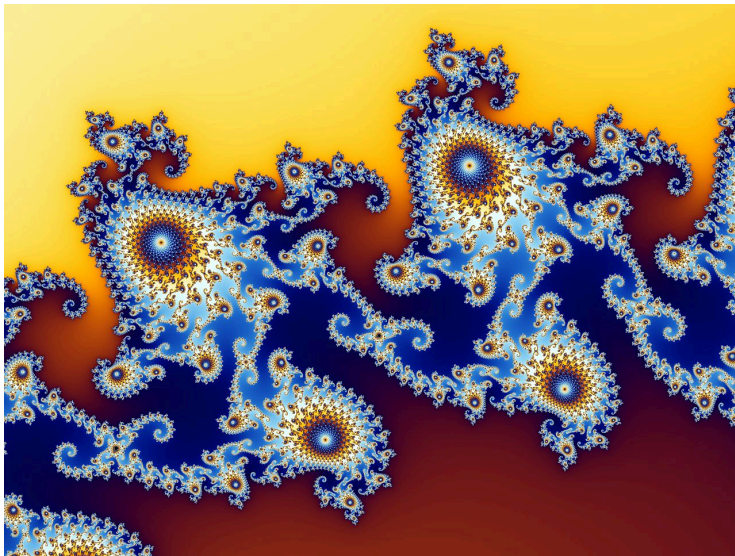
Mandelbrot set ([Wikipedia](#))

Fractals



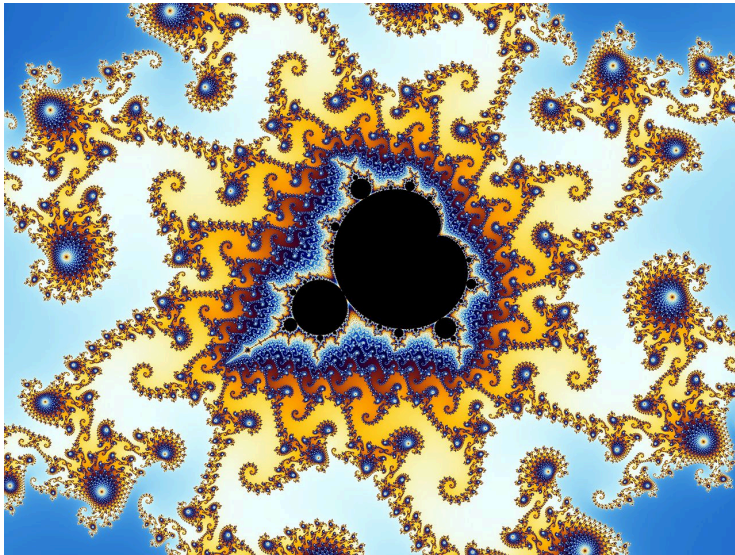
Element of Mandelbrot set (Wikipedia)

Fractals

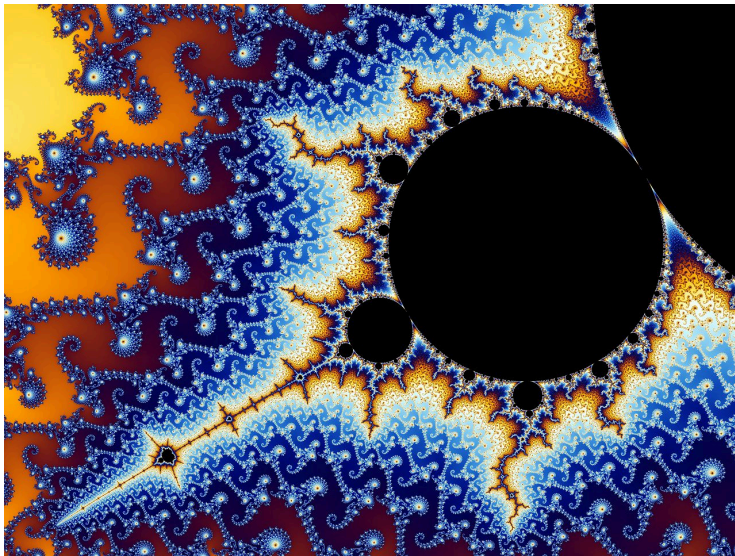


Element of Mandelbrot set ([Wikipedia](#))

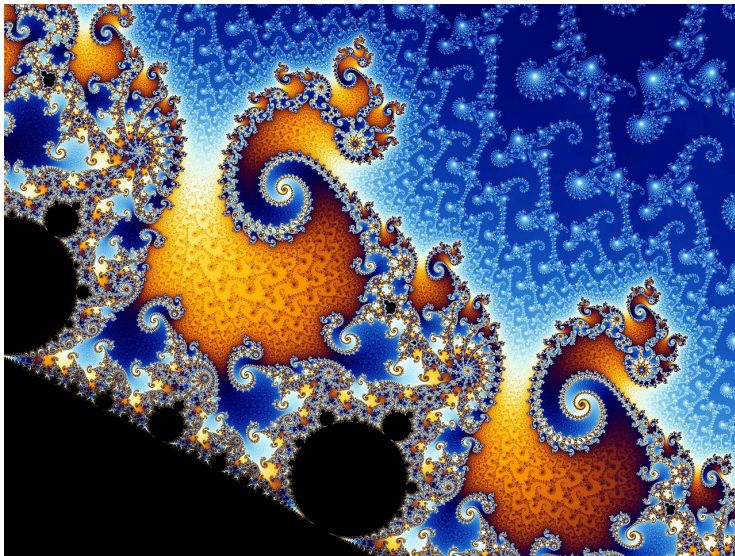
Fractals



Element of Mandelbrot set (Wikipedia)

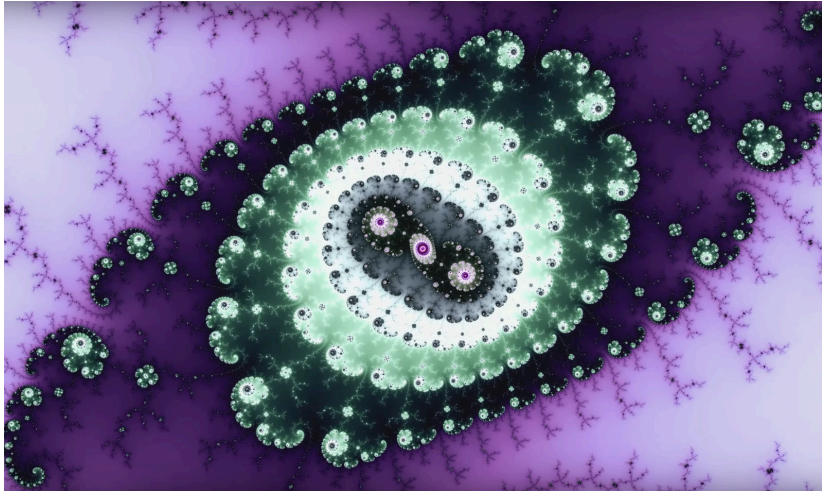


Element of Mandelbrot set (Wikipedia)



Element of Mandelbrot set ([Wikipedia](#))

