Contact mechanics and elements of tribology

Lecture 4.

Micromechanical contact: roughness

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@ Centre des Matériaux (& virtually)
February 25, 2025

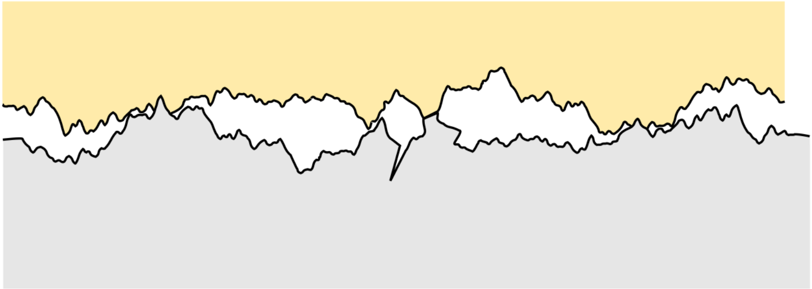
Outline

- Introduction
- Measurement techniques
- Classifications
- Main characteristics
- PDF and PSD
- Random process model of roughness
- Computational roughness models
- Reading

Contact under microscope

Contact under microscope

Contact under microscope



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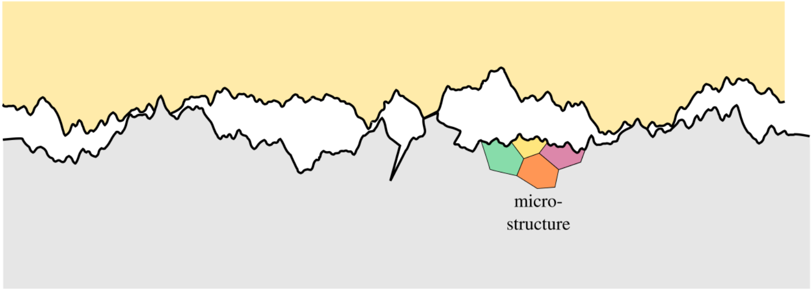

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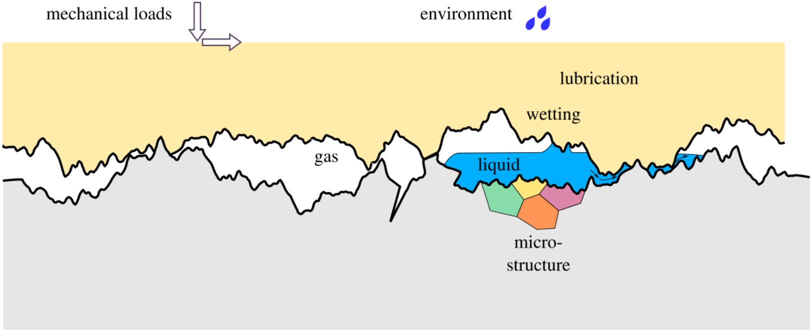

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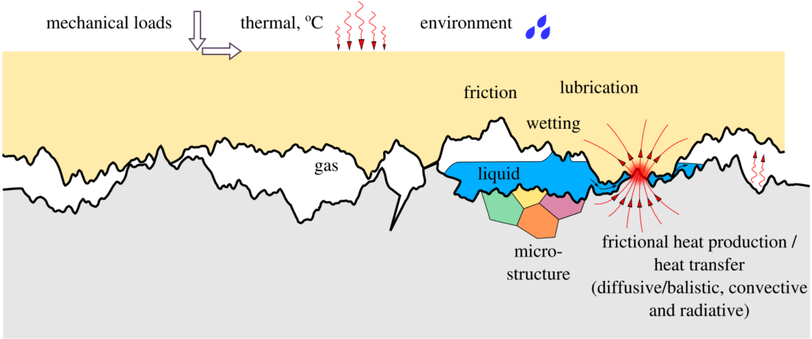
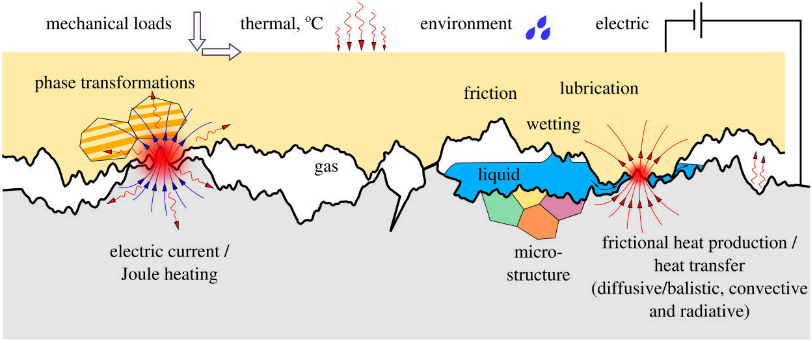


Contact under microscope



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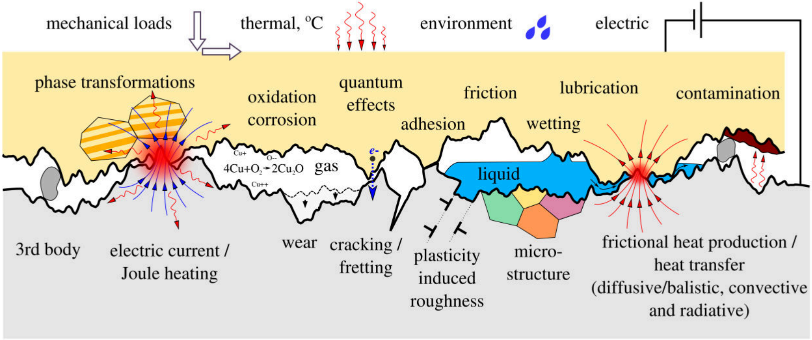

- **Natural and industrial surfaces are *rough*:**
  - processing
  - polishing
  - coating
  - microstructure
  - surface energy
  - deformation
  - aging
  - environment

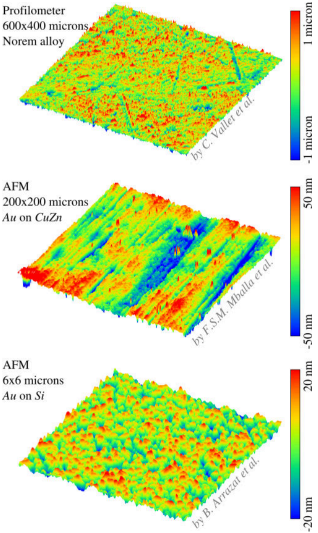

Fig. Examples of rough surfaces

Roughness

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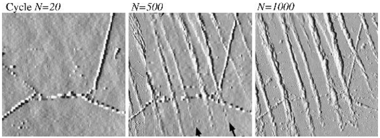

Fig. Persistent slip marks [1]

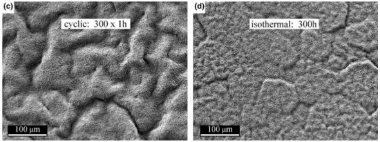

Fig. Rumpling (thermal cycling induced roughness in air)[2]

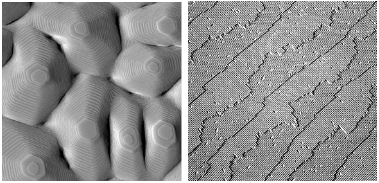

Fig. Epitaxial surface growth [3,4]

[1] J.Polák, J. Man & K. Orbtlík, Int J Fatigue 25 (2003)
[2] V.K. Tolpygo, D.R. Clarke, Acta Mat 52 (2004)
[3] M. Einax, W. Dieterich, P. Maass, Rev Mod Phys 85 (2013)
[4] J.R. Arthur, Surf Sci 500 (2002)

Roughness

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- **Roughness affects:**
  - stress-strain state
  - dry friction
  - wear
  - adhesion
  - fluid flow
  - sealing
  - energy transfer

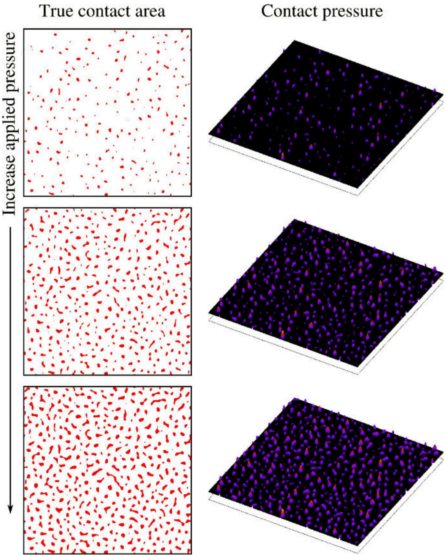

Fig. True contact area and stress fluctuations

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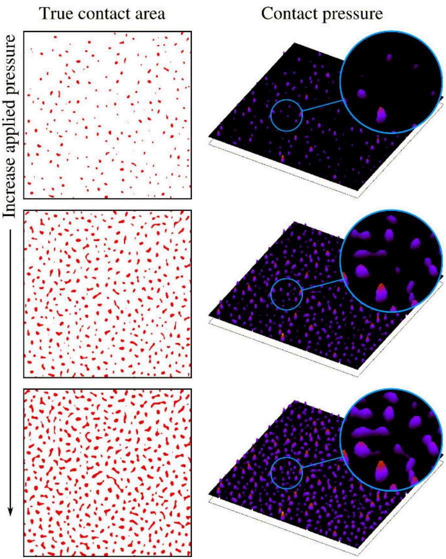

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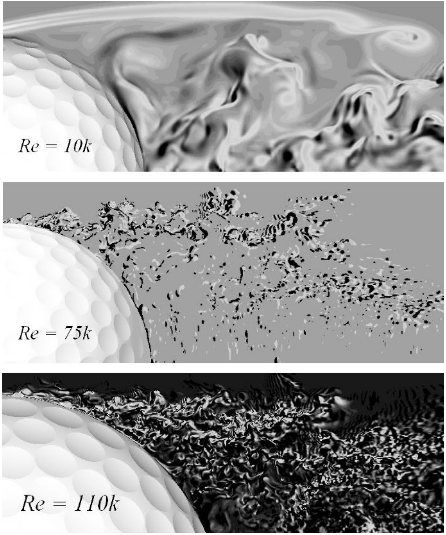

Fig. Numerical simulation of airflow around a (dimpled) golf ball [5]

[5] C.E. Smith, PhD thesis (2011)

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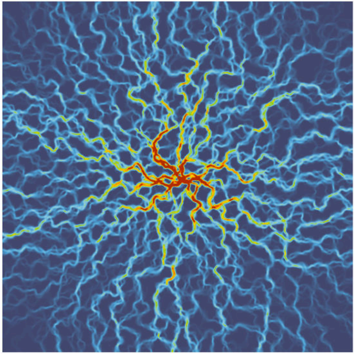


Fig. Fluid passage through free volume between rough surfaces

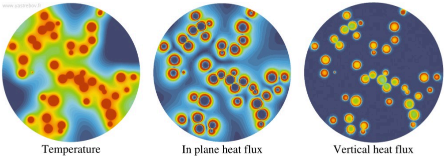
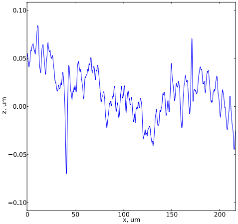


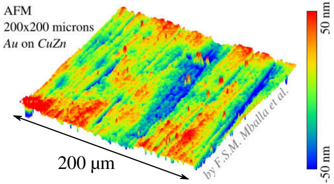
Fig. Heat transfer between rough surfaces (asperity-based model)

Roughness: in plane and out of plane scales

$$\frac{\text{in-plane factor}}{\text{out-of-plane factor}} = 1000$$



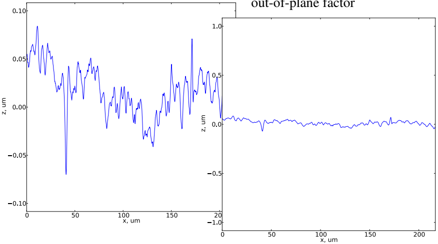
Au on CuZn



Roughness: in plane and out of plane scales

$$\frac{\text{in-plane factor}}{\text{out-of-plane factor}} = 1000$$

$$\frac{\text{in-plane factor}}{\text{out-of-plane factor}} = 100$$



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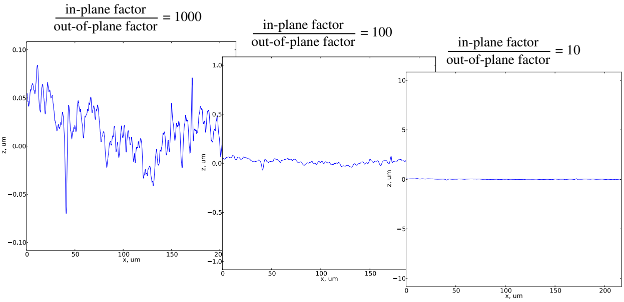

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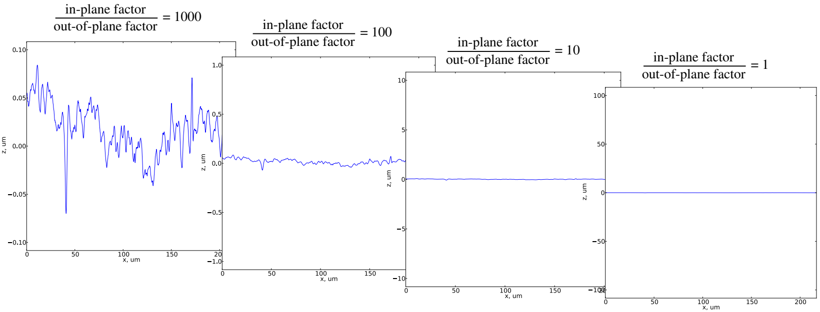
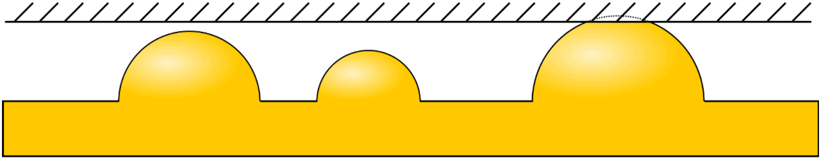


