Contact mechanics and elements of tribology

Lecture 4
Contact at small scales

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@ Centre des Matériaux (& virtually)
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Outline

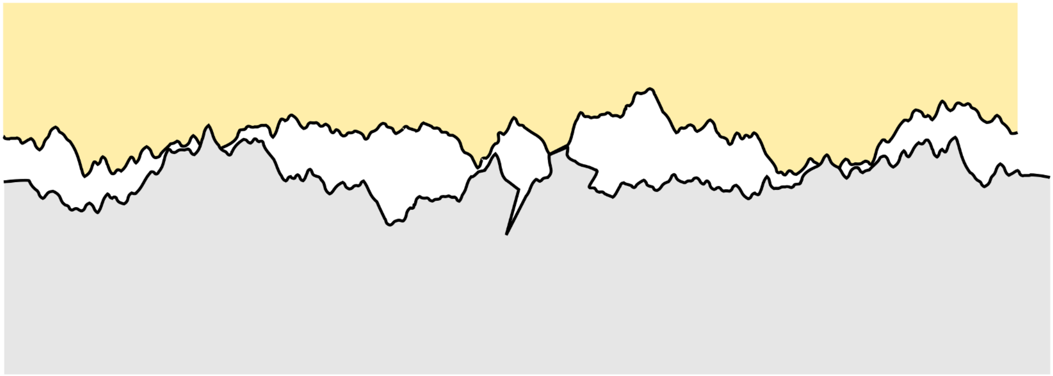
Objective:

link **roughness** parameters with the evolution of the true **contact area**

Contact between rough surfaces

Contact under microscope

Contact under microscope

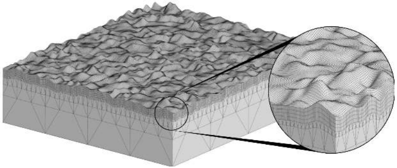


Problem

- Solve contact problem for two elastic half-spaces E_1, ν_1 and E_2, ν_2
- With surface roughnesses $z_1(x,y)$ and $z_2(x,y)$
- Balance of momentum $\nabla \cdot \underline{\underline{\sigma}} = 0$,
- Boundary conditions $-\sigma_z^\infty = p_0$
- Contact constraints $g \geq 0, \quad p \geq 0, \quad g\,p = 0$,
  where $g(x,y)$ is the gap between surfaces,
  $p = -\underline{n} \cdot \underline{\underline{\sigma}} \cdot \underline{n}$ is the contact pressure.

Methods

- Finite element method

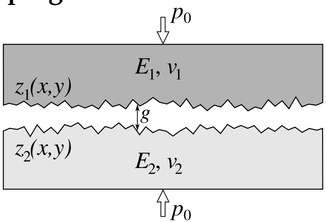


[1] Yastrebov, Wiley/ISTE (2013)

- Boundary element method

[2] Stanley & Kato, J Tribol 119 (1997)

Problem mapping



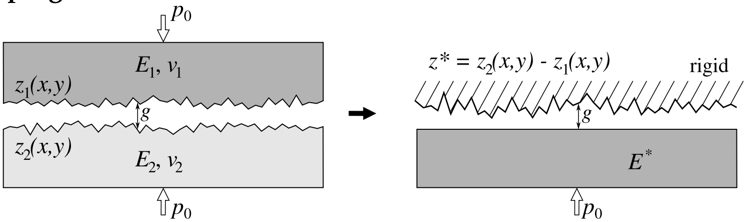
- **Flat** elastic[1] half-space with $E^* = \dfrac{E_1 E_2}{E_2(1-\nu_1^2) + E_1(1-\nu_2^2)}$
- **Rough** rigid[1] surface with $z^* = z_2 - z_1$
- Optimization problem[2]: $\min \mathcal{F}$

  under constraints $p \ge 0$ and $\frac{1}{A_0}\int\limits_A p dA = p_0$,

  with $\mathcal{F} = \int\limits_A p[u_z/2 + g]dA$, vertical displacement u_z and gaps g

[1] Barber, Bounds on the electrical resistance between contacting elastic rough bodies, PRSL A 459 (2003)
[2] Kalker, Variational Principles of Contact Elastostatics, J Inst Maths Applics (1977)

Problem mapping



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Analytical models

Multi-asperity models

[1] Greenwood, Williamson. *P Roy Soc Lond A Mat* (1966)
[2] Bush, Gibson, Thomas. *Wear* (1975)
[3] Mc Cool. *Wear* (1986)
[4] Thomas. *Rough Surfaces* (1999)
[5★] Greenwood. *Wear* (2006)
[6] Carbone. *J. Mech. Phys. Solids* (2009)
[7] Ciavarella, Greenwood, Paggi. *Wear* (2008)

Persson's model

[8] Persson. *J. Chem. Phys.* (2001)
[9] Persson. *Phys. Rev. Lett.* (2001)
[10] Persson, Bucher, Chiaia. *Phys. Rev. B* (2002)
[11] Müser. *Phys. Rev. Lett.* (2008)

Cross-link studies

[12] Manners, Greenwood. *Wear* (2006)
[13] Carbone, Bottiglione. *J. Mech. Phys. Solids* (2008)
[14] Paggi, Ciavarella. *Wear* (2010)

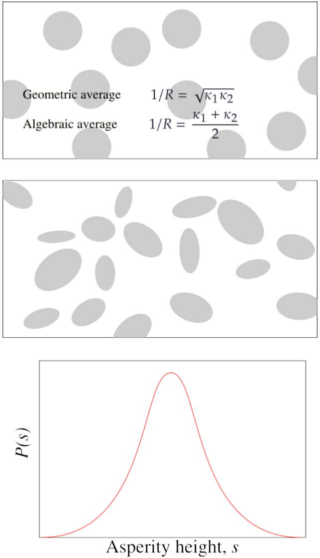

Fig. Multi-asperity models

Analytical models

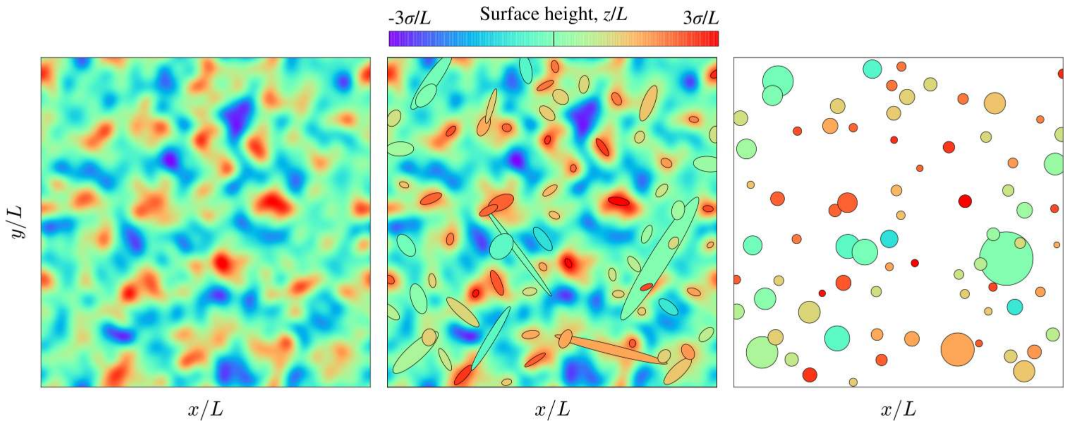

Fig. Roughness and detected asperities for $L/\lambda_l = 4$ and $L/\lambda_s = 16$

Analytical models

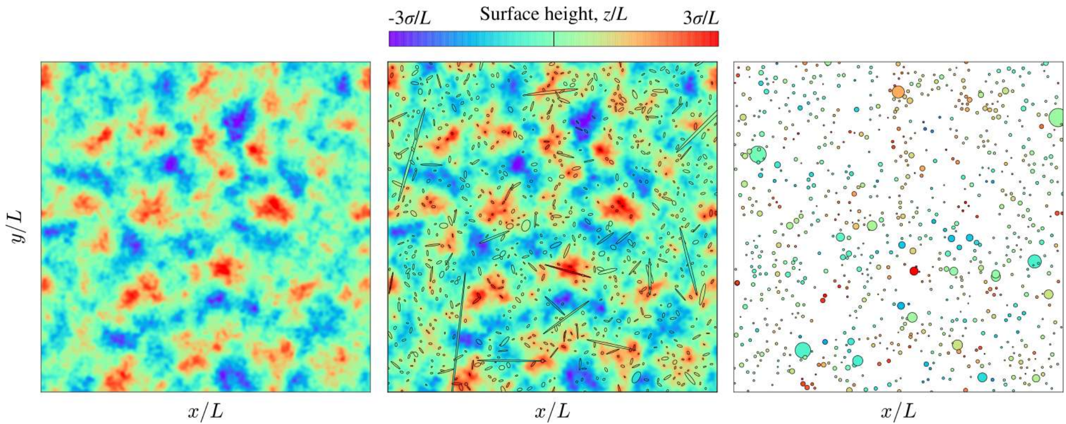

Fig. Roughness and detected asperities for $L/\lambda_l = 4$ and $L/\lambda_s = 64$