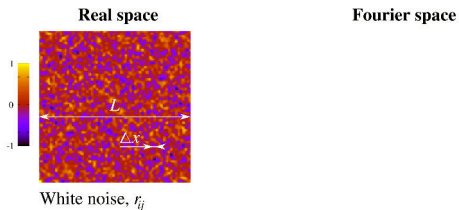
See animation



<https://youtu.be/pCpLWbHVNhk>

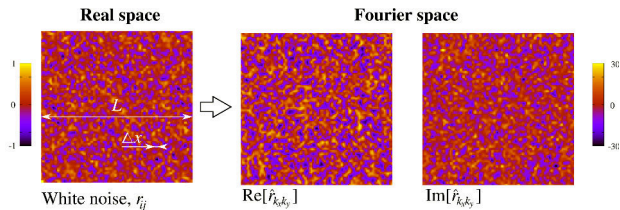
70 minutes of 4K "Eye of the Universe - Mandelbrot Fractal Zoom" e^{1091}

Synthesized rough surfaces: in pictures



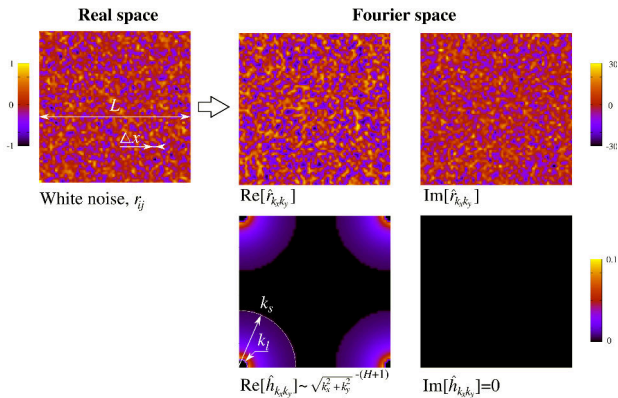
[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Synthesized rough surfaces: in pictures



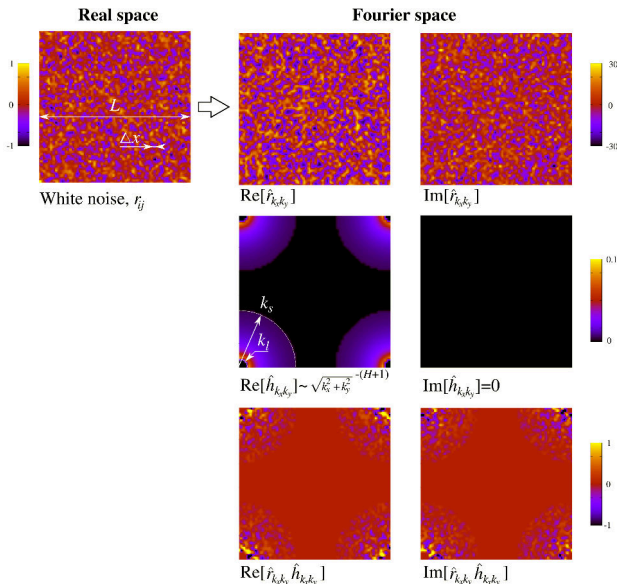
[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Synthesized rough surfaces: in pictures



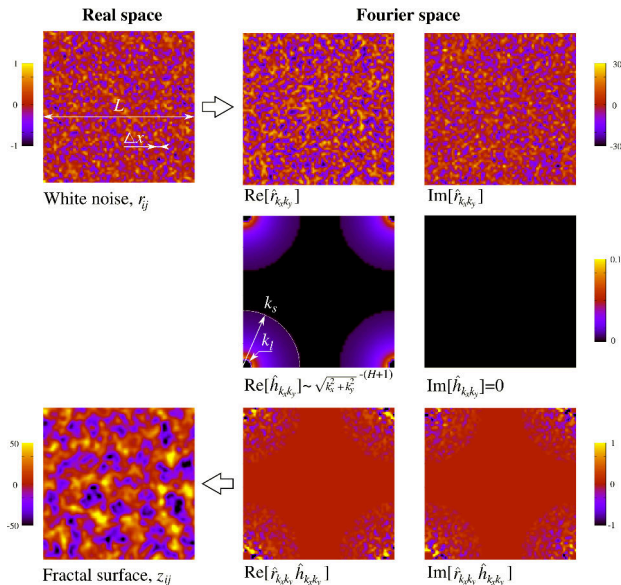
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Synthesized rough surfaces: in pictures



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Synthesized rough surfaces: in pictures



[1] Y. Z. Hu and K. Tonder, *Int. J. Machine Tools Manuf.* 32, 83 (1992)

Synthesized rough surfaces: in equations

- White noise:

$$w(x_i, y_j), \quad \langle w \rangle = 0, \quad \langle w^2 \rangle = \Phi_0$$

- Transform in Fourier space:

$$\hat{w}_{ij} = \hat{w}(k_x, k_y) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} w(x_i, y_j) \exp[-i(k_x x_i + k_y y_j)], \quad \langle \hat{w} \hat{w}^* \rangle = \langle w^2 \rangle = \Phi_0$$

- Create a filter

$$\hat{f}_{ij} = \hat{f}(k_x, k_y) = \begin{cases} \left[\frac{K_x^2 + K_y^2}{k_l^2} \right]^{-(1+H)/2}, & \text{for } 1 \leq \frac{\sqrt{K_x^2 + K_y^2}}{k_l} \leq \zeta, \\ 0, & \text{elsewhere,} \end{cases}$$

where $K_x = (s+1)\pi/L - sk_x$, $K_y = (t+1)\pi/L - tk_y$ for $s, t \in \{-1, 1\}$, $\zeta = k_s/k_l$

- Filter white noise:

$$\hat{z}_{ij} = \hat{z}(k_x, k_y) = \Re(\hat{f}_{ij}) \left[\Re(\hat{w}_{ij}) + i \Im(\hat{w}_{ij}) \right]$$

- Back to real space:

$$z(x_i, y_j) = \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} \hat{z}_{lm} \exp[i2\pi(lx_i + my_j)/L]$$

Synthesized rough surfaces: in equations II

- Power spectral density:

$$\Phi(k_x, k_y) = \hat{z}(k_x, k_y)\hat{z}^*(k_x, k_y) = \hat{f}^2(k_x, k_y)\hat{w}^2(k_x, k_y)$$

- Averaging over multiple samples:

$$\langle \Phi(k_x, k_y) \rangle = \langle \hat{w}^2(k_x, k_y) \rangle \hat{f}^2(k_x, k_y) = \begin{cases} \Phi_0 \left[\frac{\sqrt{K_x^2 + K_y^2}}{k_l} \right]^{-2(1+H)}, & \text{for } 1 \leq \frac{\sqrt{K_x^2 + K_y^2}}{k_l} \leq \zeta \\ 0, & \text{elsewhere,} \end{cases}$$

- For isotropic surface:

$$\langle \Phi(K) \rangle = \begin{cases} \Phi_0 (K/k_l)^{-2(1+H)}, & \text{if } 1 \leq K/k_l \leq \zeta \\ 0, & \text{otherwise.} \end{cases}$$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s