Roughness: in plane and out of plane scales

Confusing representation that should be avoided:



Still Hertzian solution remains valid at asperity tips

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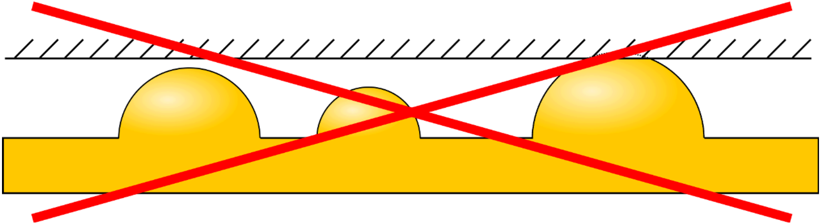
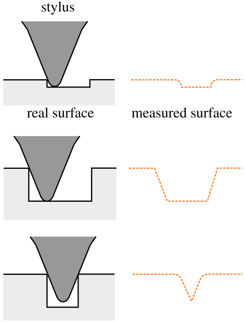


Still Hertzian solution remains valid at asperity tips

Surface metrology techniques

- Stylus measurements
  - Mechanical contact of a tip with surface
  - Force $\geq 3\mu$g, tip radius $\geq$ 50 nm
  - Mainly for profile measurements $y(x)$
- Optical measurements
  - Confocal (laser scanning) microscopy
    - *highest lateral resolution*
  - Interferometry (WLI):
    - *highest vertical resolution*
    - *10 to 100 times faster than CM*
  - Scanning Electronic Microscopy (SEM):
    - *in secondary electron emission*
    - *electrons penetrate in the matter → roughness smoothing*
    - *conducting materials*
- Nano-contact measurements
  - Atomic Force Microscopy (AFM)
    roughness + adhesive and elastic properties
  - Scanning Tunneling Microscope (STM)

Stylus profilometer



Valley measurement error

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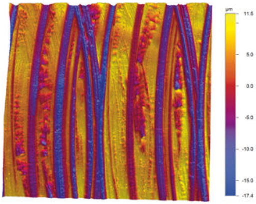
Stylus profilometer

Modern stylus profilometer
www.bruker.com



Roughness measurements ($\Delta z \approx 30\ \mu$m)
www.icryst.com

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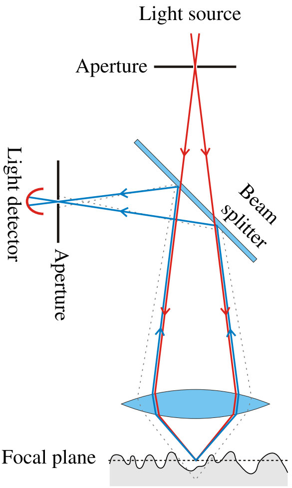
Confocal microscopy



Principle of confocal microscopy adapted from www.wikipedia.org

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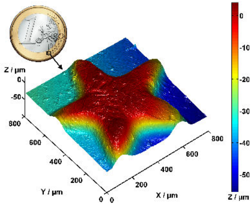
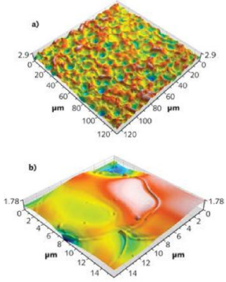


1 euro surface www.wikipedia.org



Stainless steel machined with micro-electric discharge www.laserfocusworld.org

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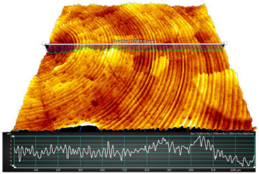
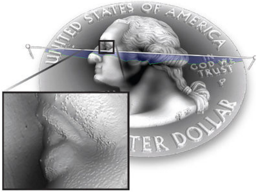
White Light Interferometry



Diamond-turned optics www.zygo.com



US quarter surface www.zygo.com

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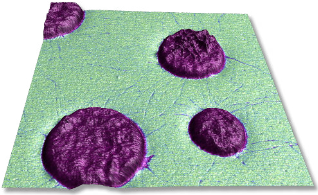
AFM

Modern AFM
www.bruker.com



Roughness and elastic moduli (color) of polymer blend
www.bruker.com

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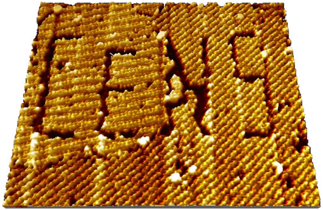
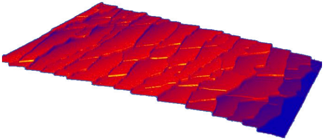


Fig. Center for NanoScience logo imprinted at atomic scale
www.cens.de



Atomic steps on platinum surface (500×500 nm)
www.icryst.com

Topography Challenge

"The Surface-Topography Challenge: A Multi-Laboratory Benchmark Study to Advance the Characterization of Topography" just submitted

Roughness: classification

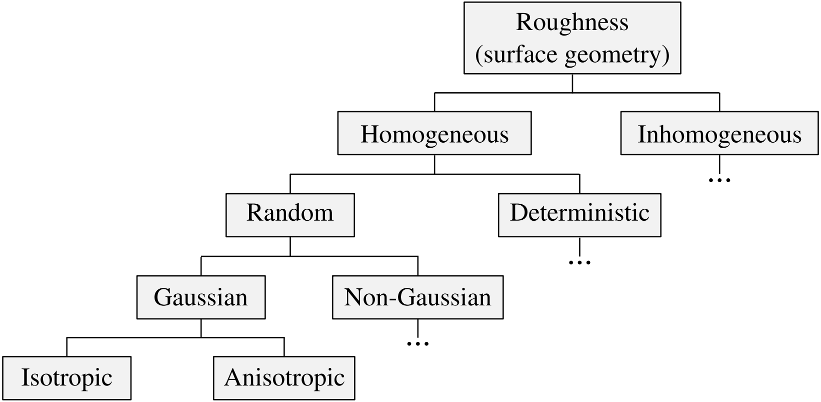
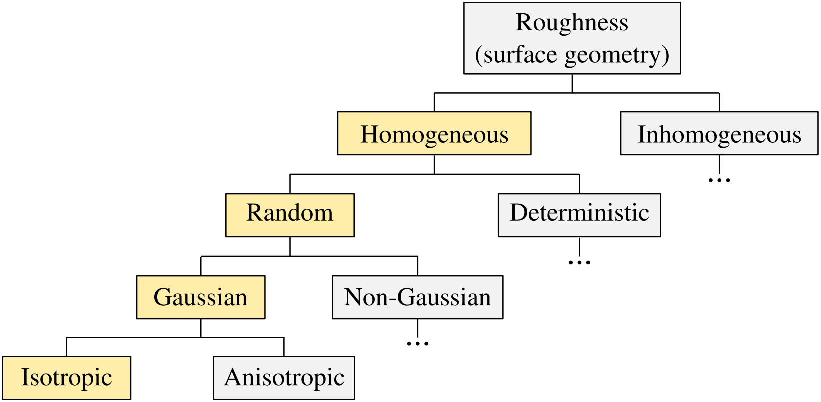


Fig. Roughness classification according to Nayak[1]

[1] Nayak, J. Lub. Tech. (ASME) 93:398 (1971)

Roughness: classification



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Roughness and geometry/form

- Roughness vs geometry of surfaces
- Sometimes macroscopic geometry is substracted (filtered out): form→error of form→wavyness→roughness
- Non-trivial to remove macroscopic shape
- Most roughness measurement tools enable shape removal

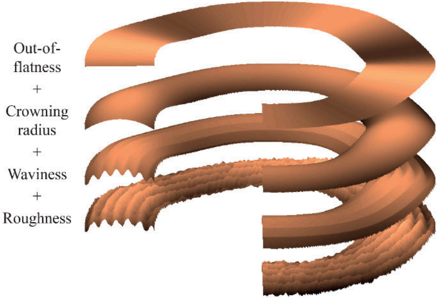

Fig. Circular metalic seal with turned copper surface[1]

[1] F.P. Rafòls, Licentiate Thesis, LTU 2016.

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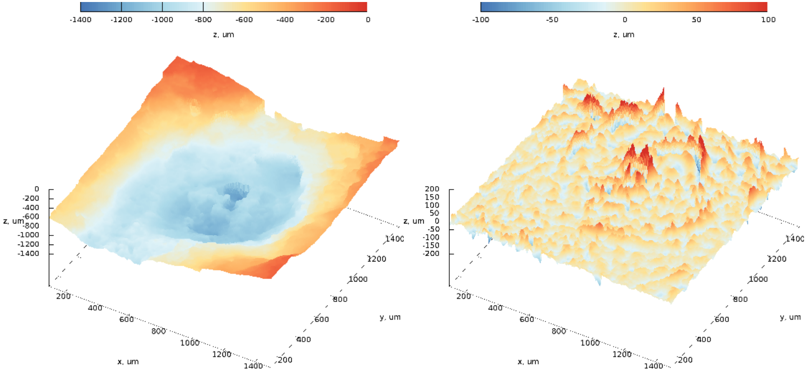

Fig. (left) impact crater, (right) shape is filtered out

Roughness and geometry/form

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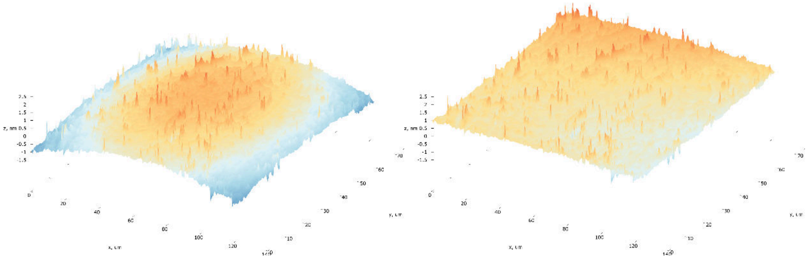

Fig. (left) spherical indenter, (right) $z = a(x^2 + y^2)$ shape is substracted

Main characteristics

Integral quantities

- Average of absolute values [l.u.] (profile - R_a, surface - S_a)

$$S_a = \frac{1}{A}\int\limits_A |z(x,y) - \bar{z}|\,dA, \quad S_a = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}|z_{ij} - \bar{z}|$$

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- Standard deviation of height [l.u.] (σ or R_q for profile, S_q for surface)

$$\sigma = \sqrt{\frac{1}{A}\int\limits_A (z(x,y) - \bar z)^2\, dA}, \quad \sigma = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}(z_{ij} - \bar z)^2}$$

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- Maximal valley depth R_v,S_v, maximal peak height R_p,S_p [l.u.]
  very sensitive to sample area

Main characteristics

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- Maximal valley depth R_v,S_v, maximal peak height R_p,S_p [l.u.]
  very sensitive to sample area
- Skewness [adim] (γ_1 or R_{sk},S_{sk})

$$\gamma_1 = \frac{1}{A\sigma^3}\int\limits_A (z(x,y) - \bar z)^3\,dA, \qquad \gamma_1 = \frac{1}{N^2\sigma^3}\sum_{i=1}^{N}\sum_{j=1}^{N}(z_{ij} - \bar z)^3$$

Integral quantities

- Average of absolute values [l.u.] (profile - R_a, surface - S_a)

$$S_a = \frac{1}{A}\int\limits_A |z(x,y) - \bar{z}|\, dA, \quad S_a = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} |z_{ij} - \bar{z}|$$

- Standard deviation of height [l.u.] (σ or R_q for profile, S_q for surface)

$$\sigma = \sqrt{\frac{1}{A}\int\limits_A (z(x,y) - \bar{z})^2\, dA}, \quad \sigma = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} (z_{ij} - \bar{z})^2}$$

- Maximal valley depth R_v,S_v, maximal peak height R_p,S_p [l.u.]
  very sensitive to sample area
- Skewness [adim] (γ_1 or R_{sk},S_{sk})

$$\gamma_1 = \frac{1}{A\sigma^3}\int\limits_A (z(x,y) - \bar{z})^3\, dA, \qquad \gamma_1 = \frac{1}{N^2\sigma^3}\sum_{i=1}^{N}\sum_{j=1}^{N} (z_{ij} - \bar{z})^3$$

- Kurtosis [adim] (κ or R_{ku},S_{ku})

$$\kappa = \frac{1}{A\sigma^4}\int\limits_A (z(x,y) - \bar{z})^4\, dA, \qquad \kappa = \frac{1}{N^2\sigma^4}\sum_{i=1}^{N}\sum_{j=1}^{N} (z_{ij} - \bar{z})^4$$

Main characteristics II

Integral quantities II

- Average of absolute value of gradient (slope) [adim] (profile - R_{da}, surface - S_{da})

$$S_{da} = \langle\left|\nabla z(x,y) - \overline{\nabla z}\right|\rangle = \frac{1}{A}\int\limits_A \left|\nabla z(x,y) - \overline{\nabla z}\right| dA$$

$$S_{da} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left|\frac{z_{i+1,j} - z_{i,j} - \overline{\Delta z_x}}{\Delta x}\right| + \left|\frac{z_{i,j+1} - z_{i,j} - \overline{\Delta z_y}}{\Delta y}\right|$$

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- Standard deviation of gradient (slope) [adim] (profile - R_{dq}, surface - S_{dq})

$$S_{dq} = \sqrt{\langle \left| \nabla z(x,y) - \overline{\nabla z} \right|^2 \rangle} = \sqrt{\frac{1}{A} \int\limits_A \left| \nabla z(x,y) - \overline{\nabla z} \right|^2 \, dA}$$

$$S_{dq} = \sqrt{\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left| \frac{z_{i+1,j} - z_{i,j} - \overline{\Delta z_x}}{\Delta x} \right| + \left| \frac{z_{i,j+1} - z_{i,j} - \overline{\Delta z_y}}{\Delta y} \right|^2}$$