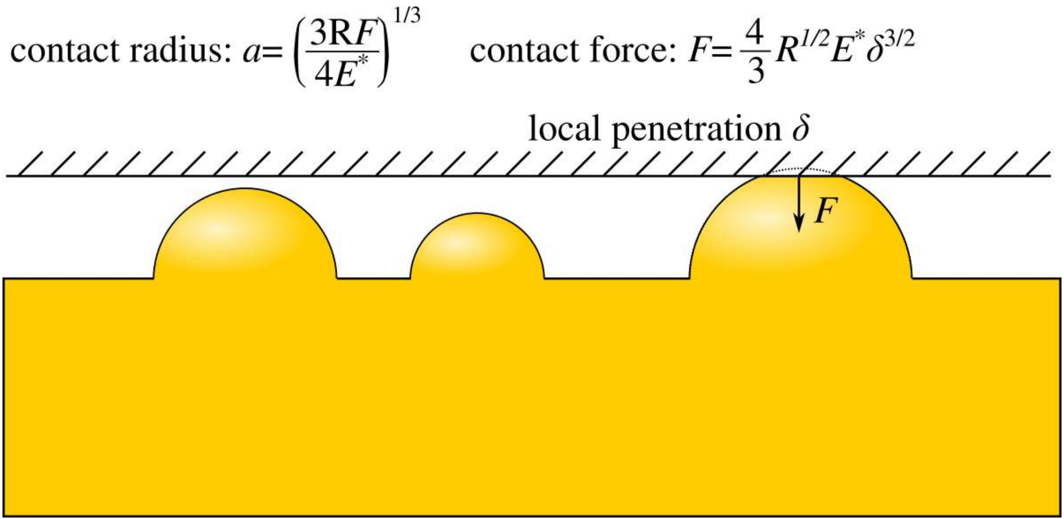

Fig. Hertz's theory of contact

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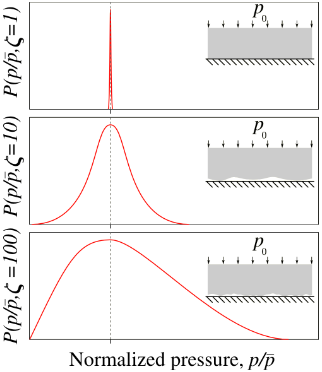


Fig. Persson's model

$$\frac{\partial P(p,\zeta)}{\partial V(\zeta)} = \frac{1}{2}\frac{\partial^2 P(p,\zeta)}{\partial p^2} \qquad P(0,\zeta) = 0$$

$$V(\zeta) = \frac{1}{2}E^* m_2(\zeta) = \frac{\pi E^*}{2}\int\limits_{k_l}^{\zeta k_l} k^3 \Phi^p(k)dk$$

Comparison of models

Multi-asperity models

Person's model

1. Evolution of the real contact area $A(p_0)$ for $A/A_0 \to 0$

$$\frac{A}{A_0} = \frac{\kappa}{\sqrt{\langle|\nabla z|^2\rangle}} \frac{p_0}{E^*}$$

$\kappa_{BGT} = \sqrt{2\pi} \approx 2.5$ *according to* [2-5]

$\kappa_P = \sqrt{8/\pi} \approx 1.6$ *according to* [6-7]

2. Evolution of the real contact area $A(p_0)$ for $\forall A/A_0$

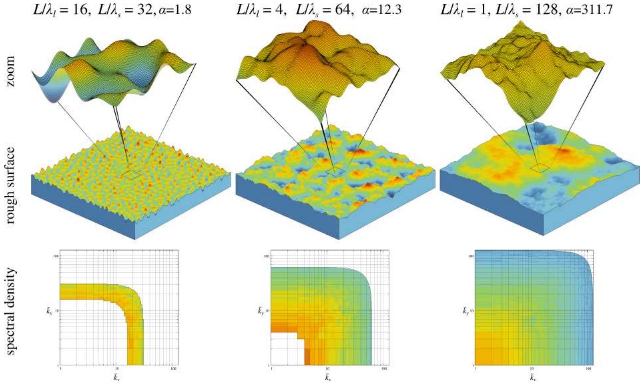
$\frac{A}{A_0} = A(p_0, \alpha)/A_0$ *according to* [2-5]

$\frac{A}{A_0} = \mathrm{erf}\left(\sqrt{\frac{2}{\langle|\nabla z|^2\rangle}} \frac{p_0}{E^*}\right)$ *according to* [6-7]

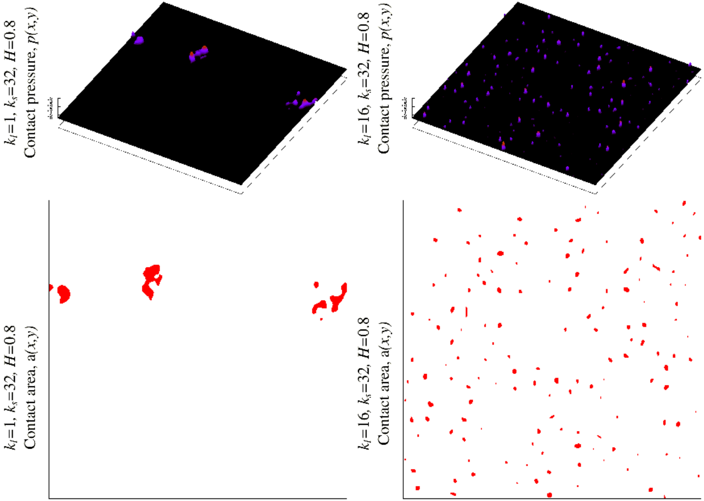
[1] Greenwood, Williamson, P Roy Soc Lond A Mat 295 (1966)
[2] Bush, Gibson, Thomas, Wear 35 (1975)
[3] Mc Cool, Wear 107 (1986)
[4] Thomas, Rough Surfaces (1999)
[5] Greenwood, Wear 261 (2006)
[6] Persson, J. Chem. Phys. 115 (2001)
[7] Persson, Phys. Rev. Lett. 87 (2001)
[8] Persson, Bucher, Chiaia, Phys. Rev. B 65 (2002)
[9] Müser, Phys. Rev. Lett. 100, (2008)

Simulations set-up

- Cut-off parameters: $L/\lambda_l \otimes L/\lambda_s = \{1,2,4,8,16\} \otimes \{32,64,128,256,512\}$
- Hurst exponent $H = \{0.4, 0.8\}$
- 10 random surface realizations per combination of parameters
- Discretization: $\{L/\Delta x\} \times \{L/\Delta x\} = 2048 \times 2048$
- Search for contact area A', gap field $g(x,y)$ and gap PDF $P(g)$



Contact area and contact pressure evolution



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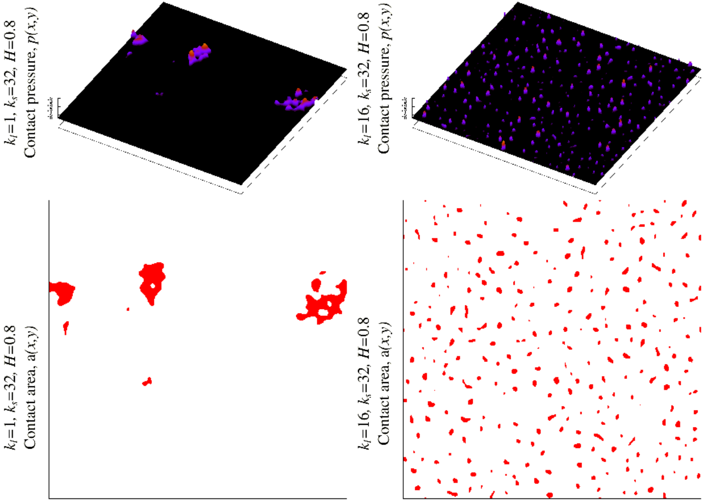

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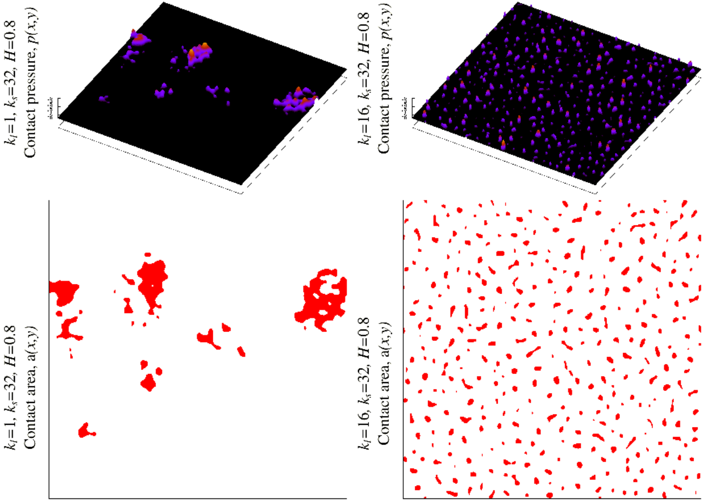

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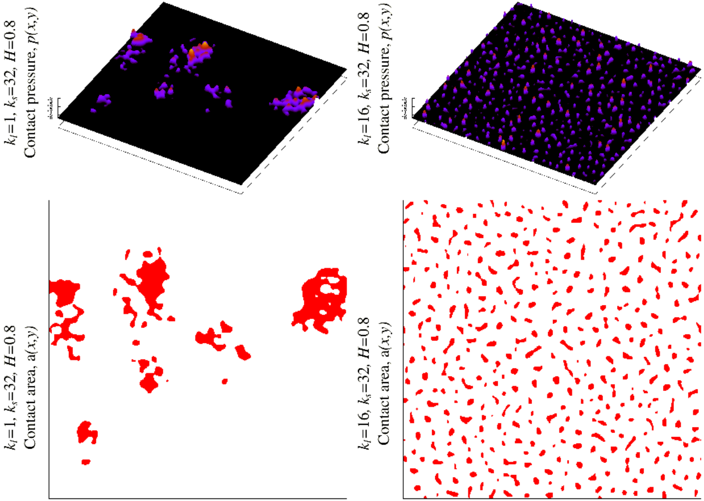

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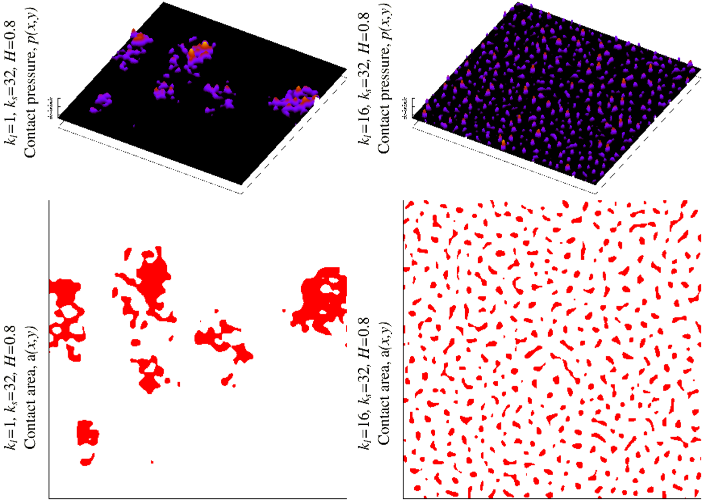