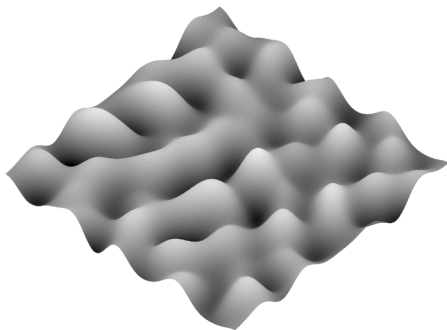
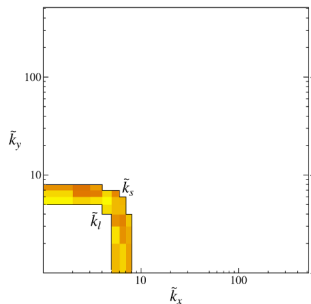


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
 $k_l = 4, \quad k_s = 8$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s

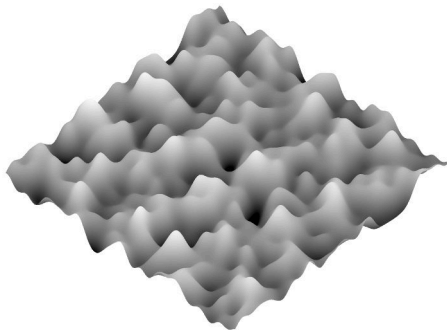
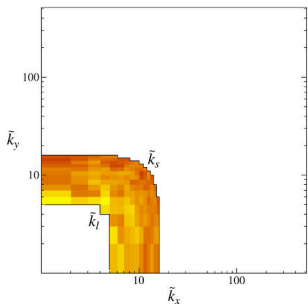


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
 $k_l = 4$, $k_s = 16$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s

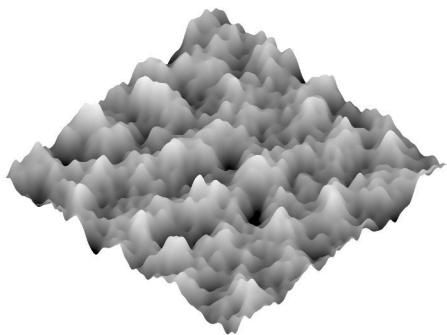
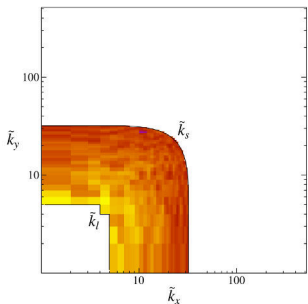


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
 $k_l = 4, \quad k_s = 32$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s

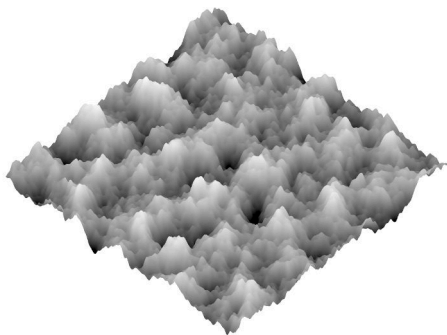
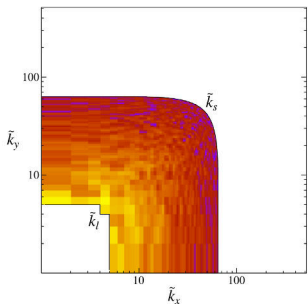


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
 $k_l = 4, \quad k_s = 64$

- Effect of the high frequency cutoff k_s

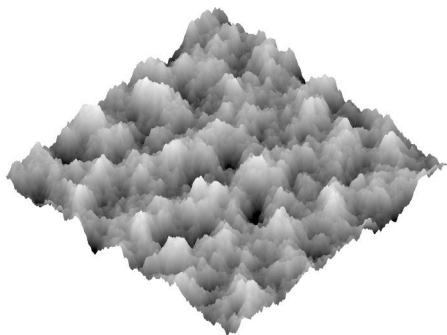
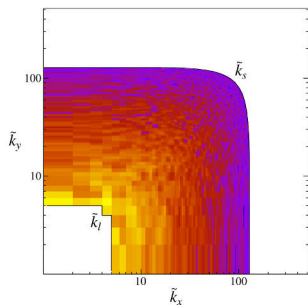


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
 $k_l = 4, \quad k_s = 128$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$

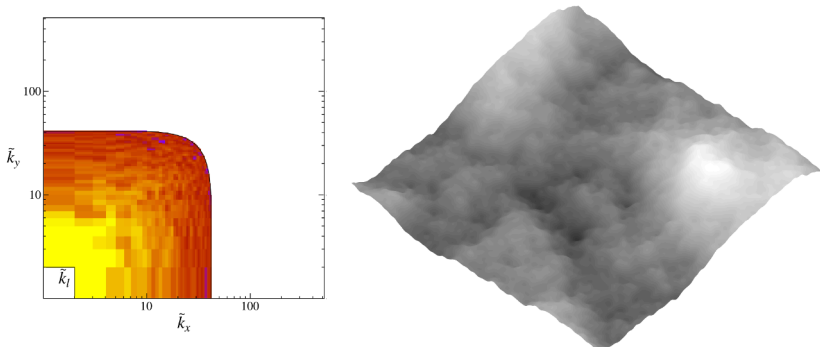


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
 $k_l = 1, \quad k_s = 43$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$

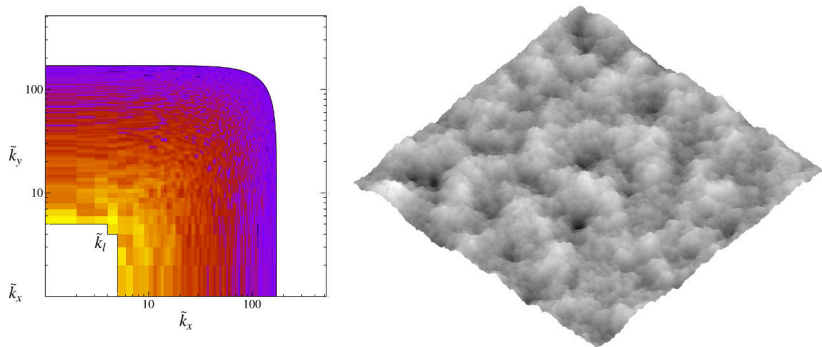


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
 $k_l = 4, \quad k_s = 171$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$

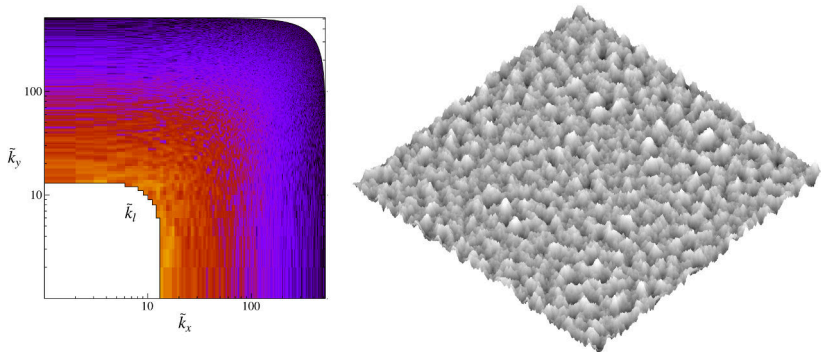


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
 $k_l = 12, \quad k_s = 512$