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- Often in integrated slope measurements a smoothing filter is used, for example, according to ASME B46.1 standard

$$\frac{\partial z}{\partial x} \approx \frac{1}{60\Delta x}\left(z_{i+3,j} - 9z_{i+2,j} + 45z_{i+1,j} - 45z_{i-1,j} + 9z_{i-2,j} - z_{i-3,j}\right)$$

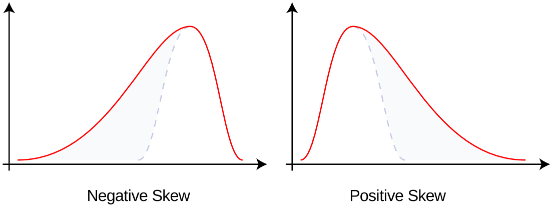
Main characteristics: probability density

- Probability density of heights $P(z)$
- Properties and moments

$$1 = \int\limits_{-\infty}^{\infty} P(z)\,dz, \qquad \bar z = \int\limits_{-\infty}^{\infty} zP(z)\,dz, \qquad \sigma = \sqrt{\int\limits_{-\infty}^{\infty} (z-\bar z)^2 P(z)\,dz}$$

$$\mu_q = \int\limits_{-\infty}^{\infty} z^q P(z)\,dz \quad \text{then} \quad \mu_0 = 1,\ \mu_1 = \bar z,\ \mu_2 = \sigma^2 + \bar z^2$$

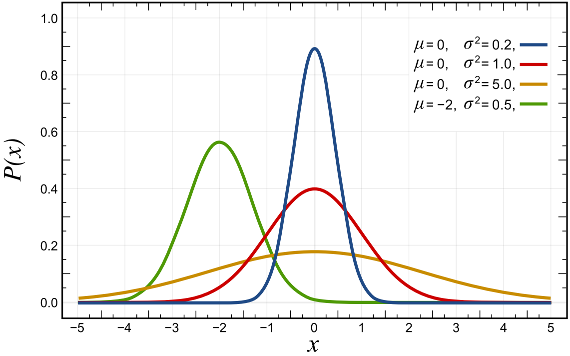
- Link to skewness



Main characteristics: probability density II

Distribution examples

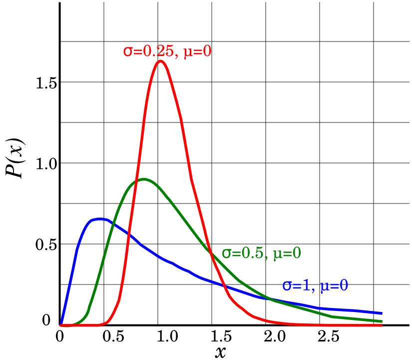
- Normal (Gaussian): $P(x,\mu,\sigma) = \dfrac{1}{\sigma\sqrt{2\pi}}\exp\left[-\dfrac{(x-\mu)^2}{2\sigma^2}\right], x \in \mathbb{R}$



adapted from wikipedia.org

Distribution examples

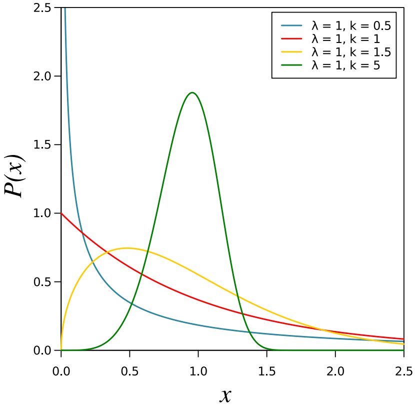
- Lognormal: $P(x,\mu,\sigma) = \dfrac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\dfrac{(\log(x)-\mu)^2}{2\sigma^2}\right] x \in \mathbb{R}^+$



adapted from wikipedia.org

Distribution examples

- Weibull: $P(x,k,\lambda) = \dfrac{k}{\lambda}\left(\dfrac{x}{\lambda}\right)^{k-1} \exp -(x/\lambda)^k \; x \in \mathbb{R}^+$

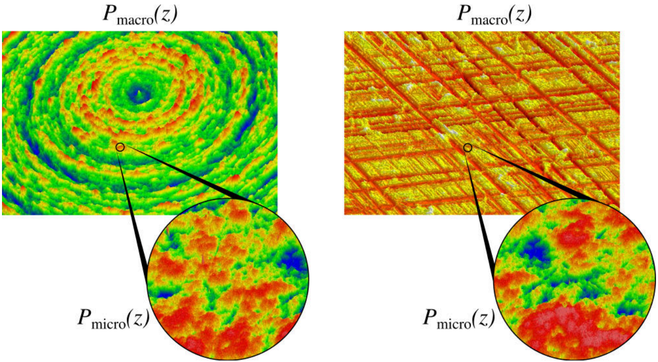


adapted from wikipedia.org

Main characteristics: comments on PDF

Real rough surfaces

- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
- Wear, polishing, flattening results in removal of the right distribution tail $P(z > z_0) \to 0$: negative skewness



Macro- and microscopic roughness (left: diamond-turned surface, right: cross-hatched surface)
Images from www.zygo.com used

Real rough surfaces

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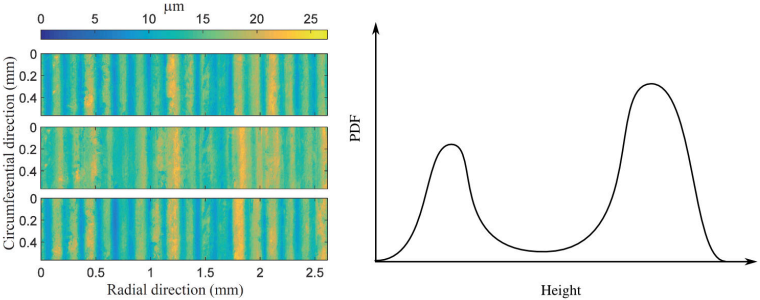

Turned surface topography and a sketch of height PDF
Pérez-Ràfols, Larsson, Almqvist, Tribol Int 94 (2016)

Real rough surfaces

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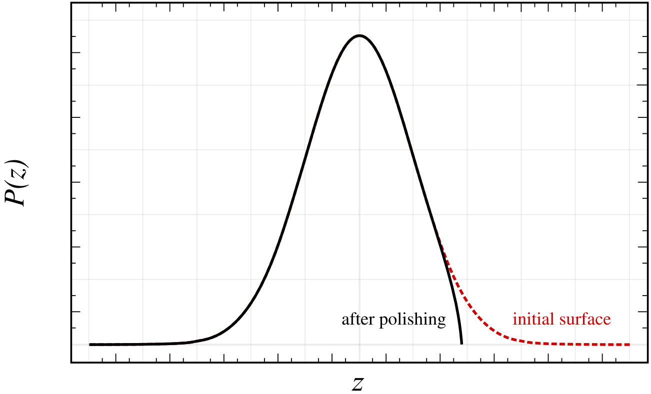

Initial surface with Gaussian PDF and surface after polishing

Real rough surfaces

- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
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Very fresh asphalt concrete


Normal asphalt concrete

Main characteristics: comments on PDF

Real rough surfaces

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Very fresh asphalt concrete

Normal asphalt concrete . . . with a bolt ☺

Main characteristics: comments on PDF

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Old asphalt concrete with worn out bitumen

Real rough surfaces

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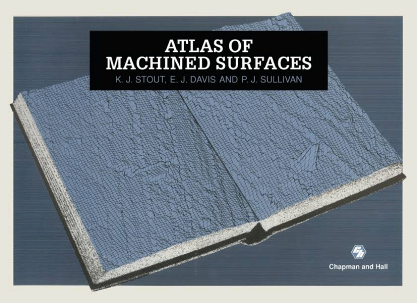


Atlas of machined surfaces (with height distributions)

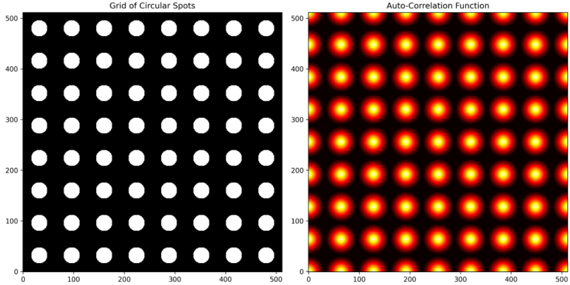
Autocorrelation function

- Continuous autocorrelation function

$$R(\Delta x,\Delta y) = \lim_{L\to\infty}\frac{1}{L^2}\int\limits_0^L\int\limits_0^L z(x+\Delta x, y+\Delta y)z(x,y)\,dxdy$$

- Discrete autocorrelation function for a surface $N\times N$

$$R(\Delta x,\Delta y) = \frac{1}{N^2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} z(x+\Delta x, y+\Delta y)z(x,y)$$



Autocorrelation function

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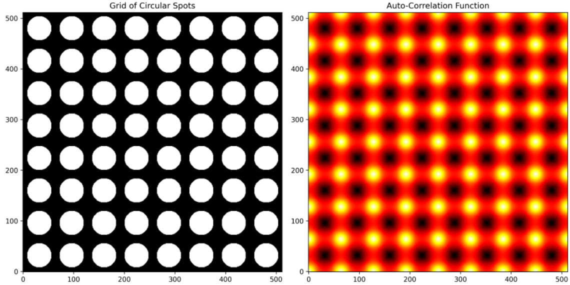

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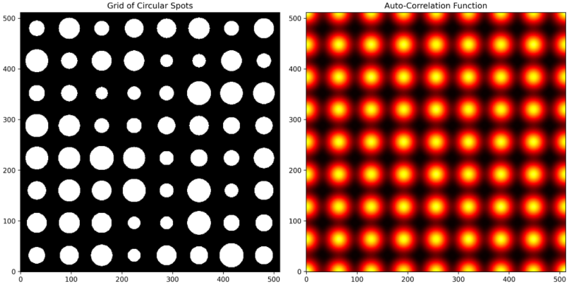

- Continuous autocorrelation function

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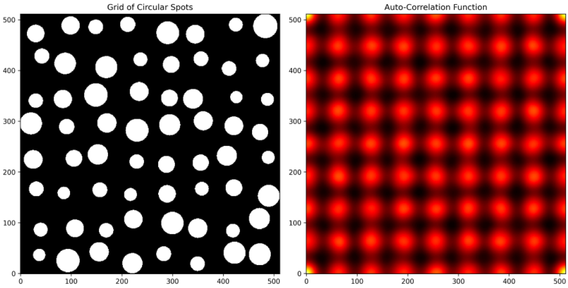

Power spectral density (PSD)

- Recall: Fourier Transform: $\hat{f}(k) = \dfrac{1}{2\pi}\displaystyle\int\limits_{-\infty}^{\infty} f(x)\exp(-ikx)\,dx$
- Recall: Discrete Fourier Transform: $\hat{f}_k = \displaystyle\sum_{n=0}^{N-1} x_n \exp(-ikn/N)$
- where x is the spatial coordinate, $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength.
- PSD is the Fourier Transform of R

  $\Phi(k_x, k_y) \equiv \hat{R}(k_x, k_y) = \mathrm{FFT}\left[z(x+\Delta x, y+\Delta y) * z(x,y)\right]$
- Using convolution theorem

  $\Phi(k_x, k_y) = \hat{z}(k_x, k_y)\,\hat{z}^*(k_x, k_y) = \hat{z}^2(k_x, k_y)$
- Interpretation: energy distribution by frequencies
- Usage: signal analysis, seismology, microstructure characterization, roughness.

Spectral moments

- Spectral moment m_{pq}, $p,q \in \mathbb{N}$:

$$m_{pq} = \iint\limits_{-\infty}^{\infty} k_x^p k_y^q \Phi(k_x,k_y)\, dk_x dk_y$$

- Generalized spectral moment m_{pq}, $p,q \in \mathbb{R}^+$
- For isotropic surface: $m_2 = m_{20} = m_{02}, \quad m_4 = 3m_{22} = m_{40} = m_{04}$
- Averaging:

$$m_2 = \frac{m_{20}+m_{02}}{2}, \quad m_4 = \frac{m_{40}+3m_{22}+m_{04}}{3}$$

- Physical meaning:

Height variance[1]: $m_0 = \left\langle (z - \langle z \rangle)^2 \right\rangle$

Gradient variance: $2m_2 = \left\langle (\nabla z - \langle \nabla z \rangle)^2 \right\rangle$

Curvature variance: $m_4 = \left\langle (\nabla \cdot \nabla z - \langle \nabla \cdot \nabla z \rangle)^2 \right\rangle$

[1] Longuet-Higgins, M. S. (1957). The statistical analysis of a random, moving surface. Phil. Trans. Royal Society of London. Series A 249(966):321-387.
[2] Nayak, P. R. (1971). Random Process Model of Rough Surfaces. Journal of Lubrication Technology, 93(3):398-407

[1] Variance is a squared standard deviation

Summary

- Fractal (self-affine) roughness
- Power spectral density (PSD)
  $$\Phi(k) \sim k^{-2(H+1)}$$
  k is a wavenumber,
  H is the Hurst exponent.
- **Isotropic**/anisotropic surfaces
- **Gaussian**/non-Gaussian height distribution $P(h)$

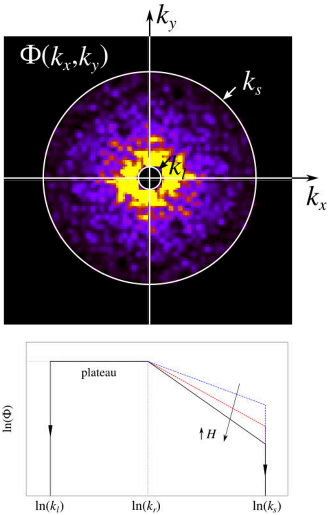

Fig. 3D and radial power spectral densities

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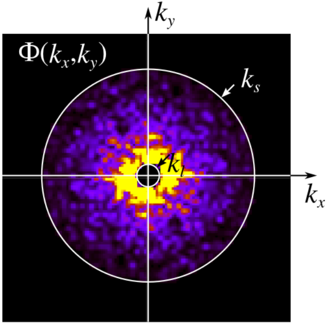