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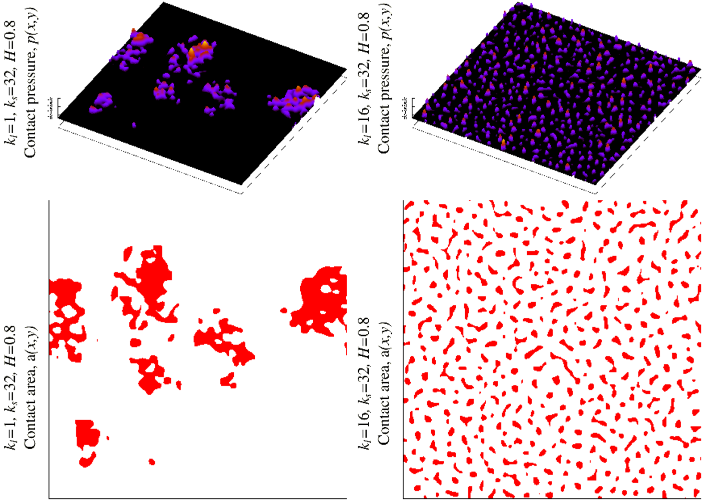

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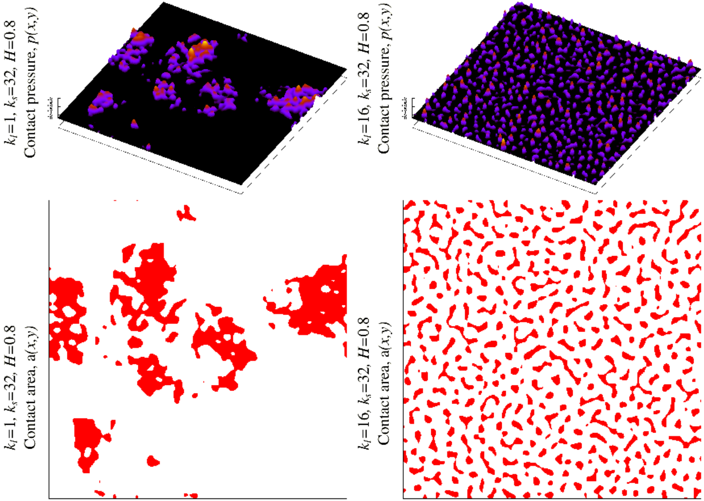

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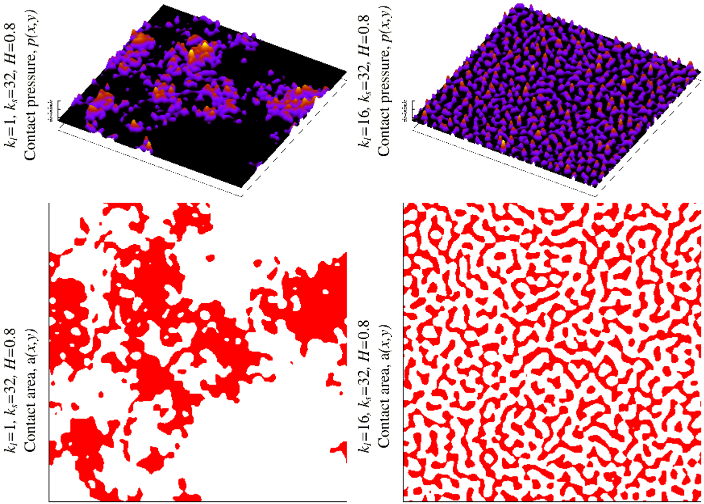

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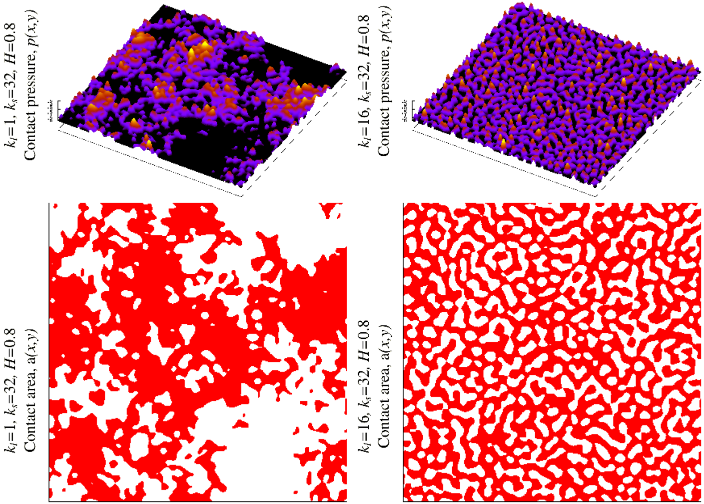

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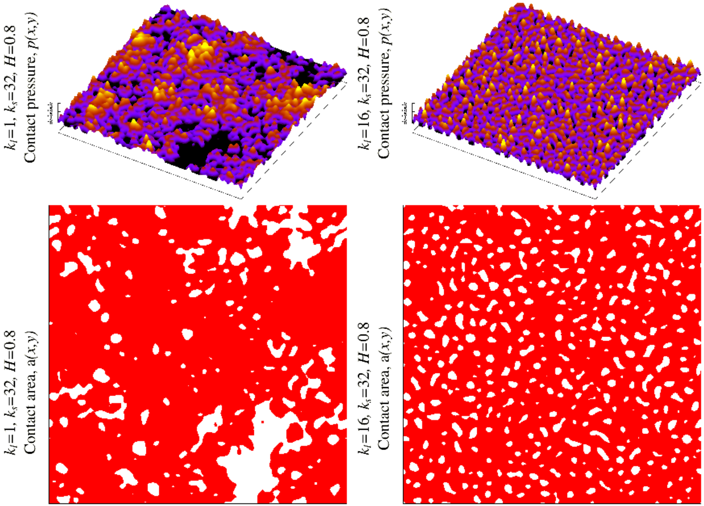

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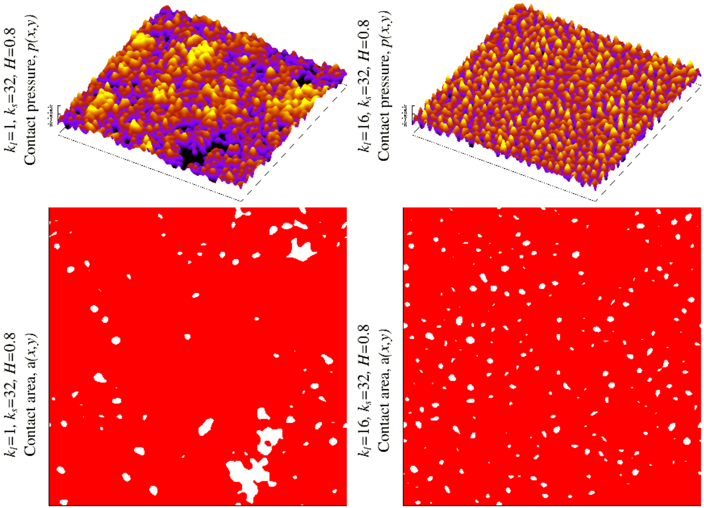

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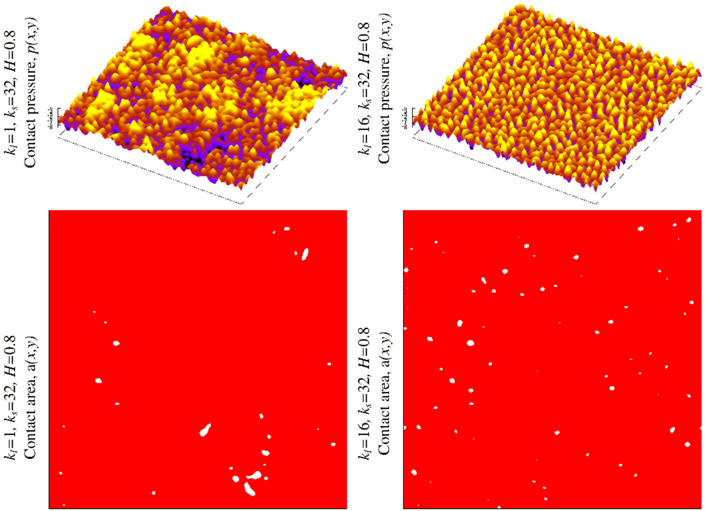

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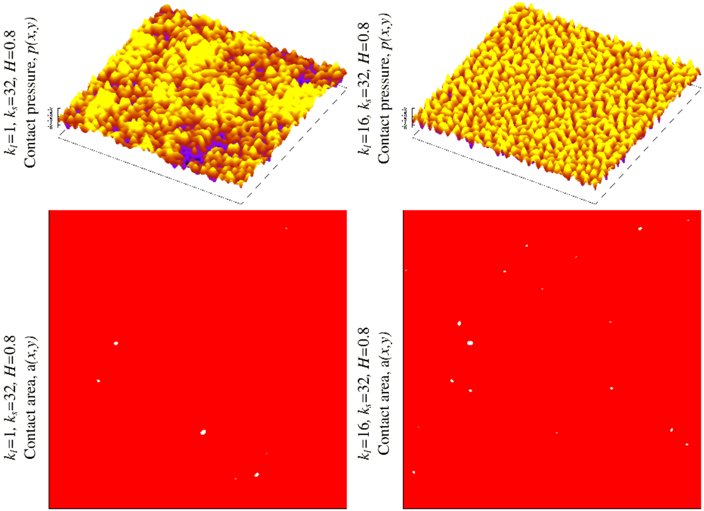