Effect of parameters: illustration

- Effect of the ratio of the higher cutoff to the discretization k_s/N

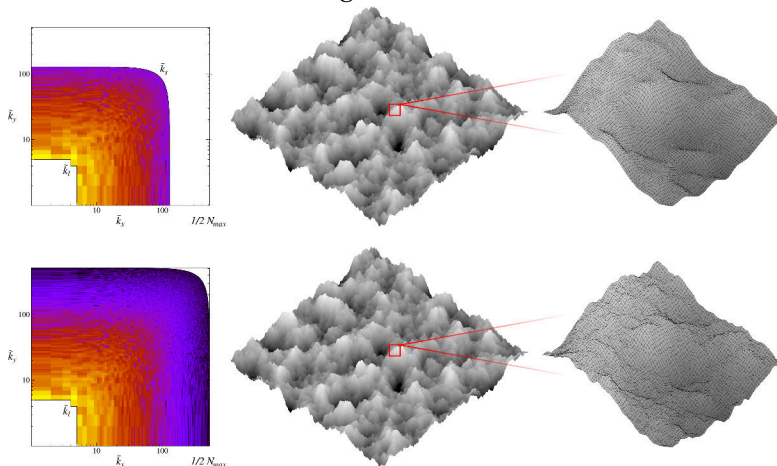


Fig. Power spectral densities (Fourier space)
and corresponding rough surfaces (real space) for
 $k_l = 12$, $k_s/N = 1/8$ **VS** $k_l = 12$, $k_s/N = 1/2$

Effect of parameters: illustration

- Effect of the ratio of the higher cutoff to the discretization k_s/N

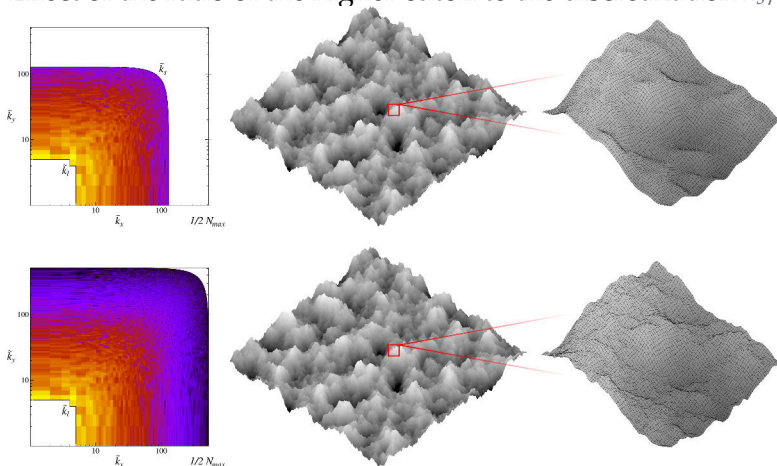


Fig. Power spectral densities (Fourier space)
and corresponding rough surfaces (real space) for
 $k_l = 12$, $k_s/N = 1/8$ (fine) VS $k_l = 12$, $k_s/N = 1/2$ (too coarse)
for mechanical simulations

Effect of parameters: illustration

- Effect of the discretisation (single asperity)

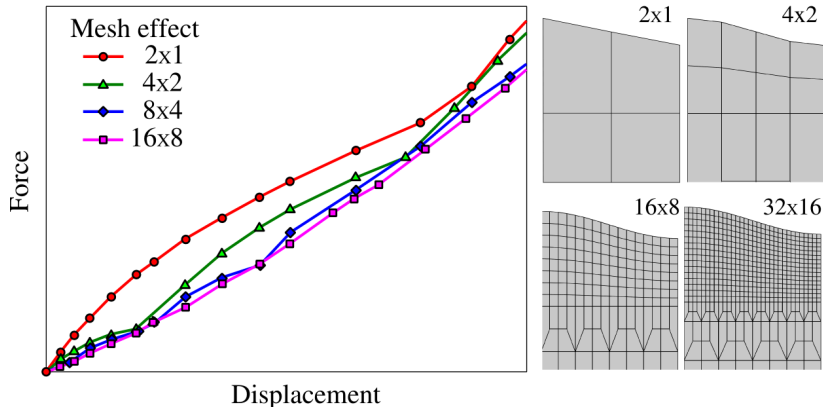


Fig. Effect of the mesh on mechanical response

Effect of parameters: illustration

- Data interpolation (Shanon, bi-cubic Bézier surfaces)

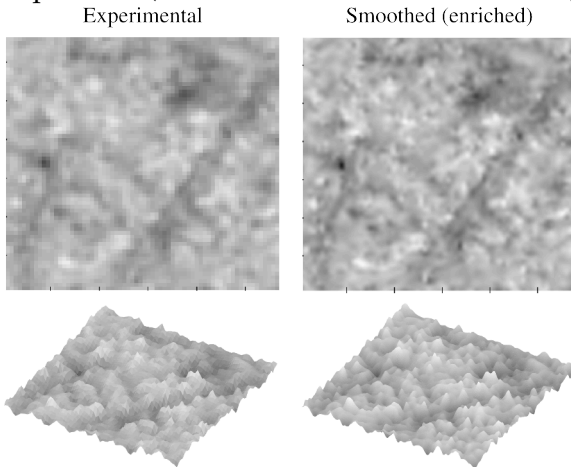


Fig. Bi-cubic Bézier interpolation of an experimental rough surface

[1]Hyun, Robbins, Tribol. Int. (2007)

[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)

Effect of parameters:

- k_l low frequency cutoff
- *representativity/normality*^[1,2,3]
- k_s high frequency cutoff
- *smoothness and density of asperities*
- $\zeta = k_s/k_l$ ratio^[3]
- *breadth of the spectrum*

$$\alpha \sim \zeta^{2H}$$

Nayak's parameter α is the central characteristic of roughness in asperity based mechanical models.

[1] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. (2009)

[2] Yastrebov, Durand, Proudhon, Caillaud, C.R. Mécan. (2011)

[3] Yastrebov, Ancaux, Molinari, Phys. Rev. E (2012)

[4] Yastrebov, Ancaux, Molinari, Int. J. Solids Struct. (2015)

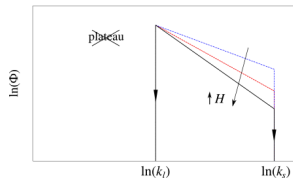
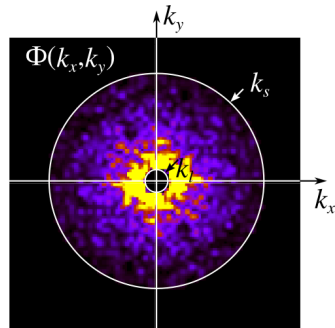


Fig. 3D and radial power spectral densities

Effect of parameters:

- k_l low frequency cutoff
- *representativity/normality*^[1,2,3]
- k_s high frequency cutoff
- *smoothness and density of asperities*
- $\zeta = k_s/k_l$ ratio^[3]
- *breadth of the spectrum*

$$\alpha \sim \zeta^{2H}$$

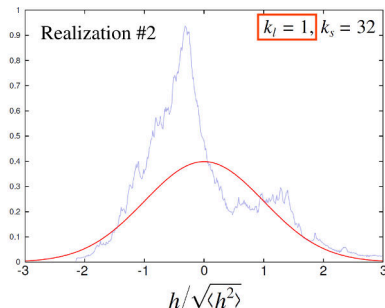
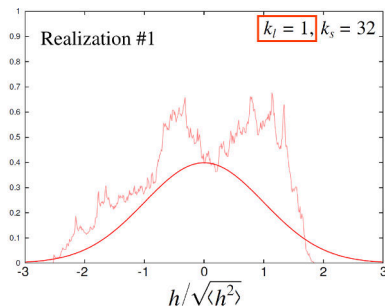
Nayak's parameter α is the central characteristic of roughness in asperity based mechanical models.