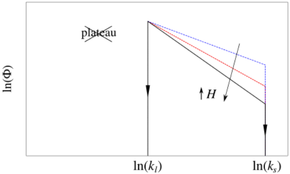

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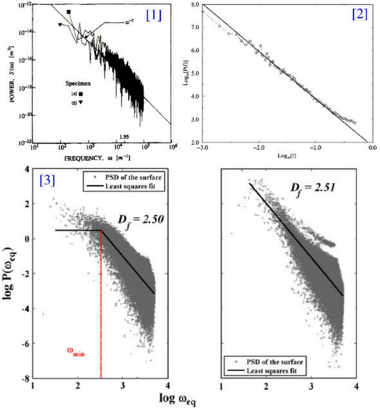

Fig. Power spectral density, measurements

[1] Majumdar, Tien, Wear 136 (1990)
[2] Schmittbuhl, Jørgen Måløy, Phys. Rev. Lett. 78 (1997)
[3] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. 42 (2009)

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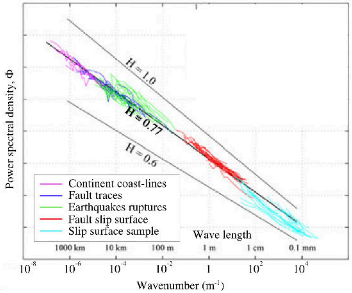

Fig. Power spectral density, geological scales

Adapted from
[4] Renard, Candela, Bouchaud, Geophys. Res. Lett. 40 (2013)

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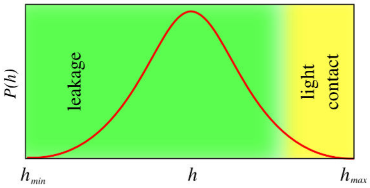

Fig. Height distribution $P(h)$

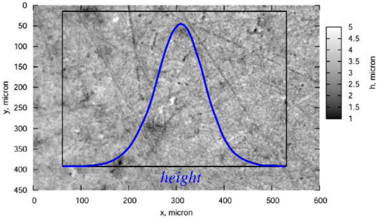

Fig. Height distribution of a polished metal surface

Summary

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- **Isotropic**/anisotropic surfaces
- **Gaussian**/non-Gaussian height distribution $P(h)$
- **Characteristics:**
  - $\sqrt{\langle z^2 \rangle}$ - *rms heights*
  - $\sqrt{\langle |\nabla z|^2 \rangle}$ - *rms slope (surface gradient)*
  - $\alpha = m_{00}m_{40}/m_{20}^2$ - *breadth of the spectrum (Nayak's parameter*[B]*),*

spectral moments $m_{pq} = \iint\limits_{-\infty}^{\infty} k_x^p k_y^q \Phi(k_x, k_y)\, dk_x dk_y$

- **Random process theory**

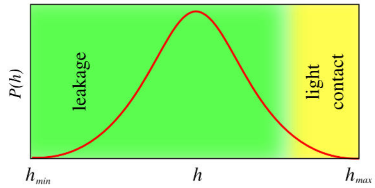

Fig. Height distribution $P(h)$

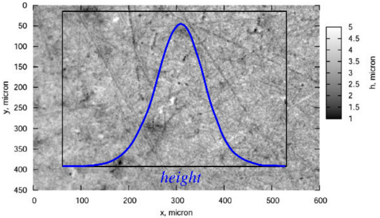

Fig. Height distribution of a polished metal surface

[A] Longuet-Higgins, Philos. Trans. R. Soc. A 250:157 (1957)
[B] Nayak, J. Lub. Tech. (ASME) 93:398 (1973)
[C] Greenwood, Wear 261: 191 (2006)
[D] Borri, Paggi, J. Phys. D Appl Phys 48:045301 (2015)

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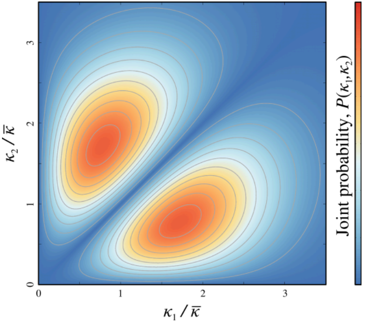

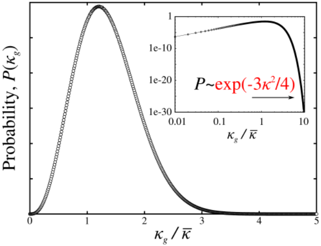

$P \sim \kappa^2 \exp(-3\kappa^2/4)\ \mathrm{erf}(3\kappa/2)$

Distribution of asperity curvatures

[A] Longuet-Higgins, Philos. Trans. R. Soc. A 250:157 (1957)
[B] Nayak, J. Lub. Tech. (ASME) 93:398 (1973)
[C] Greenwood, Wear 261: 191 (2006)
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Summary

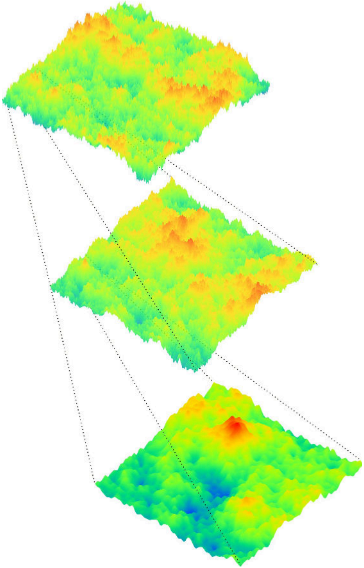


Fig. Example of a rough surface for $H = 0.3$

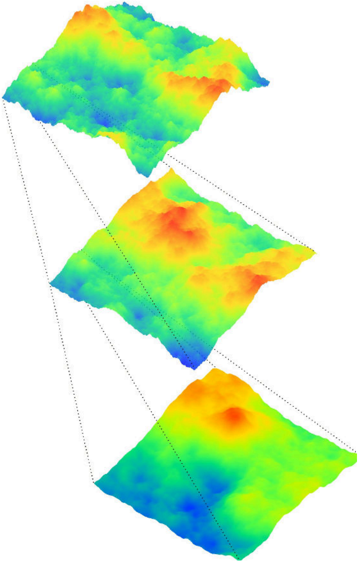
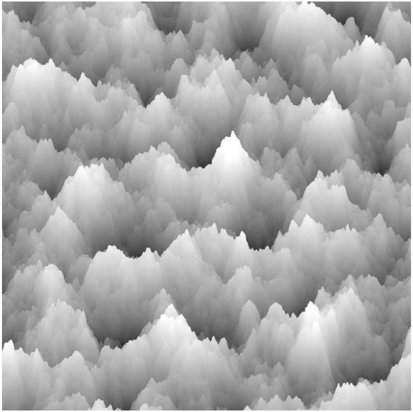


Fig. Example of a rough surface for $H = 0.8$

Recall: the Hurst exponent H and the fractal dimension D in 2D space are interconnected via $D = 3 - H$



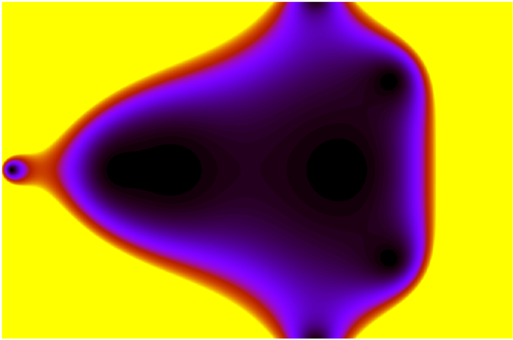
Flight over a rough surface

Romanesco broccoli

Fractals

- Mandelbrot set (not a fractal)
- Recursive function

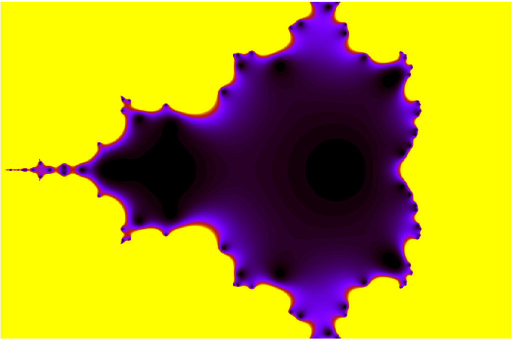
$$z_{i+1} = z_i^2 + z, \quad z \in \mathbb{C}$$



Fractals

- Mandelbrot set (not a fractal)
- Recursive function

$$z_{i+1} = z_i^2 + z, \quad z \in \mathbb{C}$$



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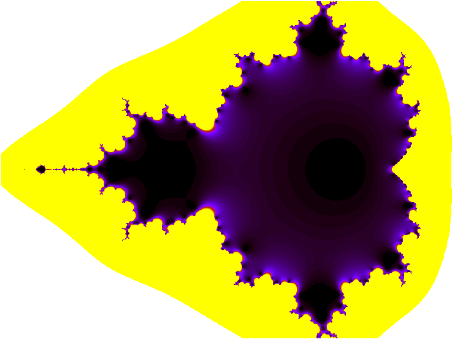

- Mandelbrot set (not a fractal)
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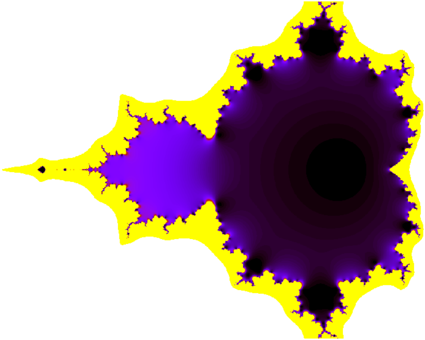

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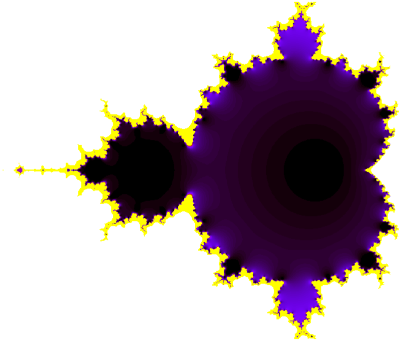

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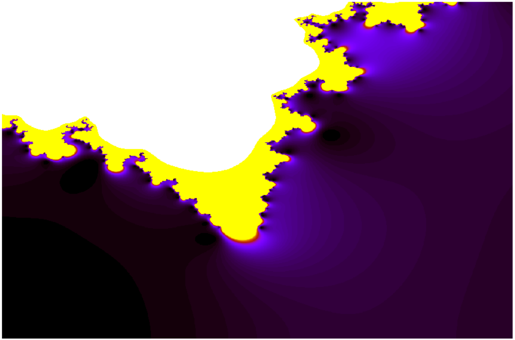

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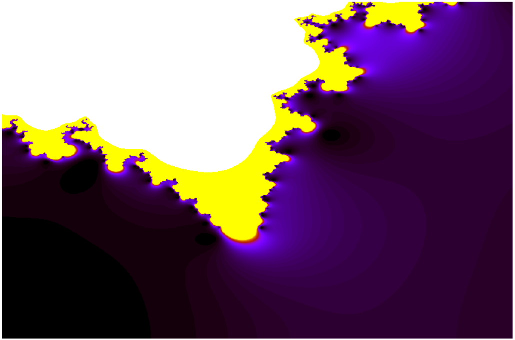
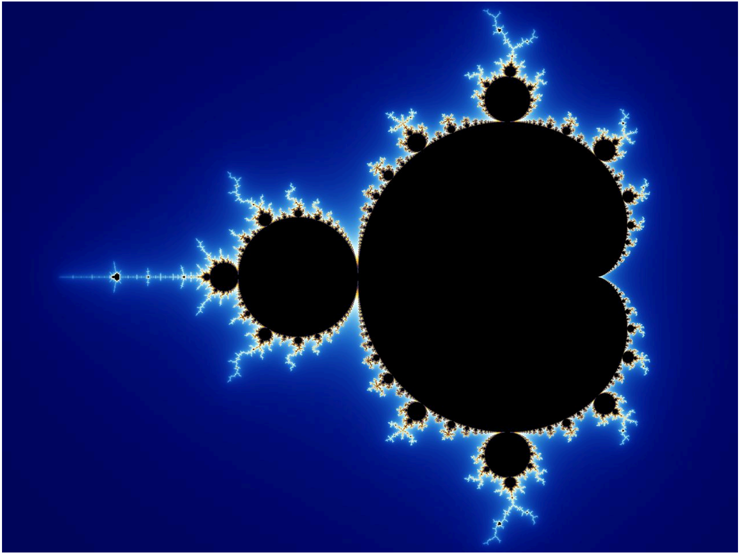
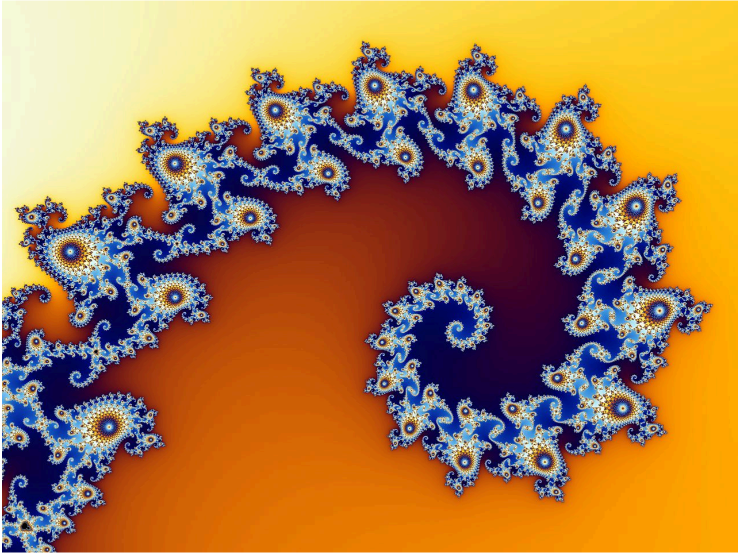