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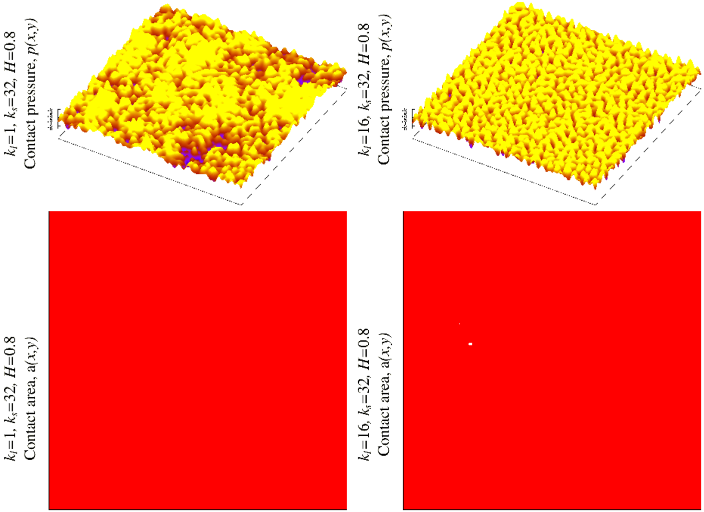
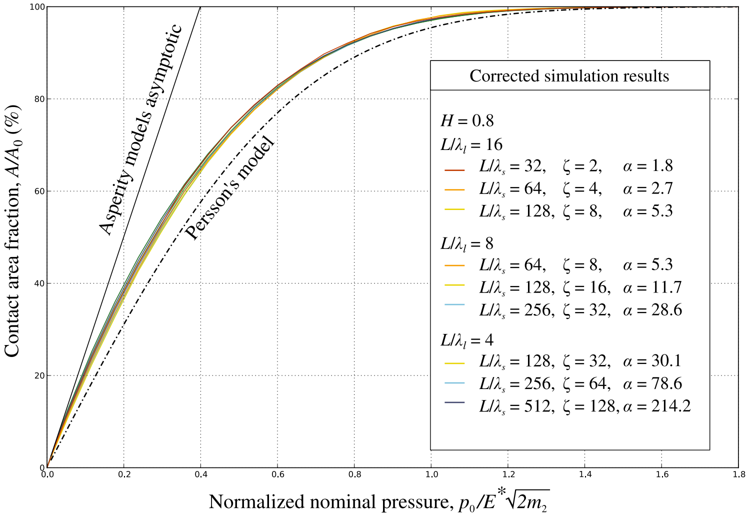


Results: contact area



Multi-asperity models asymptotic[1,2], Persson's model[3]

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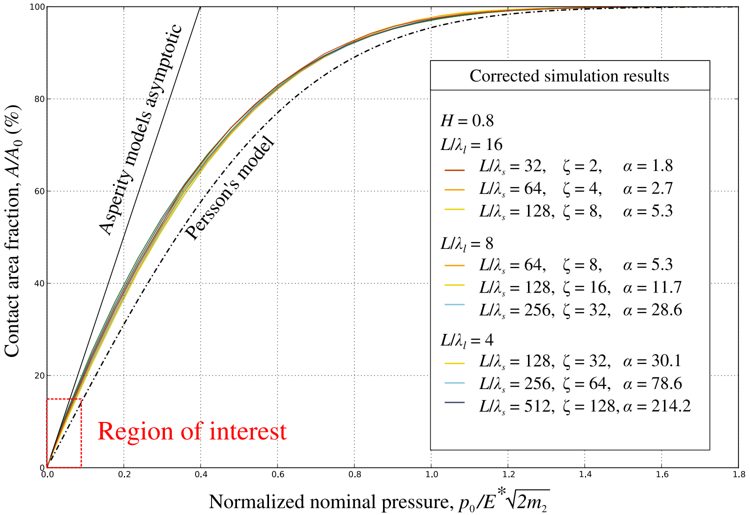

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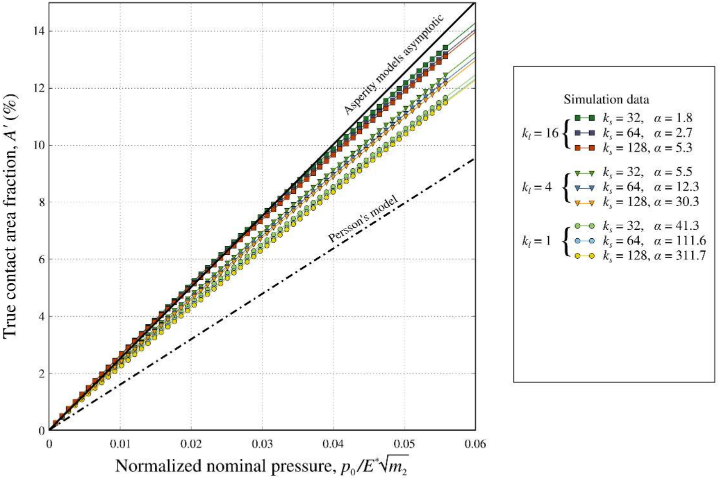
Real contact area: interpretation of results?



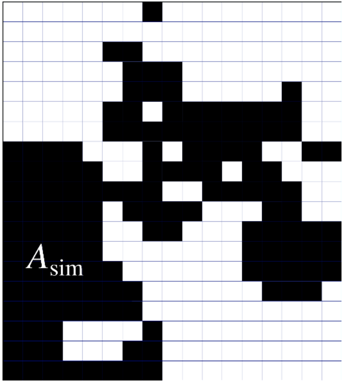
Raw data

[1] Yastrebov, Anciaux, Molinari, Int J Solids Struct 52 (2015)

Numerical error correction

- Contact area is overestimated in simulations:

  $A_{\text{sim}} > A_*$



Numerical error correction

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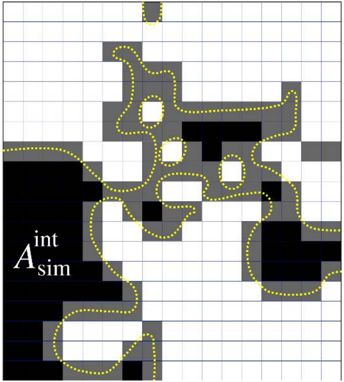
- The overestimation is localized at boundary nodes:

  $$A_{\text{sim}} > A_* > A_{\text{sim}}^{\text{int}}$$



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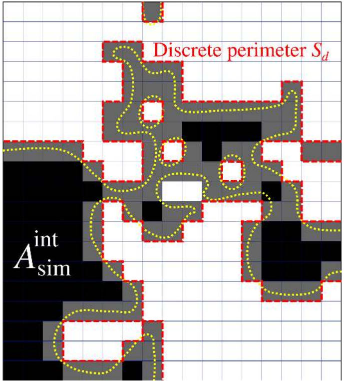
$$A_{\text{sim}} - A^{\text{int}}_{\text{sim}} = S_d \Delta x$$



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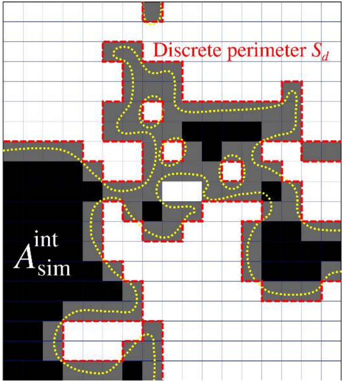
- Manhattan S_d vs Euclidean metric S:

  $\langle S \rangle = \frac{\pi}{4} \langle S_d \rangle$



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- Boundary area ~ perimeter S_d:

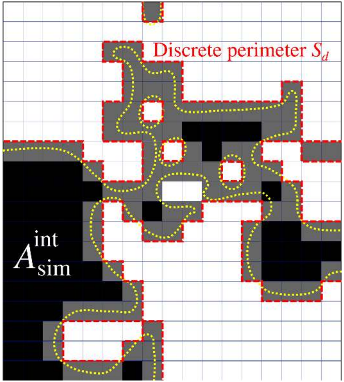
  $$A_{\rm sim} - A_{\rm sim}^{\rm int} = S_d \Delta x$$

- Manhattan S_d vs Euclidean metric S:

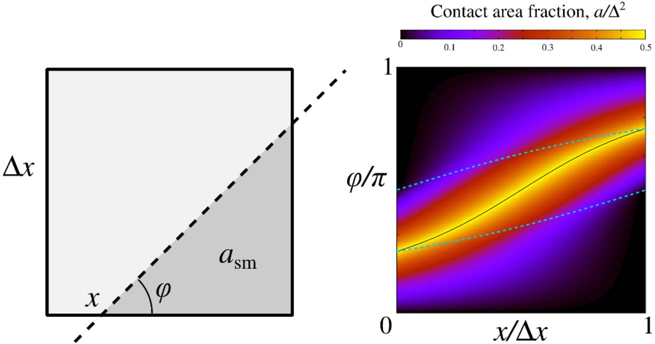
  $$\langle S \rangle = \frac{\pi}{4} \langle S_d \rangle$$

- True contact area estimation:

  $$\boxed{A_* \approx A_{\rm sim} - \beta \frac{\pi}{4} S_d \Delta x}$$



Numerical error correction: corrective factor

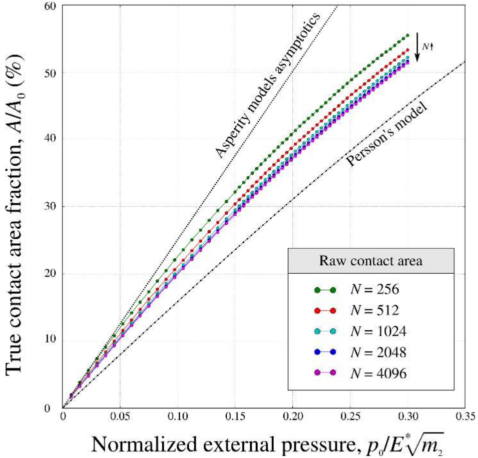


Corrective factor $\beta = \dfrac{\langle a_{\rm sm}\rangle}{\Delta x^2} = \dfrac{1}{\Delta x^2}\int\limits_0^h\int\limits_0^\pi a_{\rm sm} P(x,\phi)\,dxd\phi = \dfrac{\pi - 1 + \ln 2}{6\pi}$

$$\beta = 0.15038761899481015160 6955\ldots$$

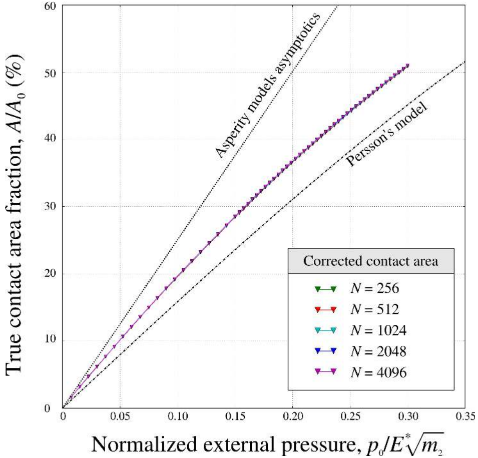
True area estimation: $A_* \approx A_{\rm sim} - \dfrac{\pi - 1 + \ln 2}{24} S_d \Delta x$