[1] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. (2009)

[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)

[3] Yastrebov, Ancaux, Molinari, Phys. Rev. E (2012)

[4] Yastrebov, Ancaux, Molinari, Int. J. Solids Struct. (2015)



Effect of parameters

Effect of parameters:

- k_l low frequency cutoff
- *representativity/normality*^[1,2,3]
- k_s high frequency cutoff
- *smoothness and density of asperities*
- $\zeta = k_s/k_l$ ratio^[3]
- *breadth of the spectrum*

$$\alpha \sim \zeta^{2H}$$

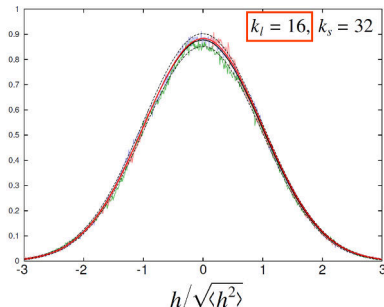
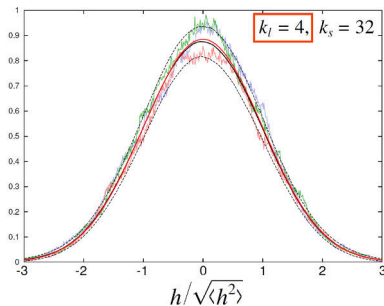
Nayak's parameter α is the central characteristic of roughness in asperity based mechanical models.

[1] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. (2009)

[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)

[3] Yastrebov, Ancaux, Molinari, Phys. Rev. E (2012)

[4] Yastrebov, Ancaux, Molinari, Int. J. Solids Struct. (2015)



Interconnection of parameters

- Spectral moment and k_l, k_s, H :

$$m_{0p} \approx m_{p0} \approx \Phi_0 \int_{k_l}^{k_s} \int_0^{2\pi} [k \cos(\varphi)]^p (k/k_l)^{-2(1+H)} k dk d\varphi = \Phi_0 k_l^{p+2} \frac{\zeta^{p-2H} - 1}{p - 2H} T(p)$$

$$\text{with } T(p) = \int_0^{2\pi} \cos^p(\varphi) d\varphi = \begin{cases} 2\pi, & \text{if } p = 0; \\ \pi, & \text{if } p = 2; \\ 3\pi/4, & \text{if } p = 4. \end{cases}$$

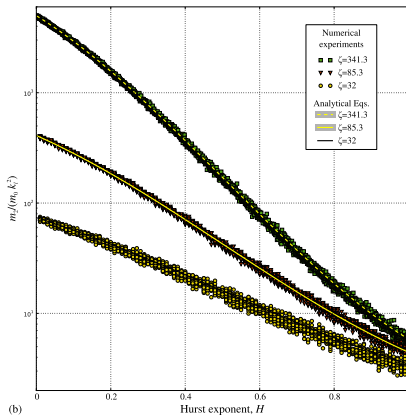
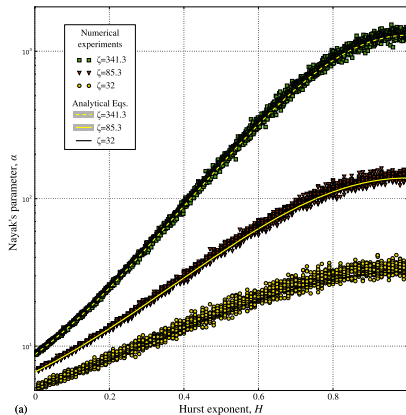
- Nayak's parameter

$$\alpha(H, \zeta) = \frac{3}{2} \frac{(1-H)^2}{H(H-2)} \frac{(\zeta^{-2H} - 1)(\zeta^{4-2H} - 1)}{(\zeta^{2-2H} - 1)^2}$$

- Asperity density

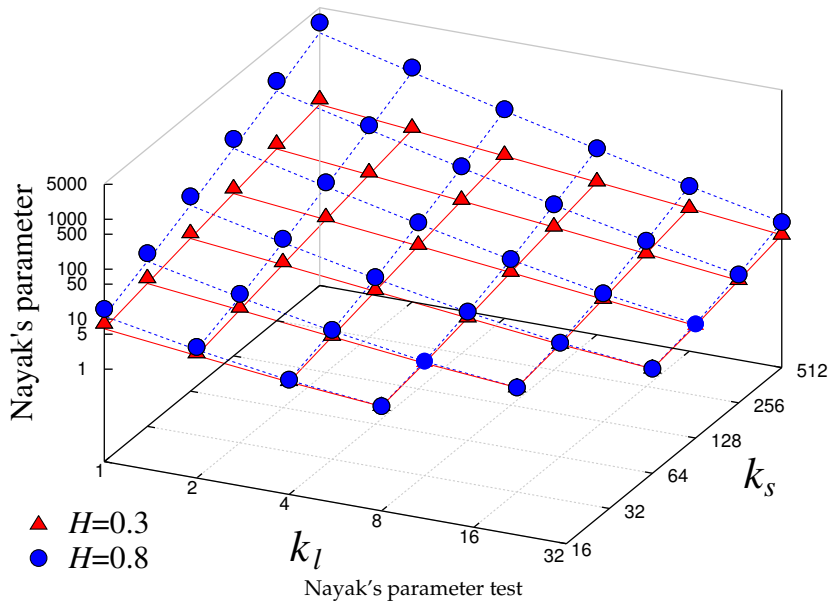
$$D = \frac{\sqrt{3}}{18\pi} \frac{m_4}{m_2} = \frac{\sqrt{3}}{24\pi} \frac{1-H}{2-H} \frac{\zeta^{4-2H} - 1}{\zeta^{2-2H} - 1} k_l^2$$

Interconnection of parameters

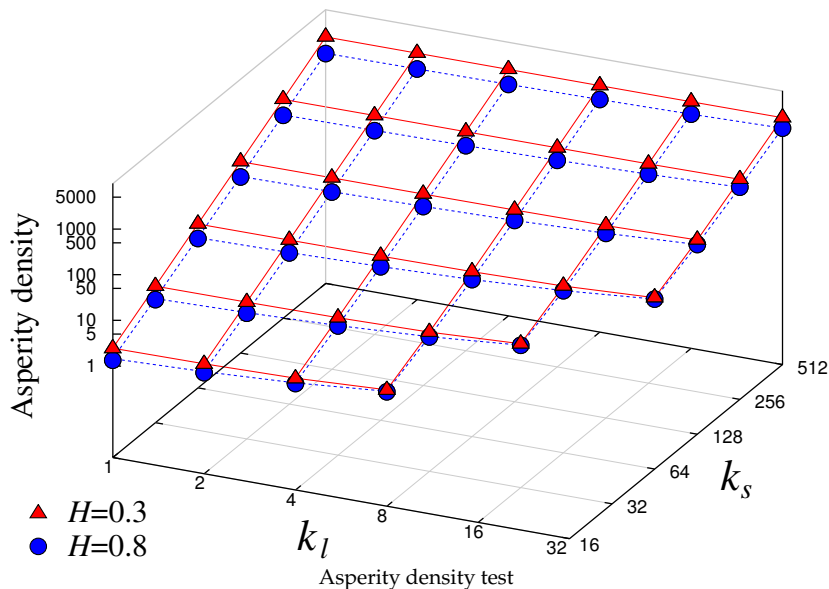


(a) Numerical verification on 100 000 generated rough surfaces with 2048×2048 points