Fractals



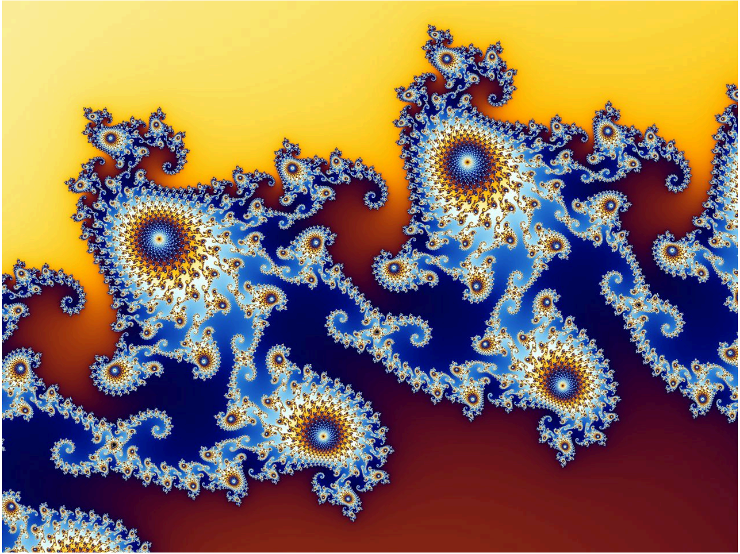
Mandelbrot set (Wikipedia)

Fractals

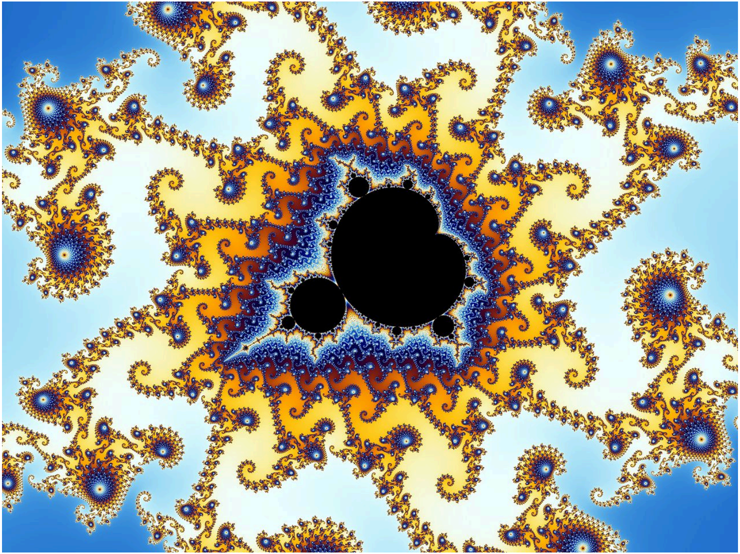


Element of Mandelbrot set (Wikipedia)

Fractals

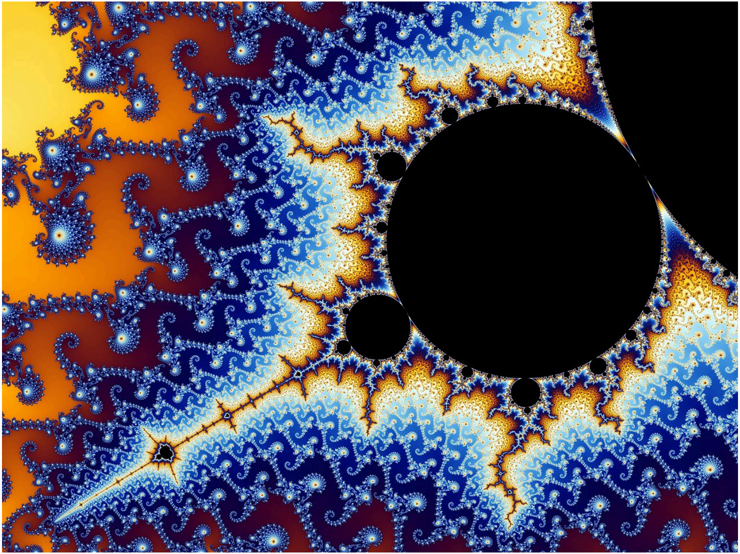


Element of Mandelbrot set (Wikipedia)

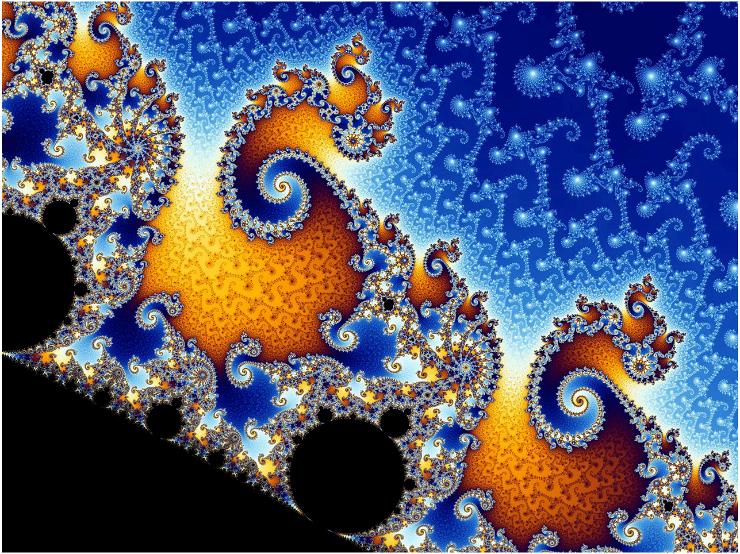


Element of Mandelbrot set (Wikipedia)

Fractals

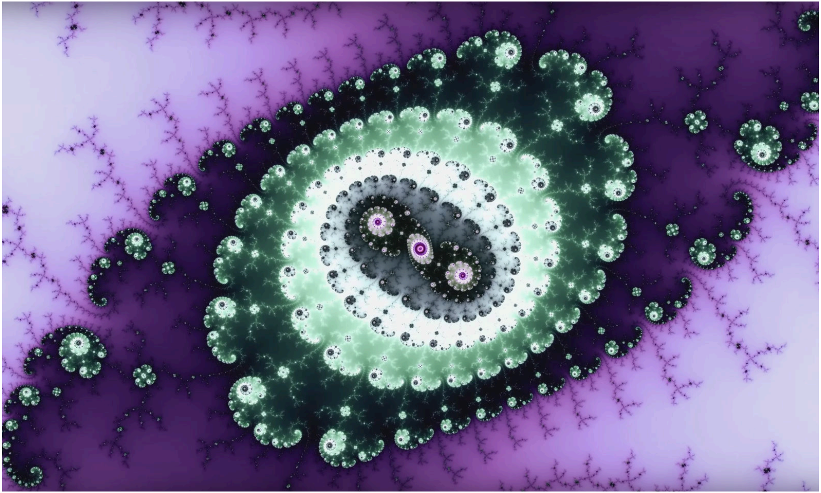


Element of Mandelbrot set (Wikipedia)



Element of Mandelbrot set (Wikipedia)

See animation

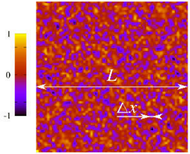


https://youtu.be/pCpLWbHVNhk

70 minutes of 4K "Eye of the Universe - Mandelbrot Fractal Zoom" e^{1091}

Synthesized rough surfaces: in pictures

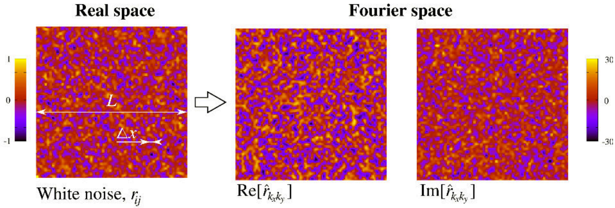
Real space **Fourier space**



White noise, r_{ij}

[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Synthesized rough surfaces: in pictures



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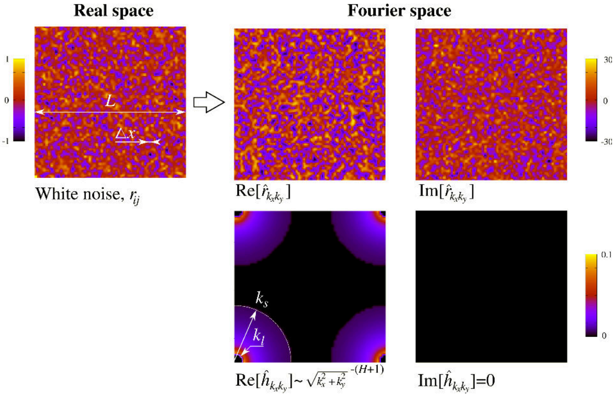

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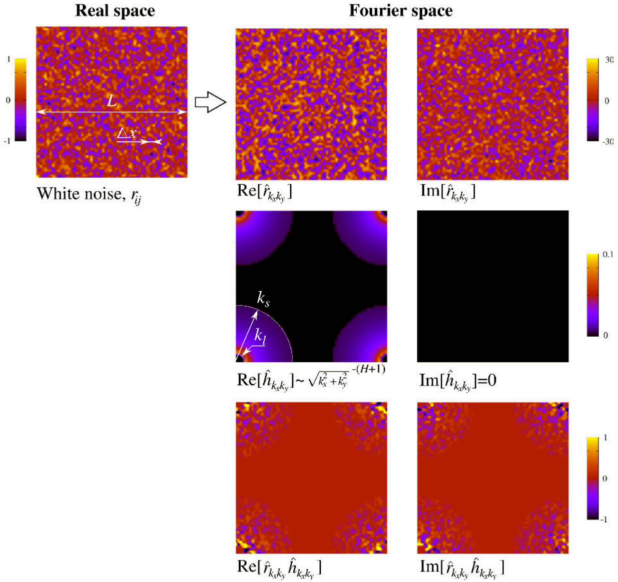
Synthesized rough surfaces: in pictures



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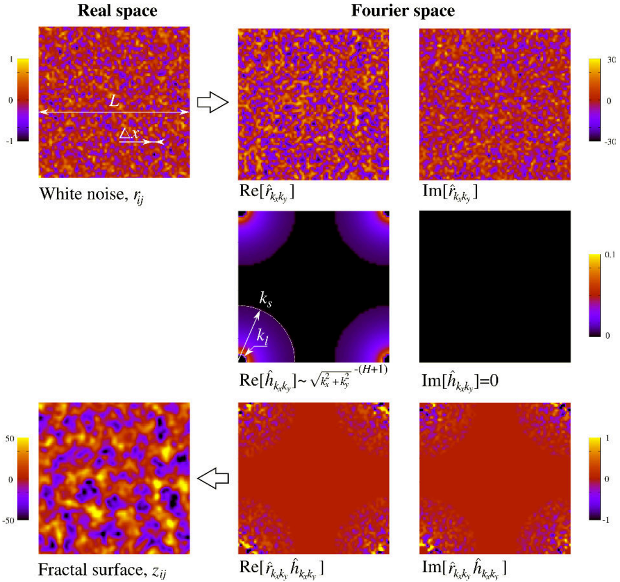
Synthesized rough surfaces: in pictures



[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Synthesized rough surfaces: in equations

- White noise:
  $w(x_i, y_j), \quad \langle w \rangle = 0, \quad \langle w^2 \rangle = \Phi_0$
- Transform in Fourier space:
  $$\hat{w}_{ij} = \hat{w}(k_x, k_y) = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} w(x_i, y_i) \exp[-\mathrm{i}(k_x x_i + k_y y_j)], \quad \langle \hat{w}\hat{w}^* \rangle = \langle w^2 \rangle = \Phi_0$$
- Create a filter
  $$\hat{f}_{ij} = \hat{f}(k_x, k_y) = \begin{cases} \left[\frac{K_x^2+K_y^2}{k_l^2}\right]^{-(1+H)/2}, & \text{for } 1 \le \frac{\sqrt{K_x^2+K_y^2}}{k_l} \le \zeta \\ 0, & \text{elsewhere}, \end{cases},$$
  where $K_x = (s+1)\pi/L - sk_x$, $K_y = (t+1)\pi/L - tk_y$ for $s, t \in \{-1, 1\}$, $\zeta = k_s/k_l$
- Filter white noise:
  $$\hat{z}_{ij} = \hat{z}(k_x, k_y) = \Re(\hat{f}_{ij})\left[\Re(\hat{w}_{ij}) + \mathrm{i}\,\Im(\hat{w}_{ij})\right]$$
- Back to real space:
  $$z(x_i, y_j) = \sum_{l=0}^{N-1}\sum_{m=0}^{N-1} \hat{z}_{lm} \exp[\mathrm{i}2\pi(lx_i + my_j)/L]$$

Synthesized rough surfaces: in equations II

- Power spectral density:
  $\Phi(k_x,k_y) = \hat{z}(k_x,k_y)\hat{z}^*(k_x,k_y) = \hat{f}^2(k_x,k_y)\hat{w}^2(k_x,k_y)$
- Averaging over multiple samples:

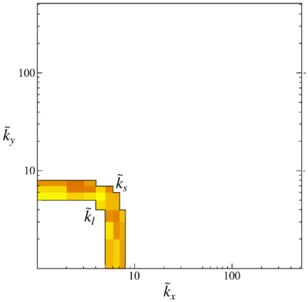
$$\langle\Phi(k_x,k_y)\rangle = \langle\hat{w}^2(k_x,k_y)\rangle\hat{f}^2(k_x,k_y) = \begin{cases}\Phi_0\left[\dfrac{\sqrt{K_x^2+K_y^2}}{k_l}\right]^{-2(1+H)}, \text{ for } 1 \le \dfrac{\sqrt{K_x^2+K_y^2}}{k_l} \le \zeta\\ 0, \text{ elsewhere,}\end{cases}$$