Numerical error correction: convergence study



[1] Yastrebov, Anciaux, Molinari, Tribol Int 114 (2017)

Numerical error correction: convergence study



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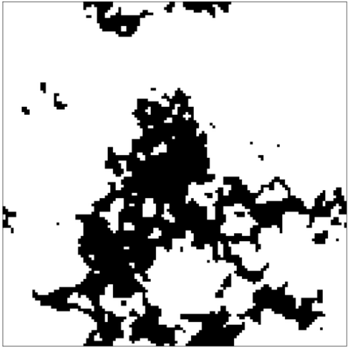
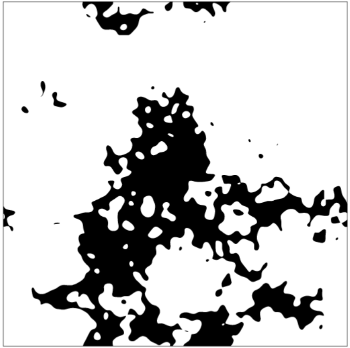
Morphological correction

- Morphology of contact clusters



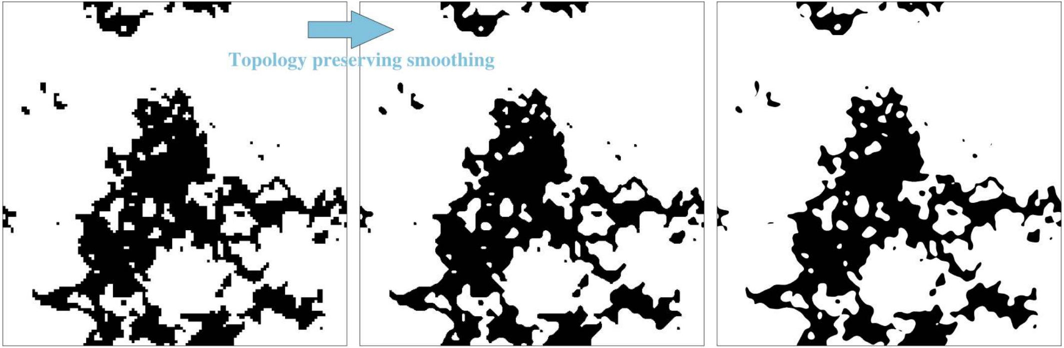
N=128, raw



N=2048, raw

Morphological correction

- Morphology of contact clusters

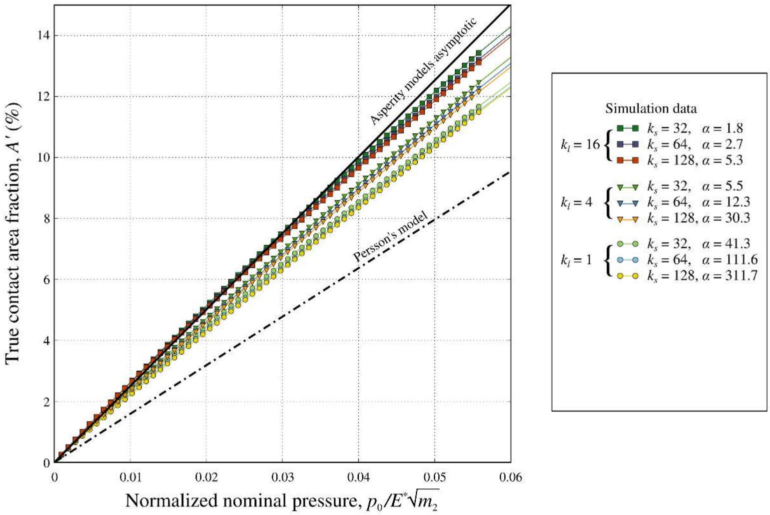


N=128, raw N=128, smoothed N=2048, raw

Topologically preserving smoothing results in realistic cluster geometry

[1] Couprie & Bertrand, *J Electr Imag* 13 (2004)

Real contact area: accurate results



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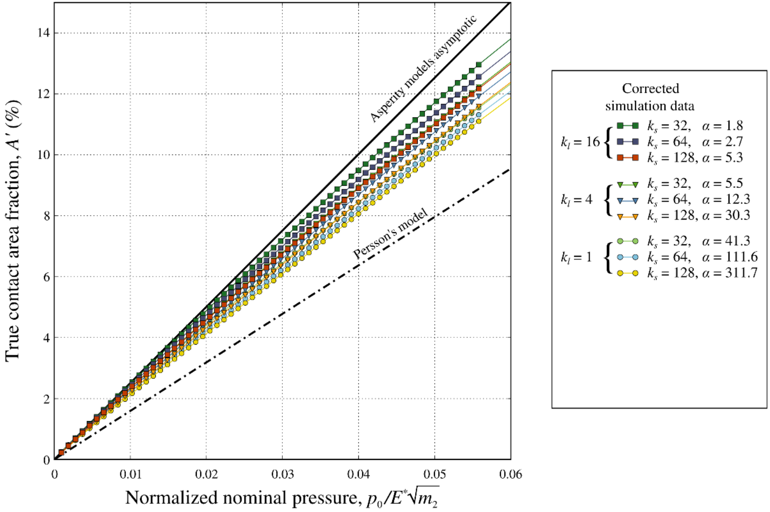

Corrected data

[2] Yastrebov, Anciaux, Molinari, *J Mech Phys Solids* 107 (2017)

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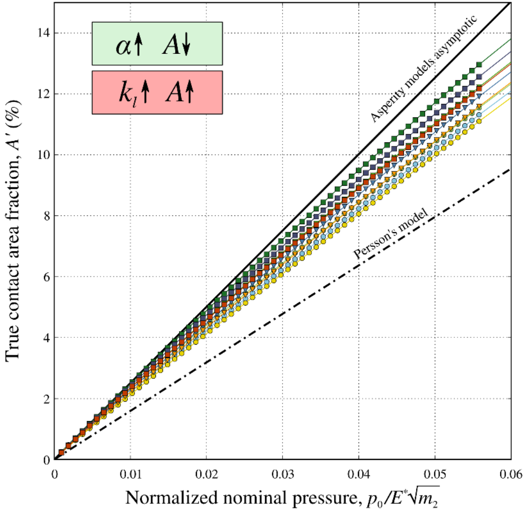

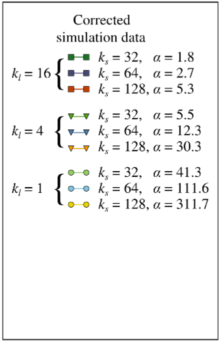

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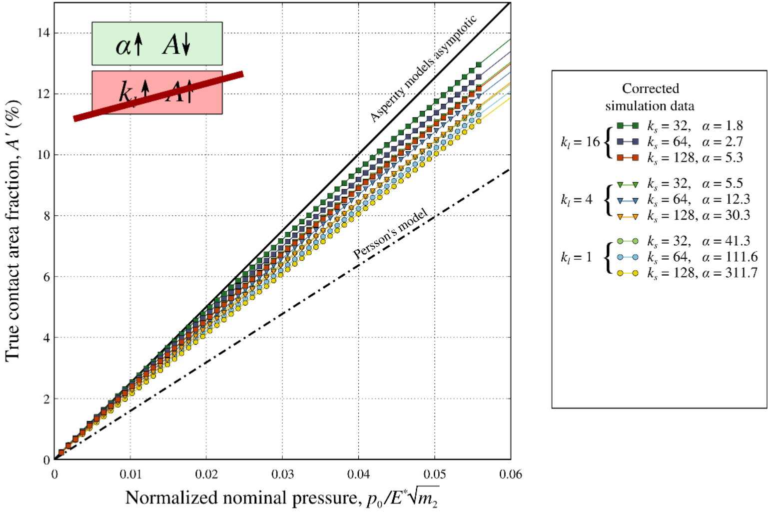