Interconnection of parameters



Interconnection of parameters



Asperity analysis

- Detect summits (z_{ij} higher than neighbouring points) and evaluate second derivatives

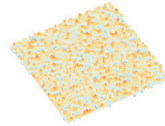
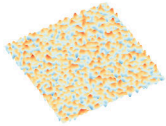
$$\frac{\partial^2 z}{\partial x^2} = \frac{z_{i+1j} + z_{i-1j} - 2z_{ij}}{2\Delta x^2}; \quad \frac{\partial^2 z}{\partial y^2} = \frac{z_{i+1j} + z_{i-1j} - 2z_{ij}}{2\Delta x^2}$$
$$\frac{\partial^2 z}{\partial x \partial y} = \frac{z_{i+1j+1} + z_{i+1j-1} - z_{i-1j+1} - z_{i-1j-1}}{4\Delta x^2}$$

- Principal curvatures $\kappa_{1,2}$:

$$\kappa_{1,2} = \frac{1}{2} \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right) \pm \sqrt{\left(\frac{\partial^2 z}{\partial x \partial y} \right)^2 + \frac{1}{4} \left(\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} \right)^2}$$

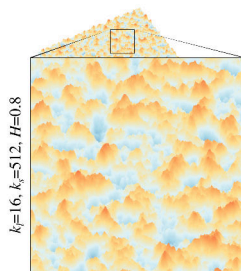
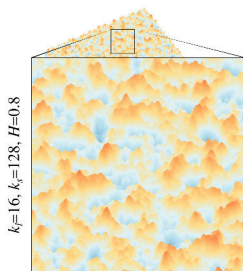
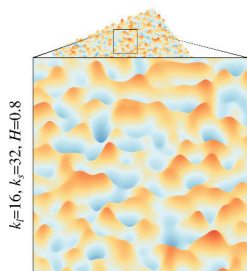
- Saddle point $\kappa_1 \kappa_2 < 0$, extrema $\kappa_1 \kappa_2 > 0$
- Mean curvature which can be safely used in Hertz theory: $\bar{\kappa} = \sqrt{\kappa_1 \kappa_2}$

Asperity analysis



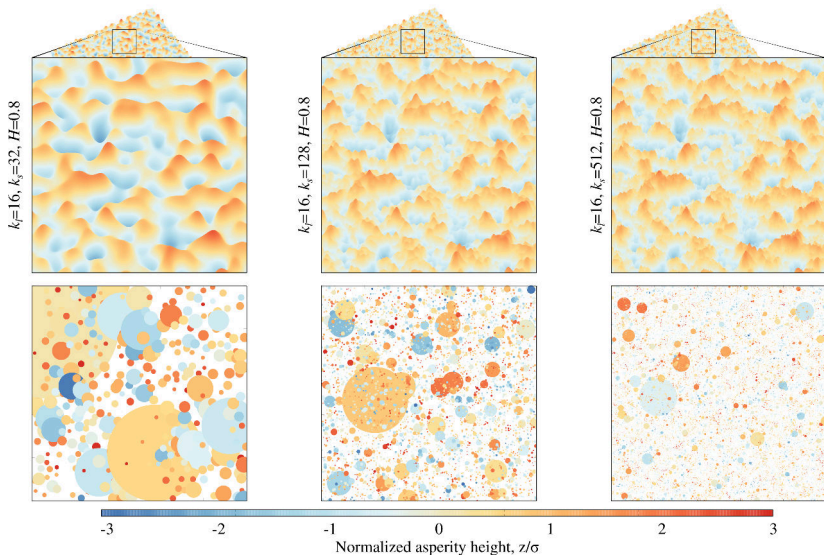
Rough surfaces and associated asperities

Asperity analysis



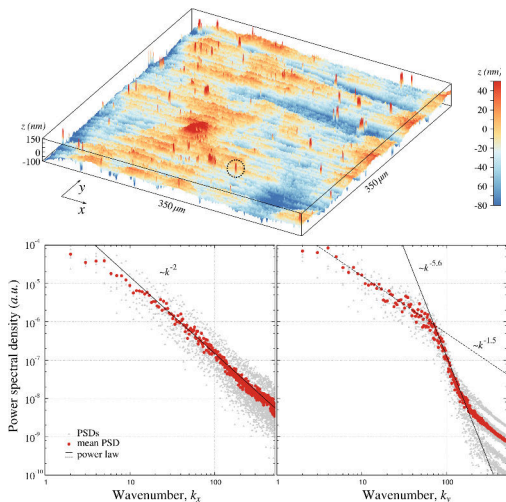
Rough surfaces and associated asperities

Asperity analysis



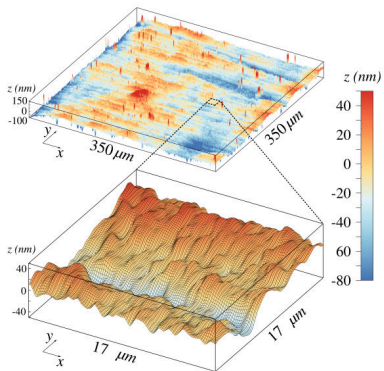
Rough surfaces and associated asperities

Examples

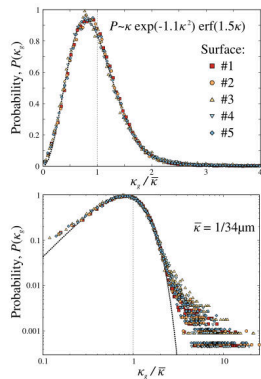


[1] Yastrebov et al, Three-level multi-scale modeling of electrical contacts sensitivity study and experimental validation, Proceedings of Holm Conference, 2015.

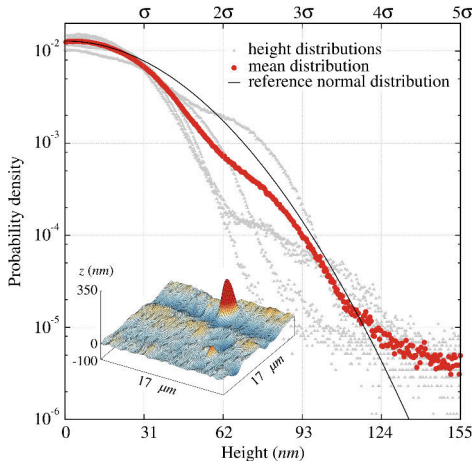
Examples



Asperity curvatures



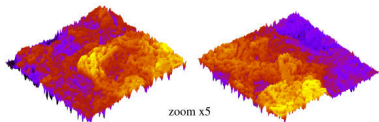
Examples



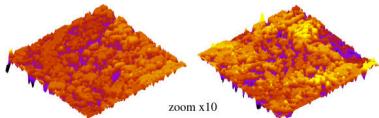
[1] Yastrebov et al, Three-level multi-scale modeling of electrical contacts sensitivity study and experimental validation, Proceedings of Holm Conference, 2015.