- For isotropic surface:

$$\langle\Phi(K)\rangle = \begin{cases}\Phi_0(K/k_l)^{-2(1+H)}, & \text{if } 1 \le K/k_l \le \zeta\\ 0, & \text{otherwise.}\end{cases}$$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



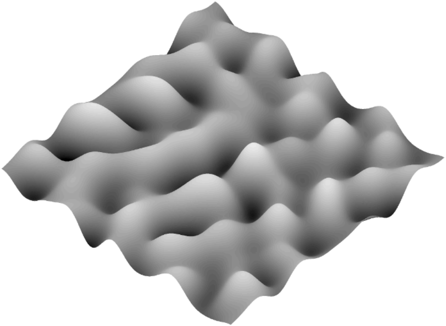
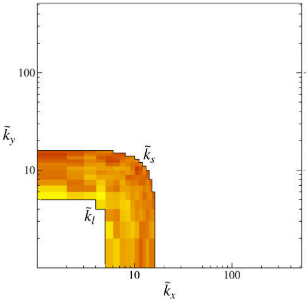


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 8$

- Effect of the high frequency cutoff k_s



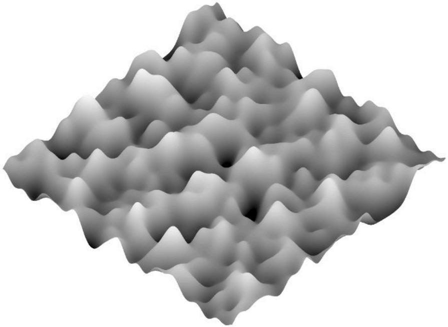


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 16$

- Effect of the high frequency cutoff k_s

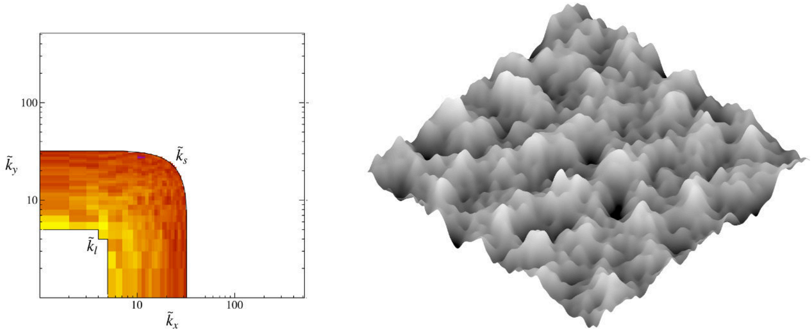


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 32$

- Effect of the high frequency cutoff k_s

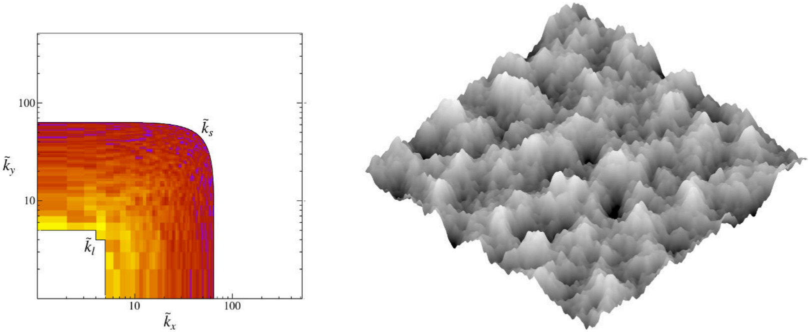
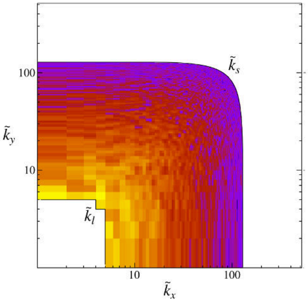


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 64$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



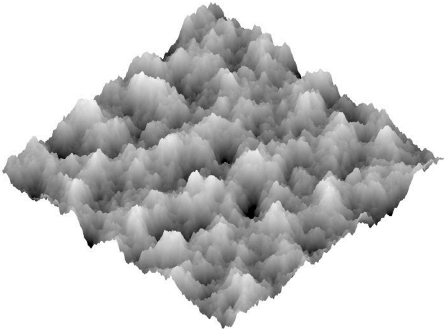


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 128$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$

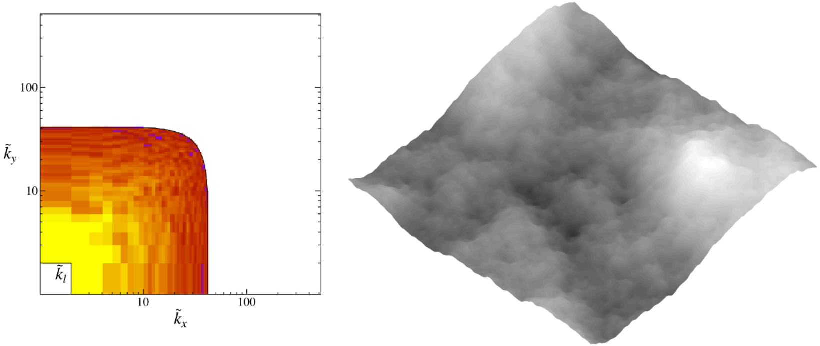
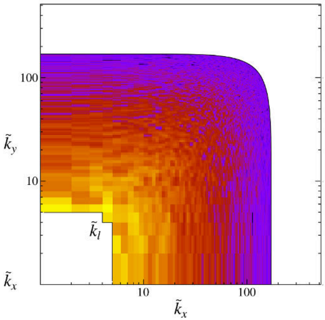


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 1, \quad k_s = 43$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$



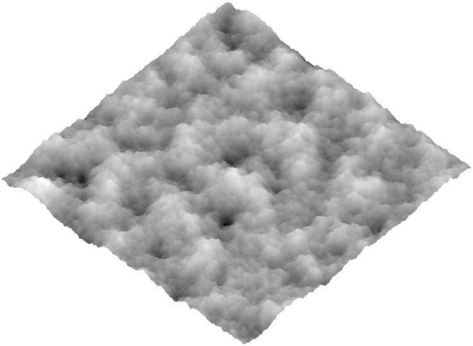
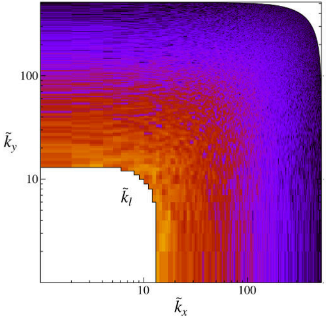


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 171$

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$



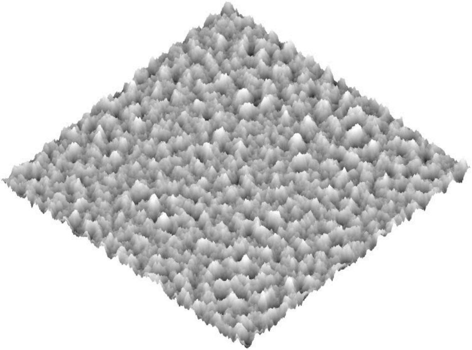


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 12, \quad k_s = 512$

- Effect of the ratio of the higher cutoff to the discretization k_s/N

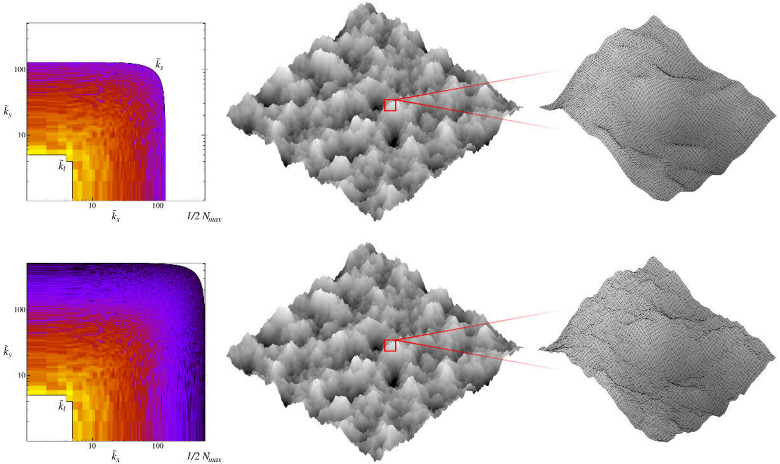


Fig. Power spectral densities (Fourier space)
and corresponding rough surfaces (real space) for
$k_l = 12, \quad k_s/N = 1/8$ **VS** $k_l = 12, \quad k_s/N = 1/2$

Effect of parameters: illustration

- Effect of the ratio of the higher cutoff to the discretization k_s/N

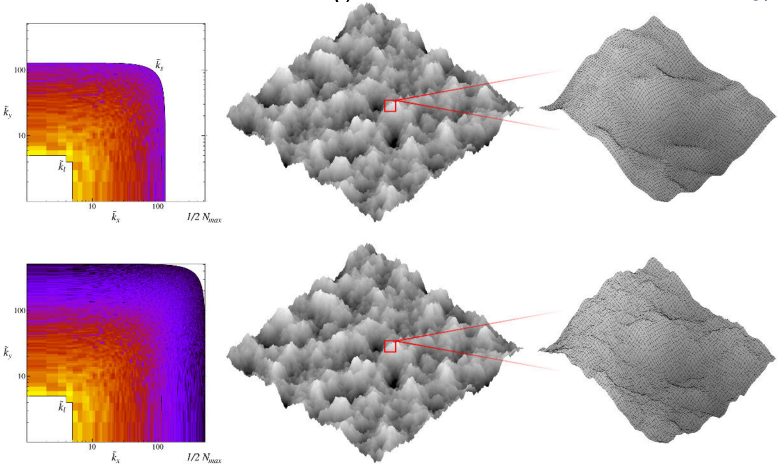


Fig. Power spectral densities (Fourier space)
and corresponding rough surfaces (real space) for
$k_l = 12, \quad k_s/N = 1/8$ **(fine)** **VS** $k_l = 12, \quad k_s/N = 1/2$ **(too coarse)**
for mechanical simulations

- Effect of the discretisation (single asperity)

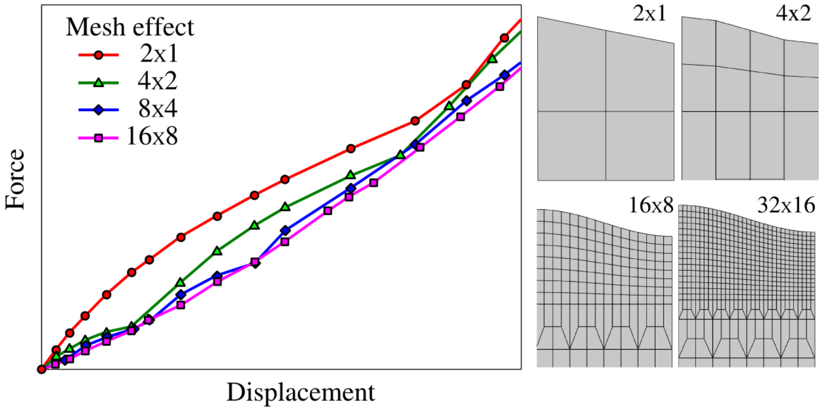


Fig. Effect of the mesh on mechanical response

Effect of parameters: illustration

- Data interpolation (Shanon, bi-cubic Bézier surfaces)

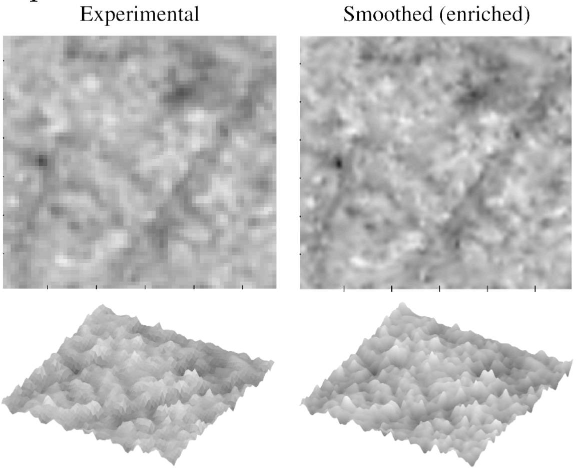


Fig. Bi-cubic Bézier interpolation of an experimental rough surface

[1]Hyun, Robbins, Tribol. Int. (2007)
[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)

Effect of parameters:

- k_l low frequency cutoff
  - representativity/normality[1,2,3]
- k_s high frequency cutoff
  - smoothness and density of asperities
- $\zeta = k_s/k_l$ ratio[3]
  - breadth of the spectrum

$$\boxed{\alpha \sim \zeta^{2H}}$$

Nayak's parameter α is the central characteristic of roughness in asperity based mechanical models.