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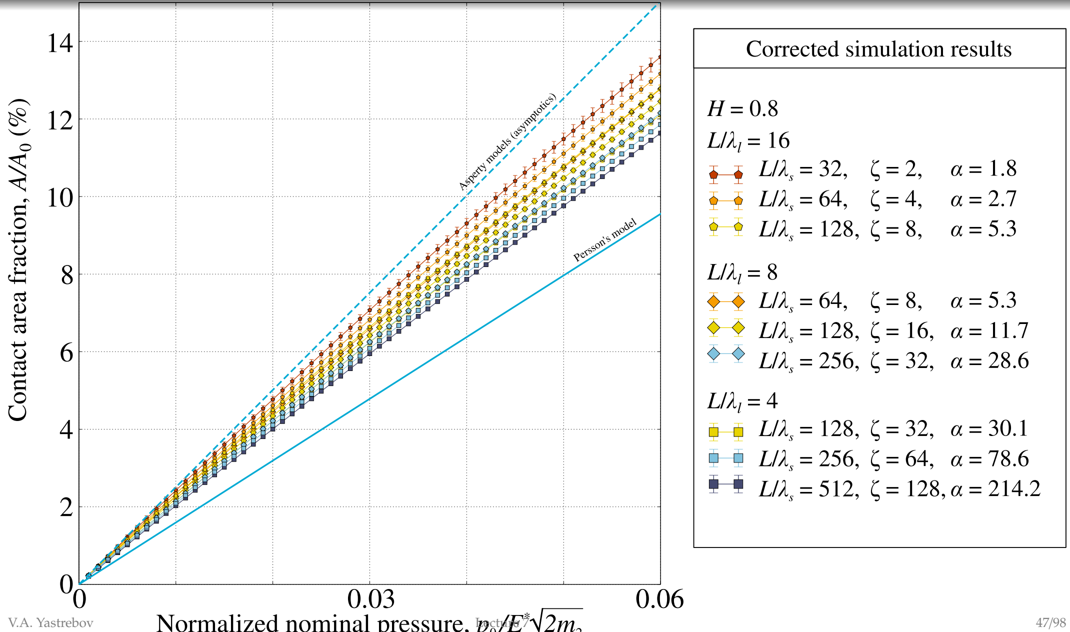
Results: contact area



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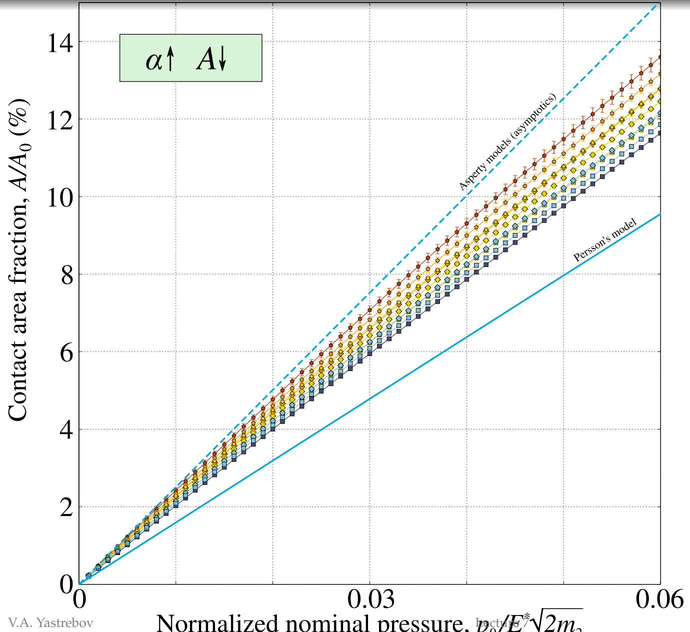


Corrected simulation results

$H = 0.8$

$L/\lambda_l = 16$

- $L/\lambda_s = 32, \quad \zeta = 2, \quad \alpha = 1.8$
- $L/\lambda_s = 64, \quad \zeta = 4, \quad \alpha = 2.7$
- $L/\lambda_s = 128, \quad \zeta = 8, \quad \alpha = 5.3$

$L/\lambda_l = 8$

- $L/\lambda_s = 64, \quad \zeta = 8, \quad \alpha = 5.3$
- $L/\lambda_s = 128, \quad \zeta = 16, \quad \alpha = 11.7$
- $L/\lambda_s = 256, \quad \zeta = 32, \quad \alpha = 28.6$

$L/\lambda_l = 4$

- $L/\lambda_s = 128, \quad \zeta = 32, \quad \alpha = 30.1$
- $L/\lambda_s = 256, \quad \zeta = 64, \quad \alpha = 78.6$
- $L/\lambda_s = 512, \quad \zeta = 128, \quad \alpha = 214.2$

Phenomenological relationship

- Contact area A grows with applied pressure p_0 as

$$\frac{A}{A_0} = a(\alpha)\frac{p_0}{E^*\sqrt{2m_2}} - b(\alpha)\left[\frac{p_0}{E^*\sqrt{2m_2}}\right]^2$$

- Contact area fraction $A' = A/A_0$ grows with normalized applied pressure $p' = p_0/E^*\sqrt{2m_2}$

$$\boxed{A' = a(\alpha)p' - b(\alpha)p'^2}$$

- With ≈universal adimensional constants:

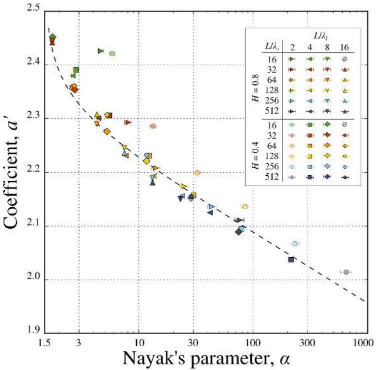
$a(\alpha) = 2.35 - 0.057\ln(\alpha - 1.5)$

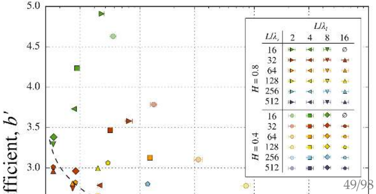
$b(\alpha) = 2.85 - 0.24\ln(\alpha - 1.5)$

- Pressure dependent friction coefficient:

$$\mu(p') = \mu_0\left[1 - \frac{b(\alpha)}{a(\alpha)}p'\right]$$

with $\mu_0 = a(\alpha)\tau_{\max}/E^*\sqrt{2m_2}$, $\tau_{\max}$ is the maximum shear traction the contact interface can bear.





Conclusions

- Contact area growth almost linearly for small pressures and saturates at bigger pressure
- The key parameter of the contact area growth is the RMS slope or its variance $2m_2$
- Contact area depends weakly on Nayak parameter $\alpha = m_0 m_4 / m_2^2$

$$\boxed{A' = a(\alpha)p' - b(\alpha){p'}^2}$$

with $a(\alpha) = 2.35 - 0.057\ln(\alpha - 1.5), \quad b(\alpha) = 2.85 - 0.24\ln(\alpha - 1.5)$

- No effect of fractal dimension D_f *per se* on the contact area
  it affects the contact area only through the Nayak parameter

Flow through the contact interface

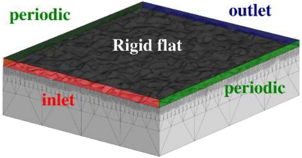
Problem statement

Problem

- **Thin creeping** flow in contact interface:
  Navier-Stokes → Stokes → Reynolds equation
- In addition: incompressible fluid, immobile walls:

$$\nabla \cdot \underline{q} = 0, \quad \underline{q} = -\frac{g^3}{12\mu}\nabla p_f$$

$\underline{q}(x,y)$ is the fluid flux,
$g(x,y)$ is the gap (opening) fields,
$p_f(x,y)$ hydrostatic fluid pressure,
μ is the dynamic viscosity.



- Gap profile $g(x,y)$ for $x,y \in (0,L)$
- At inlet: $p_f = p_{\text{in}}$
- At outlet: $p_f = p_{\text{out}}$
- At lateral sides: periodic $q_n(y=L) = -q_n(y=0)$
- Linear problem: use FEM

Analytical approach

Effective flow estimation

- Averaging over surface $\langle x \rangle = 1/A_0 \int_{A_0} x\,dA$ gives:

$$\langle \underline{q} \rangle = -\underline{\underline{K}}_{\text{eff}} \cdot \langle \nabla p_f \rangle$$

- For isotropic case, normalized scalar **effective transmissivity** along pressure drop OX:

$$K'_{\text{eff}} = -\frac{12\mu\langle q_x \rangle L}{m_0^{3/2}(p_{\text{in}} - p_{\text{out}})}$$

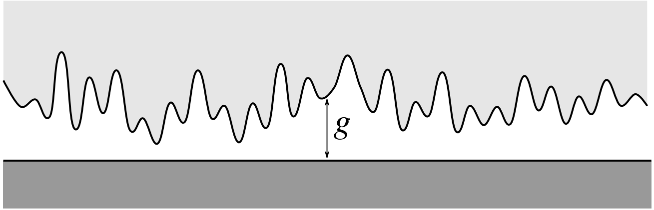
- Using effective medium[1,2] approach

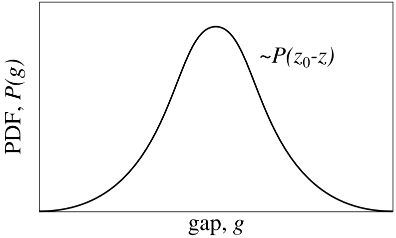
$$(1 - A')\int\limits_0^\infty \frac{g^3 P(g)}{g^3 + K'_{\text{eff}} m_0^{3/2}}\,dg = \frac{1}{2}$$

$A' = A/A_0$ is the contact area fraction, $P(g)$ is the gap probability density.