Examples

Bright fracture surface

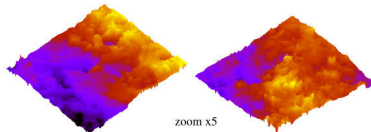


zoom x5

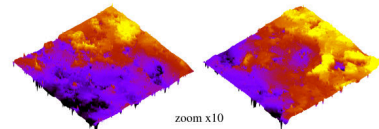


zoom x10

Dark fracture surface



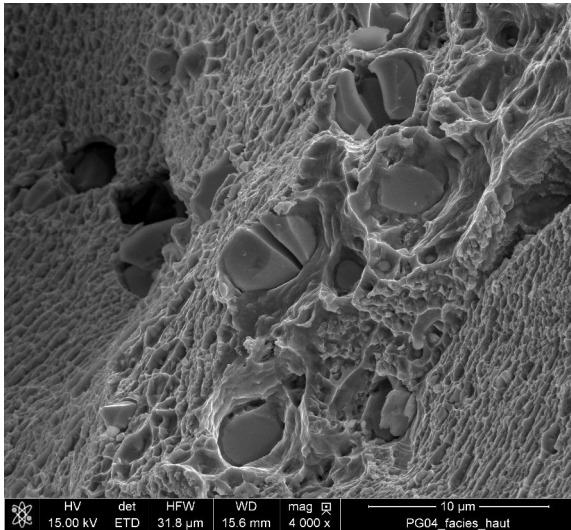
zoom x5



zoom x10

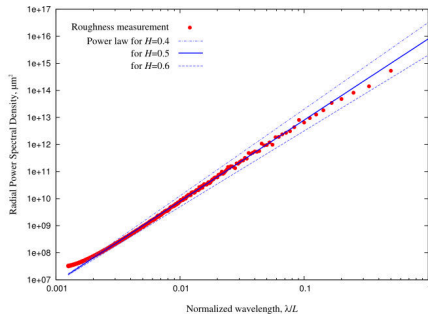
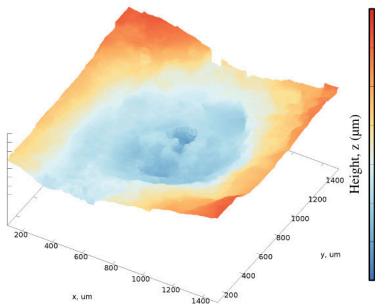
Fatigue & creep fracture surfaces (Ti-alloy)
in collaboration with A. Marchenko

Examples



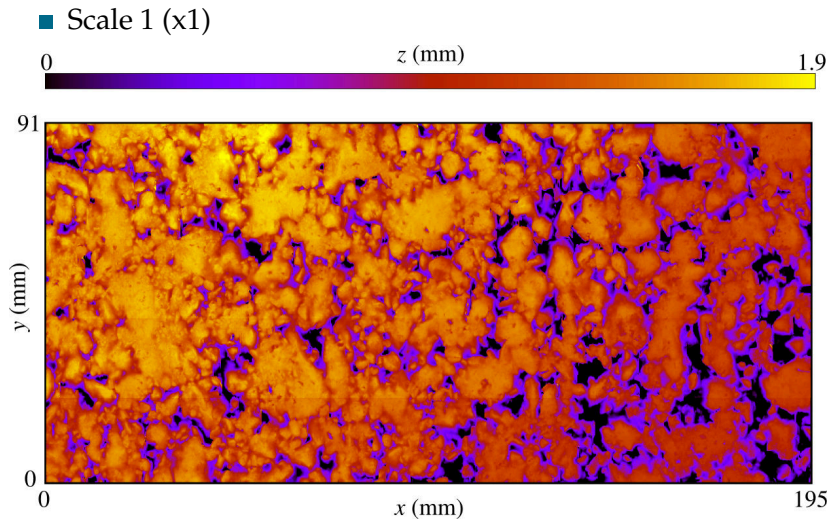
Fatigue fracture surface (Co-alloy), particles WC
Courtesy of V. Esin

Examples



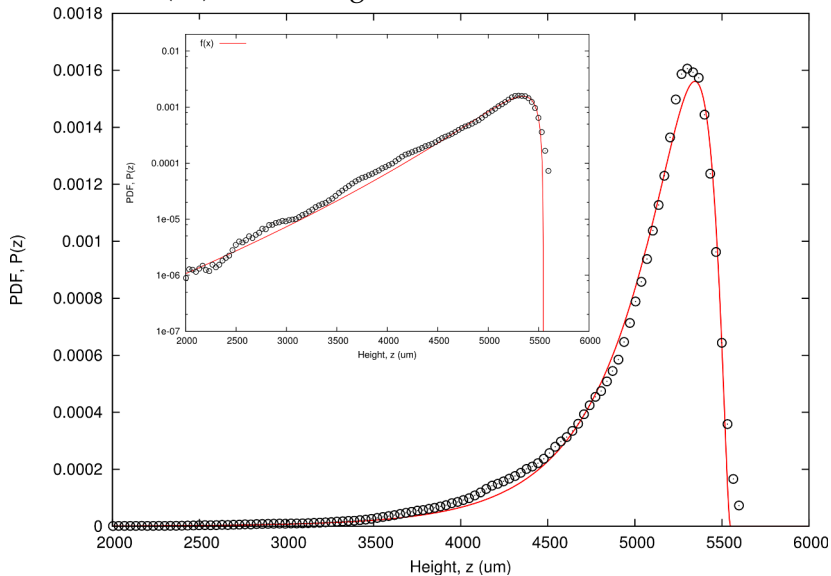
Crater topography and PSD
In collaboration with D. Tklich (NTNU, Sintef)