[1] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. (2009)
[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)
[3] Yastrebov, Anciaux, Molinari, Phys. Rev. E (2012)
[4] Yastrebov, Anciaux, Molinari, Int. J. Solids Struct. (2015)

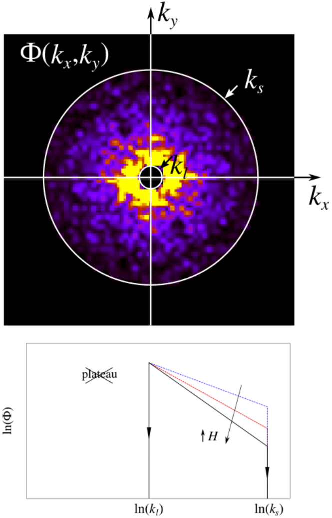


Fig. 3D and radial power spectral densities

Effect of parameters:

- k_l low frequency cutoff
  - representativity/normality[1,2,3]
- k_s high frequency cutoff
  - smoothness and density of asperities
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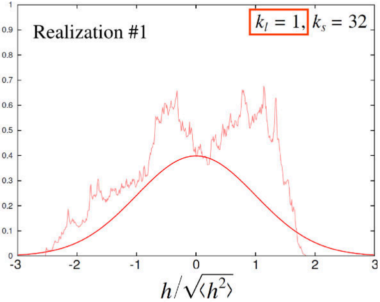

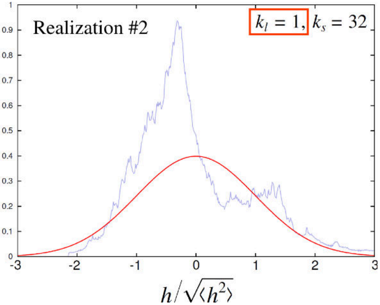

[1] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. (2009)
[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)
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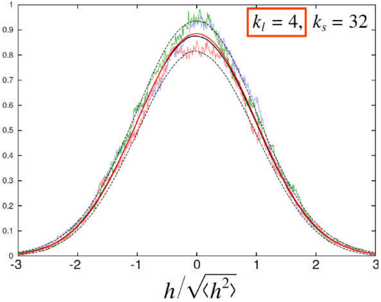
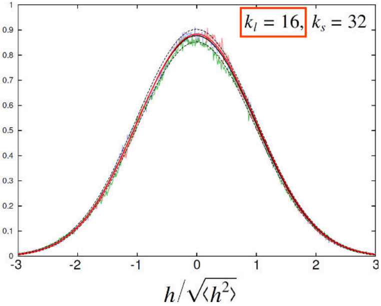




Interconnection of parameters

- Spectral moment and k_l, k_s, H:

$$m_{0p} \approx m_{p0} \approx \Phi_0 \int\limits_{k_l}^{k_s} \int\limits_0^{2\pi} [k\cos(\varphi)]^p \, (k/k_l)^{-2(1+H)} \, k dk d\varphi = \Phi_0 k_l^{p+2} \frac{\zeta^{p-2H}-1}{p-2H} T(p)$$

$$\text{with } \; T(p) = \int\limits_0^{2\pi} \cos^p(\varphi) d\varphi = \begin{cases} 2\pi, & \text{if } p = 0; \\ \pi, & \text{if } p = 2; \\ 3\pi/4, & \text{if } p = 4. \end{cases}$$

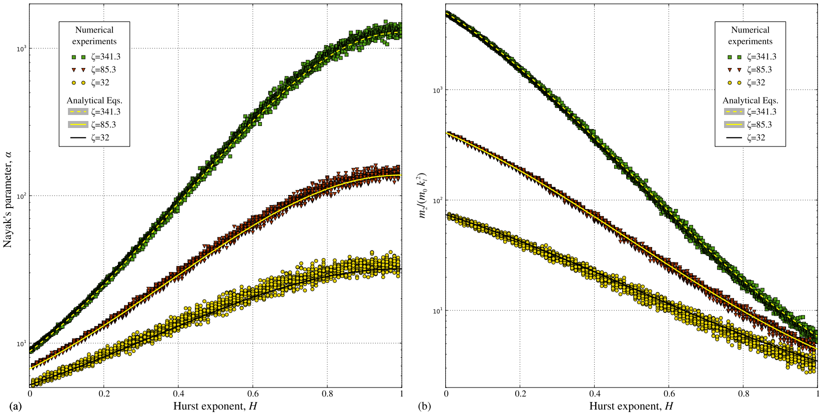
- Nayak's parameter

$$\alpha(H,\zeta) = \frac{3}{2} \frac{(1-H)^2}{H(H-2)} \frac{(\zeta^{-2H}-1)(\zeta^{4-2H}-1)}{(\zeta^{2-2H}-1)^2}$$

- Asperity density

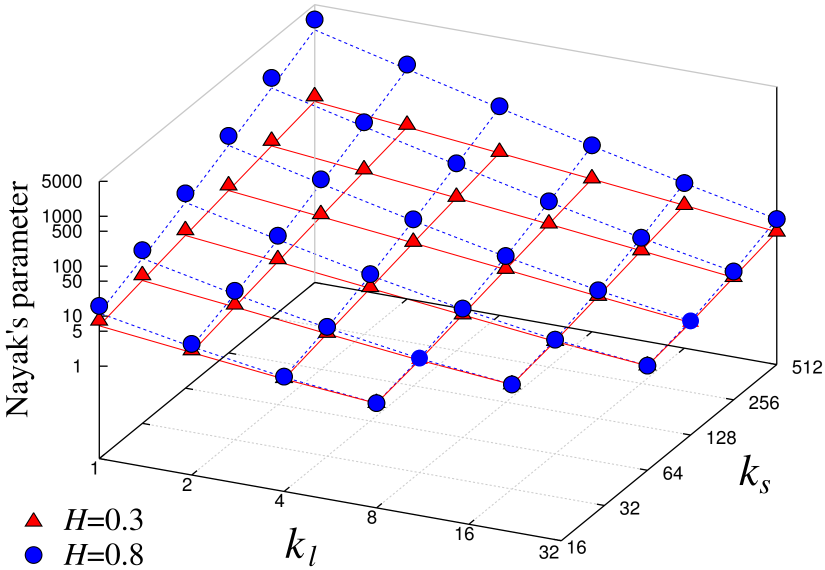
$$D = \frac{\sqrt{3}}{18\pi} \frac{m_4}{m_2} = \frac{\sqrt{3}}{24\pi} \frac{1-H}{2-H} \frac{\zeta^{4-2H}-1}{\zeta^{2-2H}-1} k_l^2$$

Interconnection of parameters



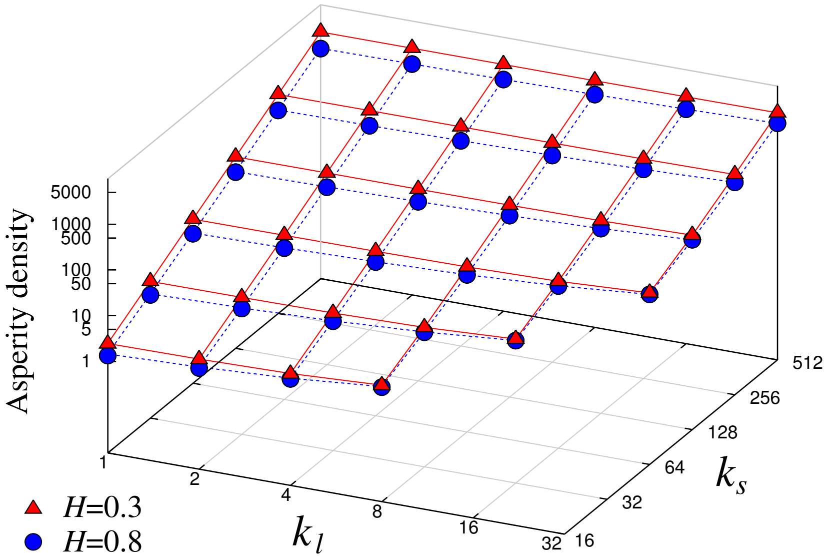
Numerical verification on 100 000 generated rough surfaces with 2048×2048 points

Interconnection of parameters



Nayak's parameter test

Interconnection of parameters



Asperity density test

Asperity analysis

- Detect summits (z_{ij} higher than neighbouring points) and evaluate second derivatives

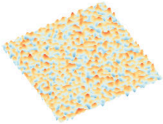
$$\frac{\partial^2 z}{\partial x^2} = \frac{z_{i+1j} + z_{i-1j} - 2z_{ij}}{2\Delta x^2}; \qquad \frac{\partial^2 z}{\partial y^2} = \frac{z_{i+1j} + z_{i-1j} - 2z_{ij}}{2\Delta x^2}$$

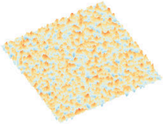
$$\frac{\partial^2 z}{\partial x \partial y} = \frac{z_{i+1j+1} + z_{i+1j+1} - z_{i+1j-1} - z_{i-1j+1}}{4\Delta x^2}$$

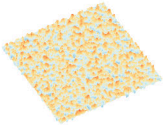
- Principal curvatures $\kappa_{1,2}$:

$$\kappa_{1,2} = \frac{1}{2}\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) \pm \sqrt{\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 + \frac{1}{4}\left(\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2}\right)^2}$$

- Saddle point $\kappa_1 \kappa_2 < 0$, extrema $\kappa_1 \kappa_2 > 0$
- Mean curvature which can be safely used in Hertz theory: $\bar{\kappa} = \sqrt{\kappa_1 \kappa_2}$

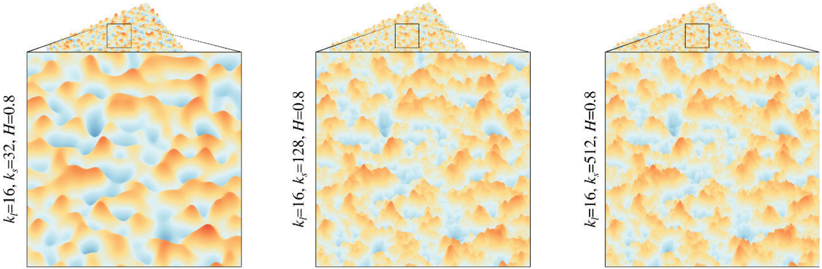




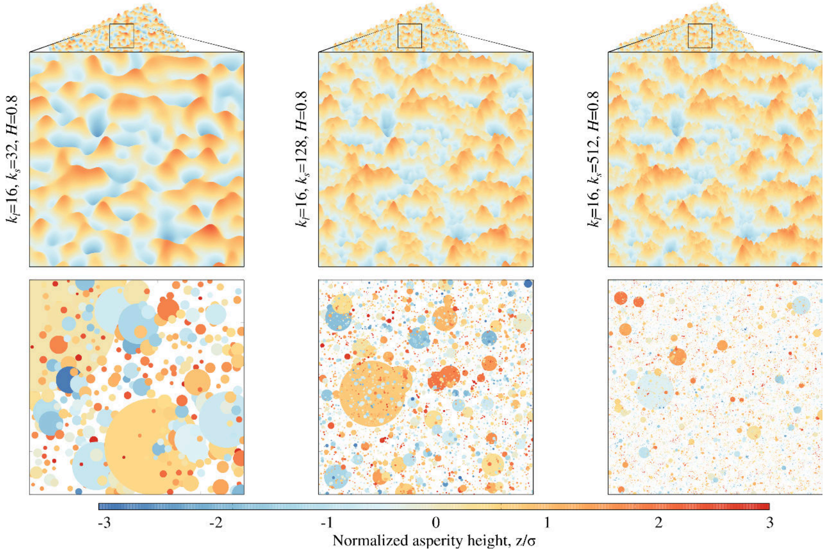


Rough surfaces and associated asperities

Asperity analysis

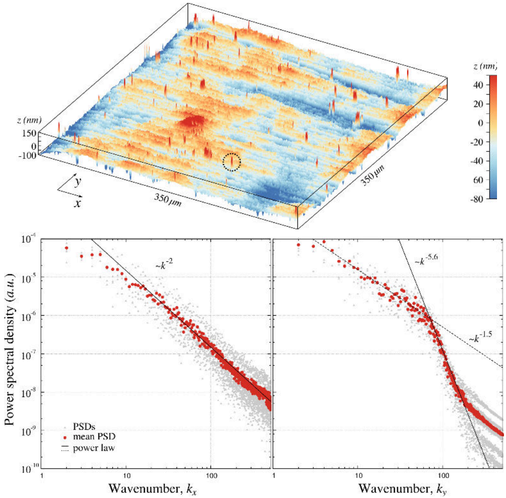


Rough surfaces and associated asperities



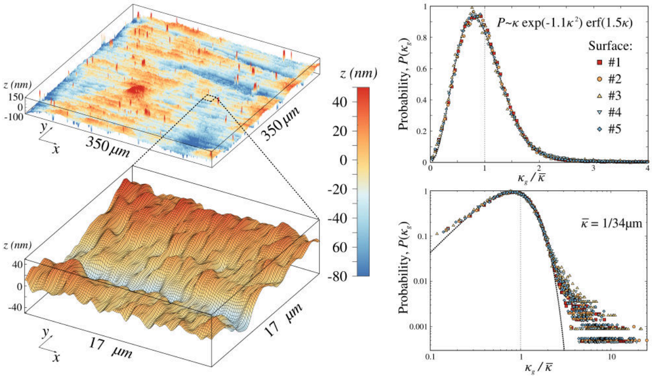
Rough surfaces and associated asperities

Examples



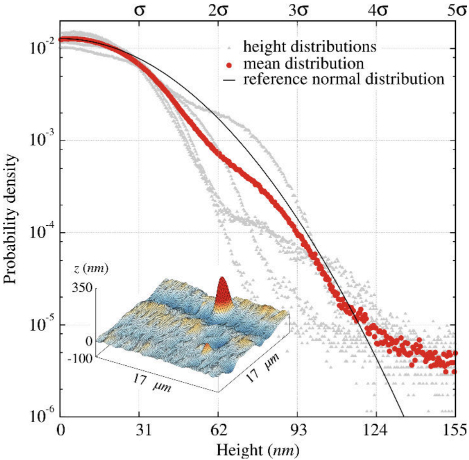
[1] Yastrebov et al, Three-level multi-scale modeling of electrical contacts sensitivity study and experimental validation, Proceedings of Holm Conference, 2015.