[1] Kirkpatrick. Rev Modern Phys, 45 (1973)
[2] Lorenz & Persson. Europ Phys J E: Soft Matter, 31 (2010)

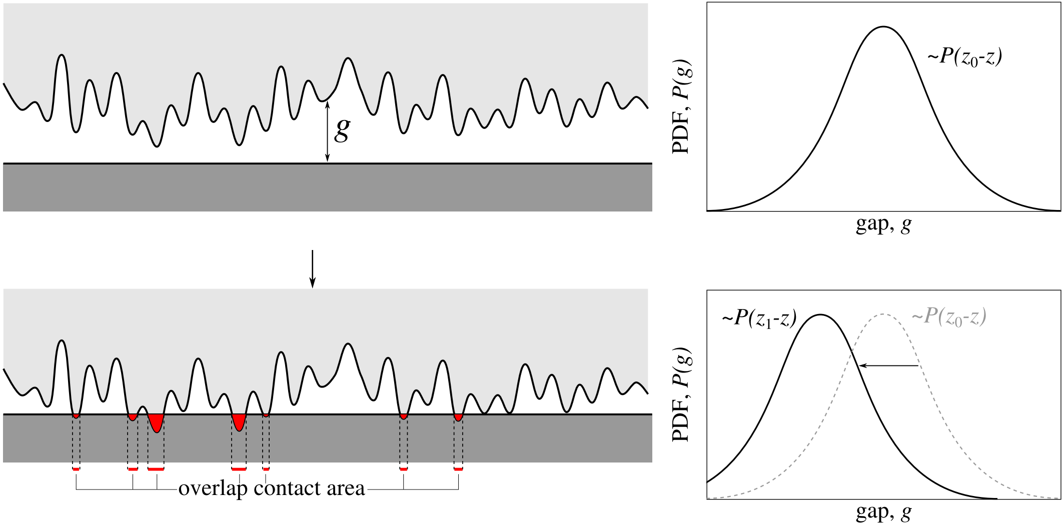
- Geometrical overlap model is highly inaccurate



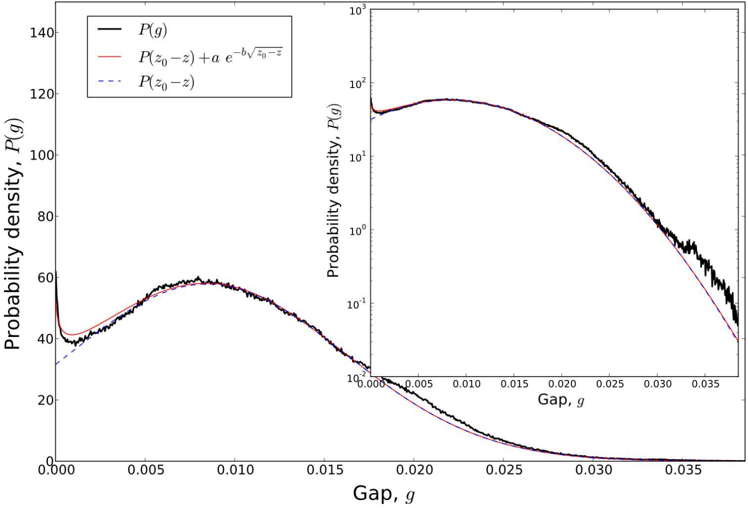


Danger: geometrical overlap

- Geometrical overlap model is highly inaccurate



Solid contact results: gap distribution

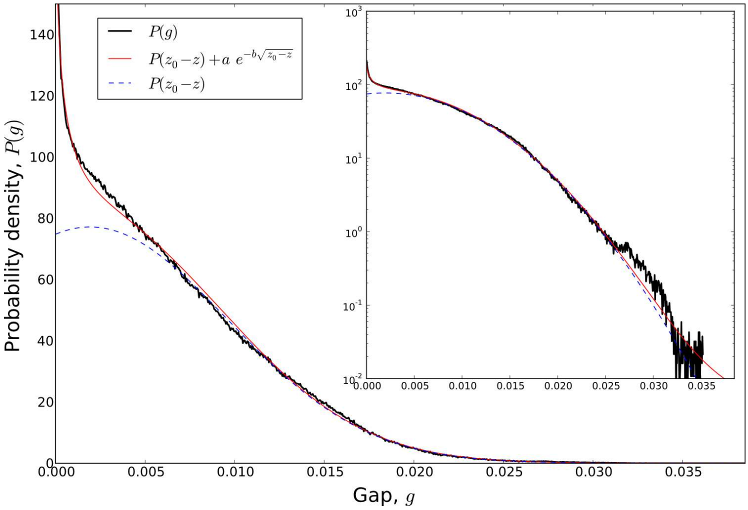


Area fraction $A' = 1.6\%$

Gap probability density VS geometrical overlap model (dashed line)

Near contact interface $P(g) \sim P(z_0 - z) + a \exp(-b\sqrt{z_0 - z})$

Solid contact results: gap distribution



Area fraction $A' = 9.5\%$

Gap probability density VS geometrical overlap model (dashed line)
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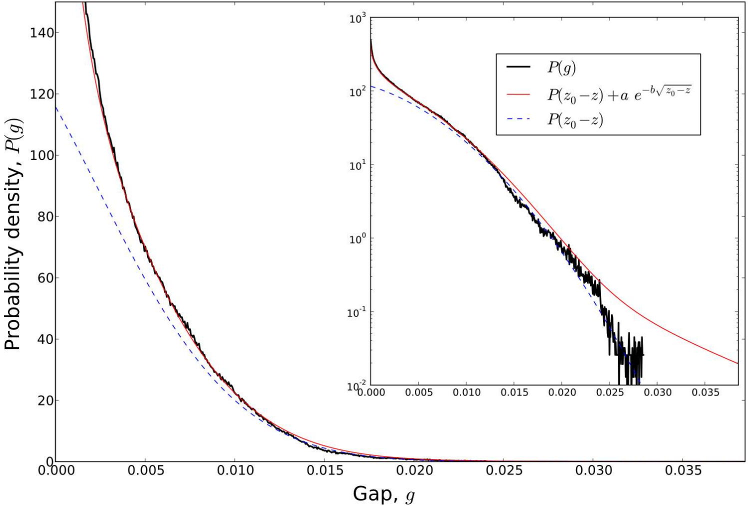
Solid contact results: gap distribution



Area fraction $A' = 24\%$

Gap probability density VS geometrical overlap model (dashed line)
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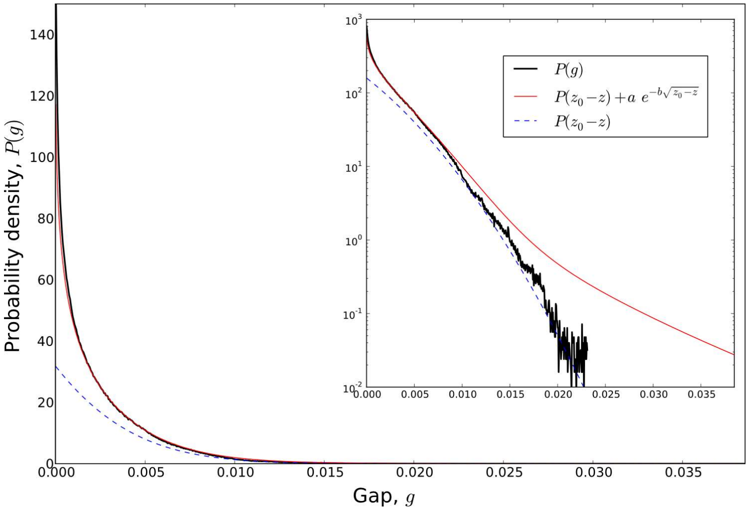
Solid contact results: gap distribution



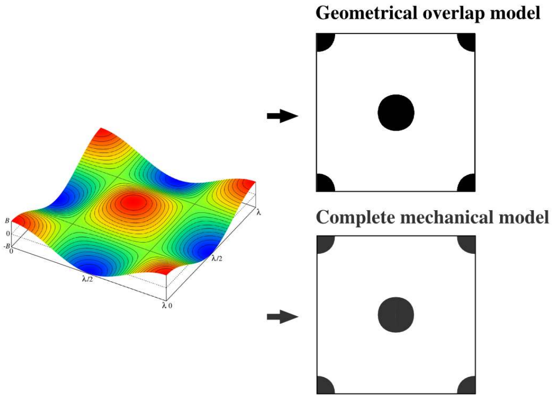
Area fraction $A' = 39\%$

Gap probability density VS geometrical overlap model (dashed line)

Near contact interface $P(g) \sim P(z_0 - z) + a \exp(-b\sqrt{z_0 - z})$

Geometrical overlap: morphology and percolation

- Geometrical overlap model is highly inaccurate[1,2]



[1] Dapp, Lücke, Persson, Müser, *Phys. Rev. Lett.* 108 (2012)

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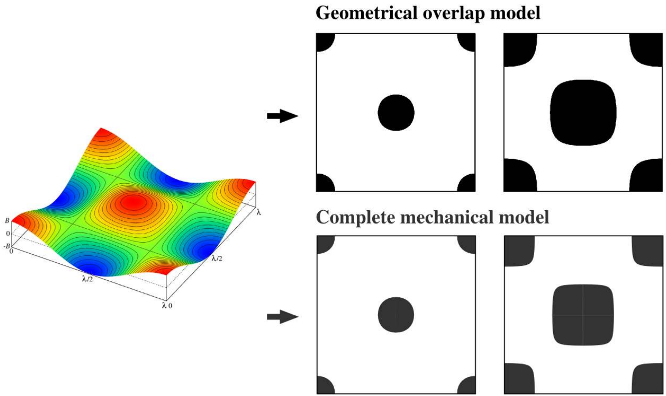

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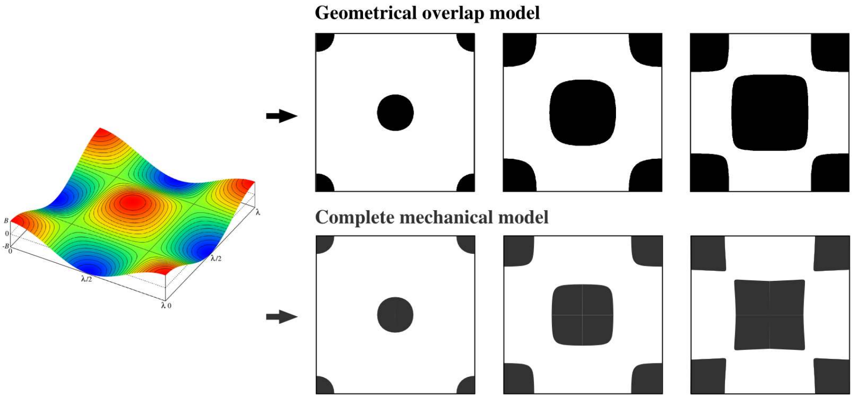