Multiscale road roughness

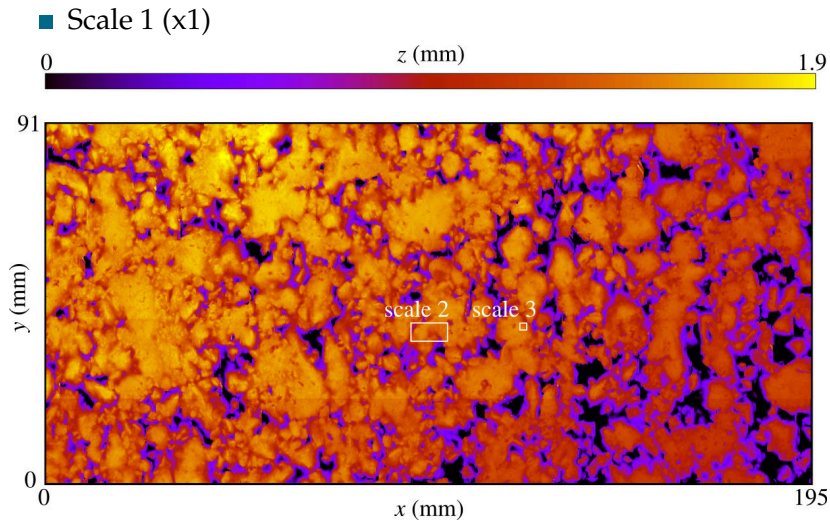


Multiscale road roughness

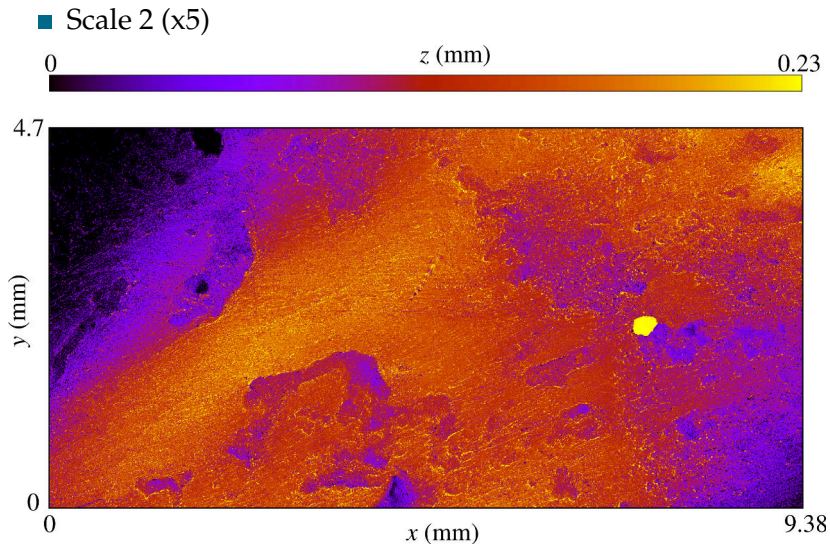
Scale 1 (x1): PDF and generalized Weibull fit



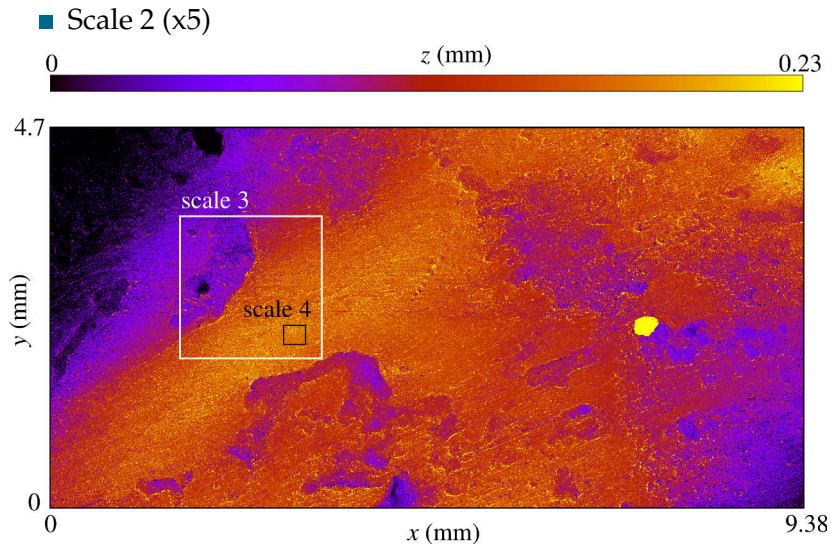
Multiscale road roughness



Multiscale road roughness

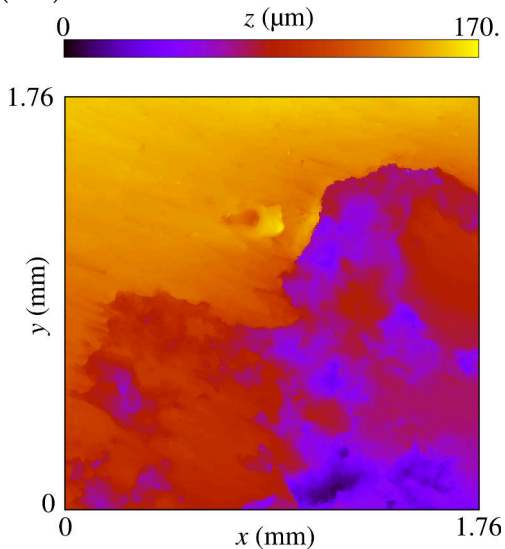


Multiscale road roughness



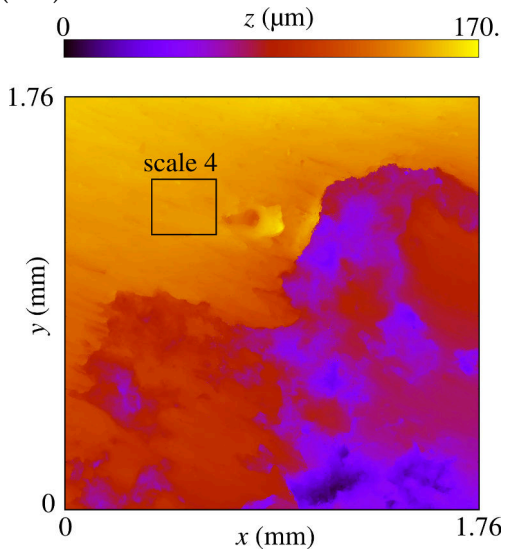
Multiscale road roughness

■ Scale 3 (x20)



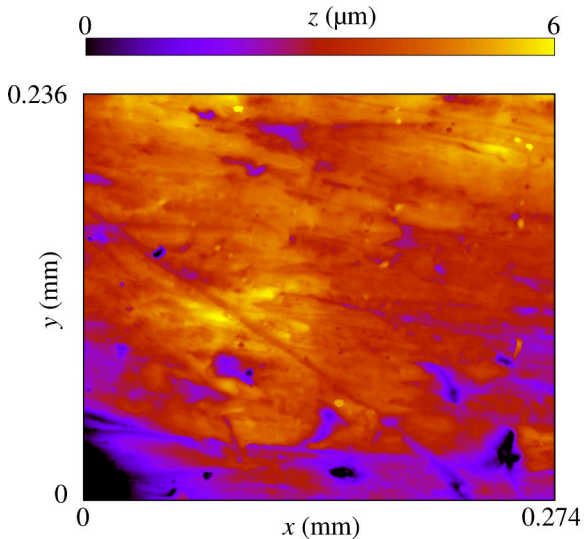
Multiscale road roughness

■ Scale 3 (x20)

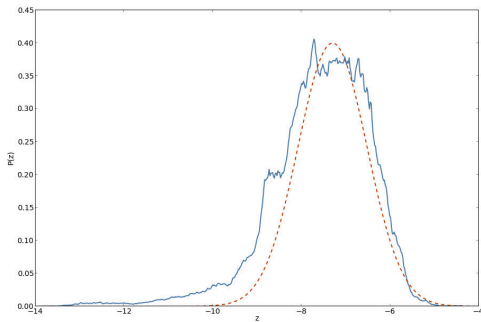


Multiscale road roughness

■ Scale 4 (x100)



■ Scale 4 (x100): PDF



What did you learn?

- Idea about the interface physics complexity
- How to measure roughness: confocal, WLI, AFM, STM
- What are the main characteristics: PDF, PSD, SQ, SDQ, Hurst, etc
- Model roughness
- Fractal aspect & Mandelbrot set
- Real roughness



Thank you for your attention!