Examples



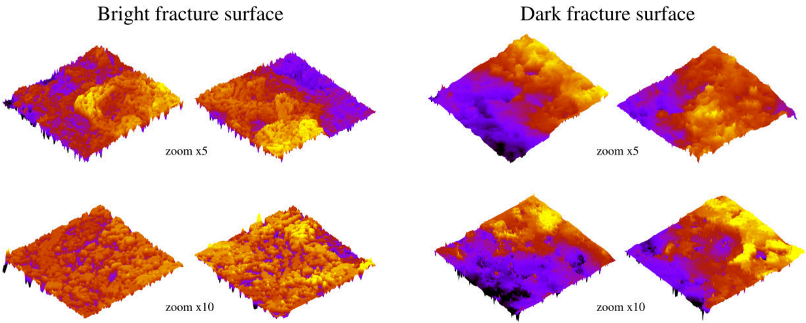
Asperity curvatures

Examples

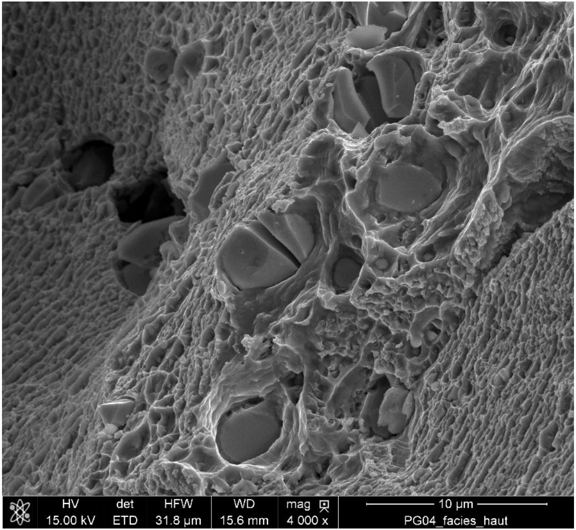


[1] Yastrebov et al, Three-level multi-scale modeling of electrical contacts sensitivity study and experimental validation, Proceedings of Holm Conference, 2015.

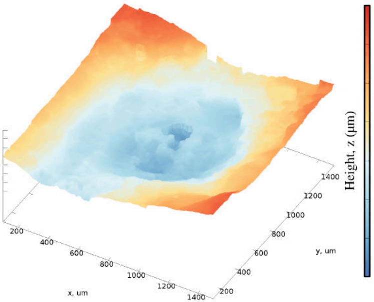
Examples

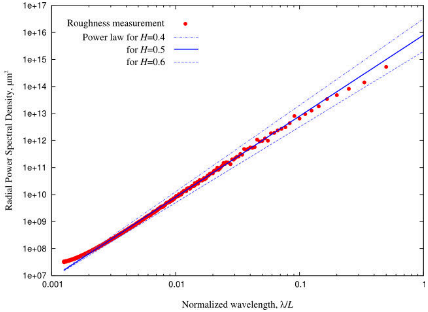


Fatigue & creep fracture surfaces (Ti-alloy)
in collaboration with A. Marchenko



Fatigue fracture surface (Co-alloy), particles WC
Courtesy of V. Esin



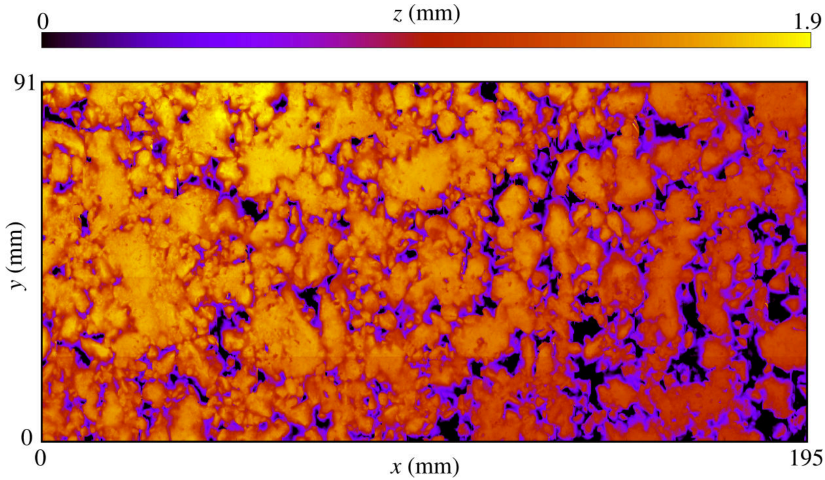


Crater topography and PSD

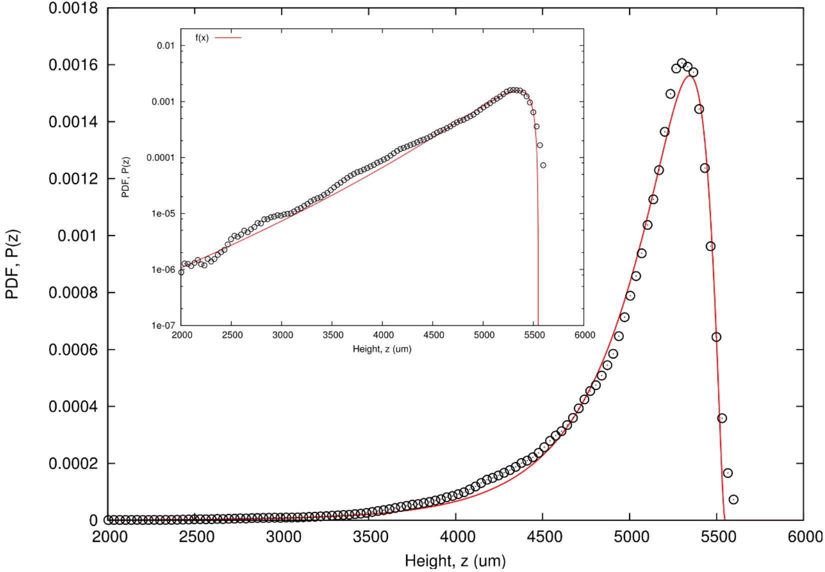
In collaboration with D. Tkalich (NTNU, Sintef)

Multiscale road roughness

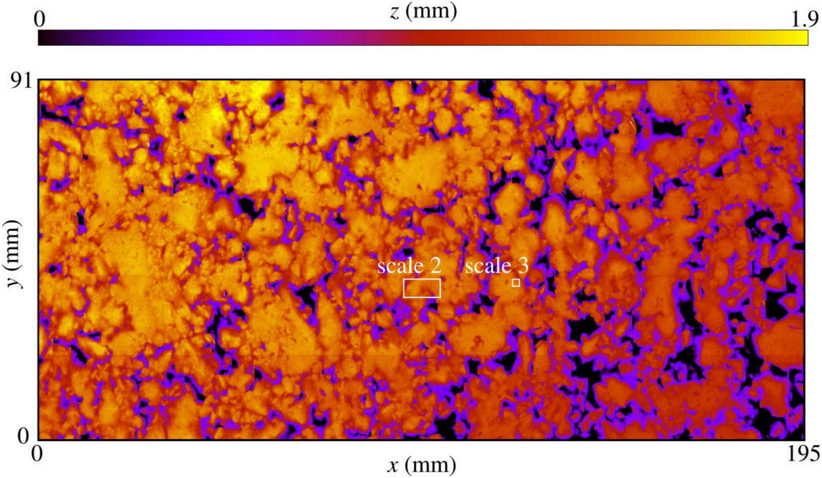
- Scale 1 (x1)



- Scale 1 (x1): PDF and generalized Weibull fit

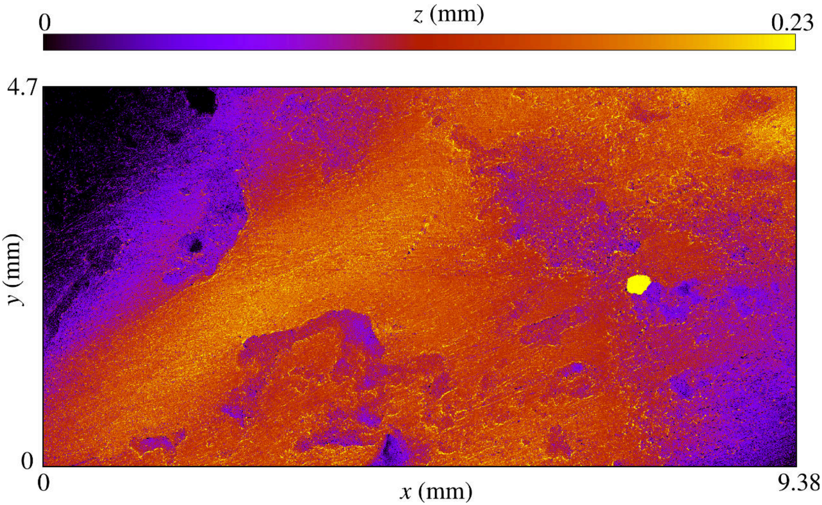


- Scale 1 (x1)

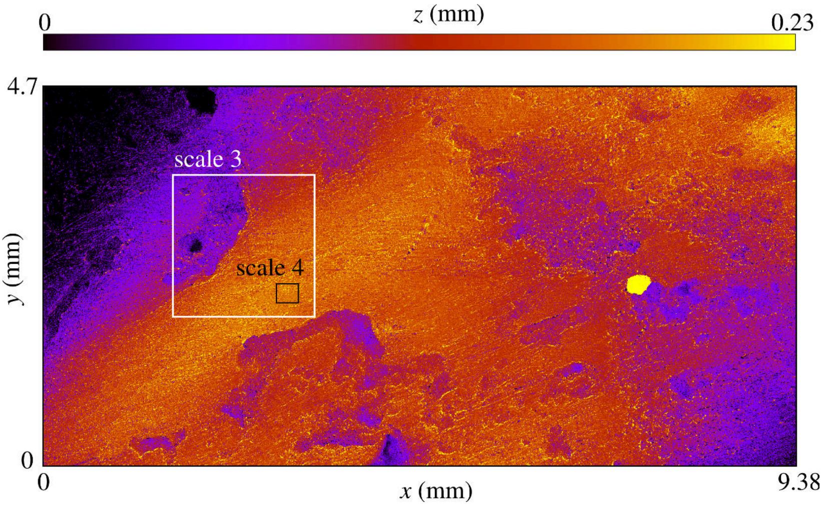


Multiscale road roughness

- Scale 2 (x5)

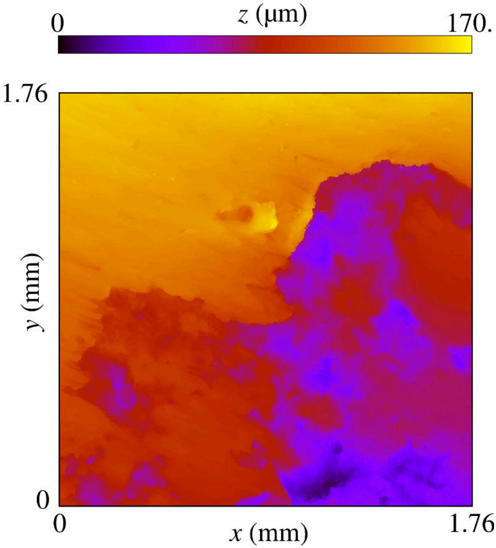


- Scale 2 (x5)



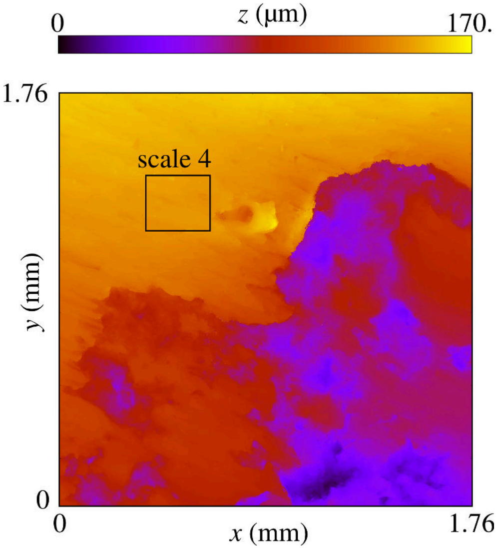
Multiscale road roughness

- Scale 3 (x20)



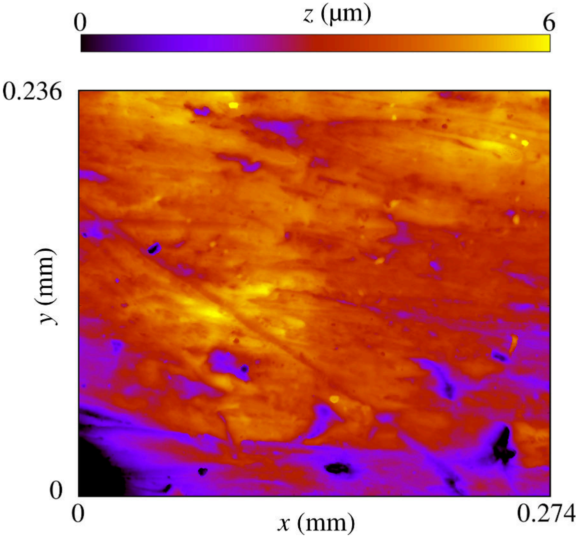
Multiscale road roughness

- Scale 3 (x20)

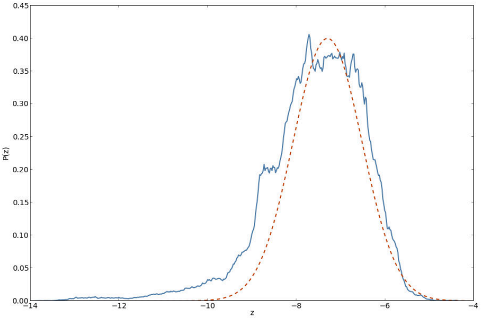


Multiscale road roughness

- Scale 4 (x100)



- Scale 4 (x100): PDF



What did you learn?

- Idea about the interface physics complexity
- How to measure roughness: confocal, WLI, AFM, STM
- What are the main characteristics: PDF, PSD, S_q, S_{dq}, Hurst, Nayak's parameter, etc
- Model roughness
- Fractal aspect & Mandelbrot set
- Real roughness

☺ Thank you for your attention!