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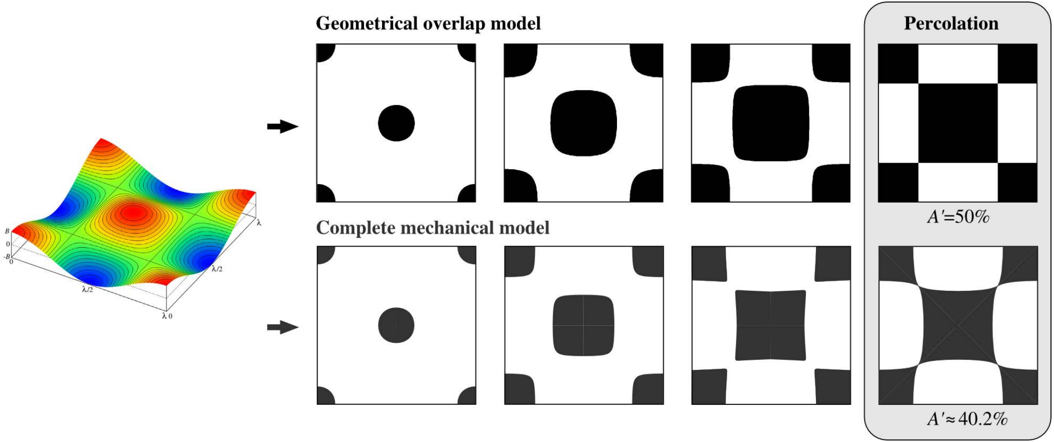

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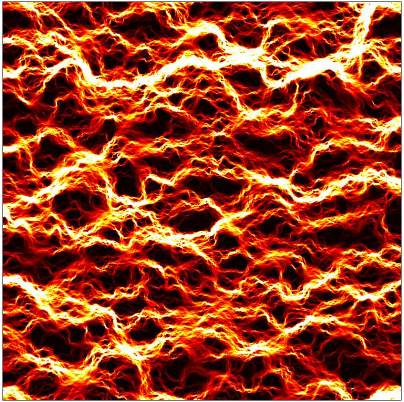
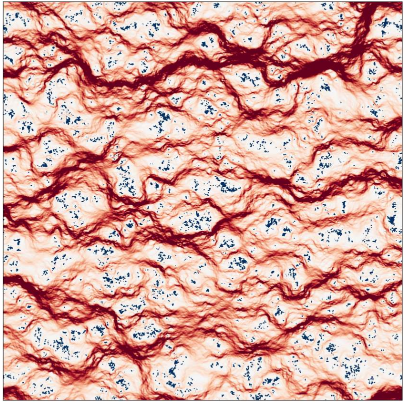
Creeping fluid transport





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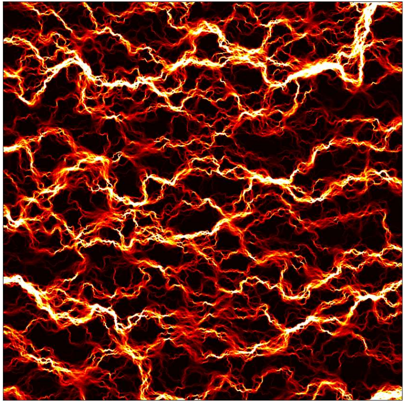

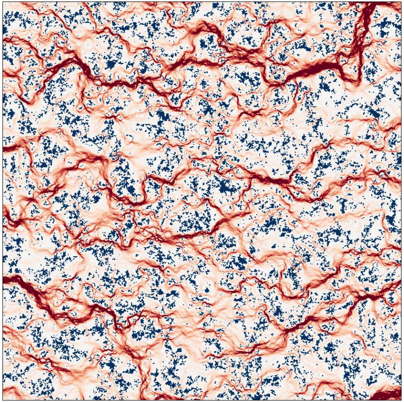

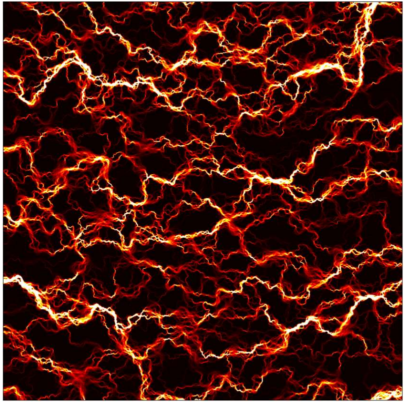

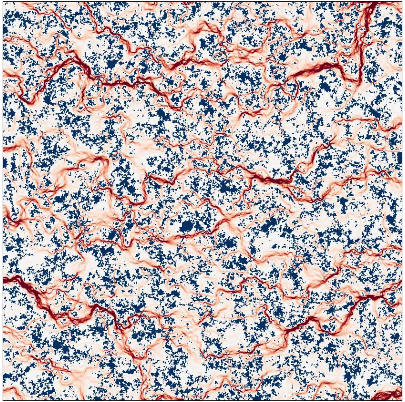

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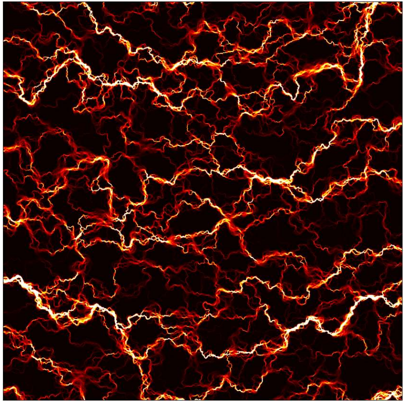

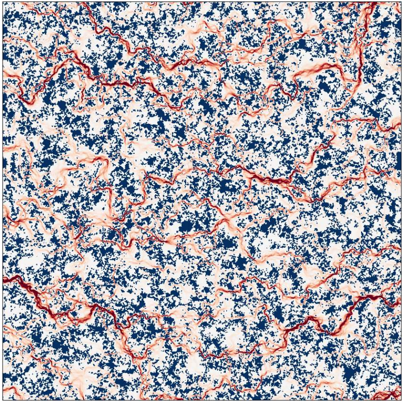

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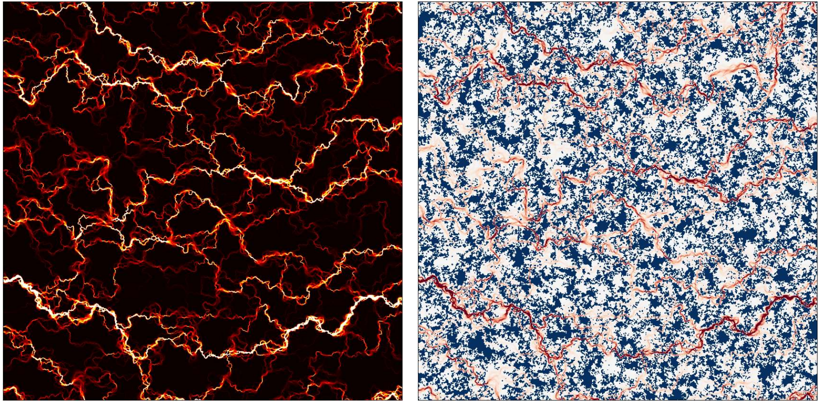

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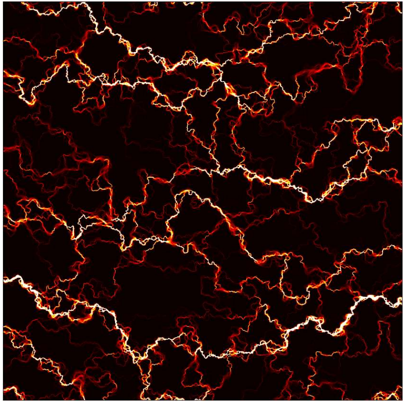

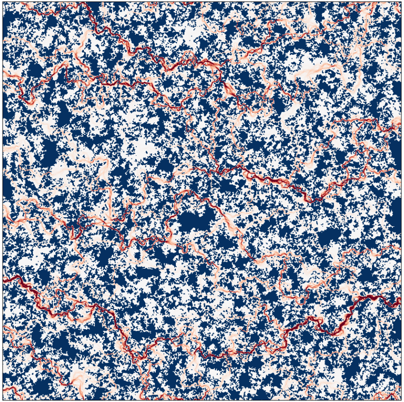

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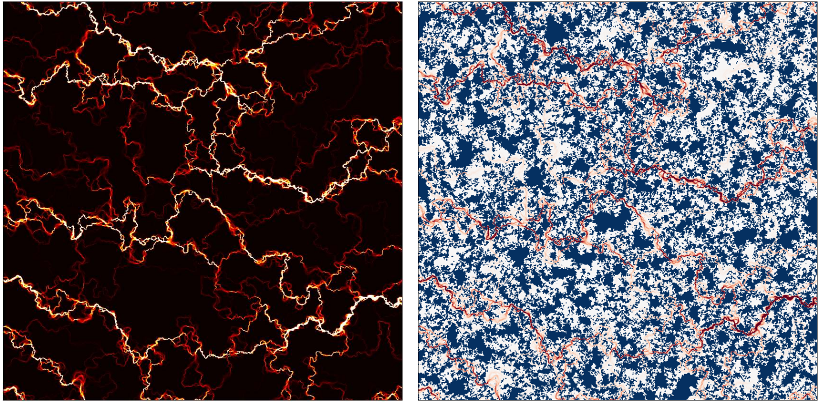

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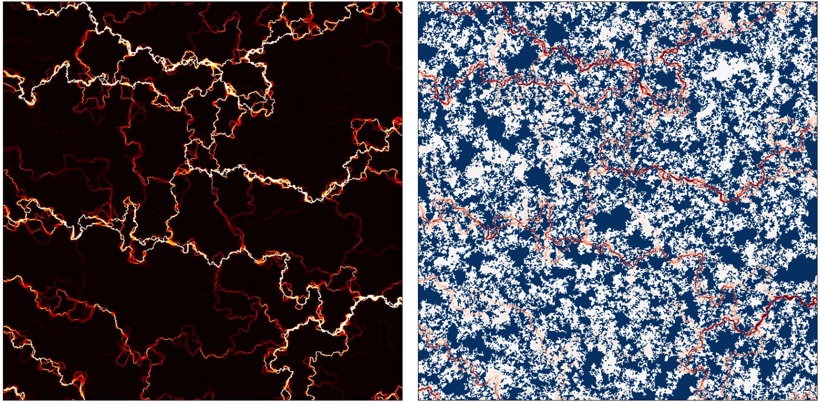

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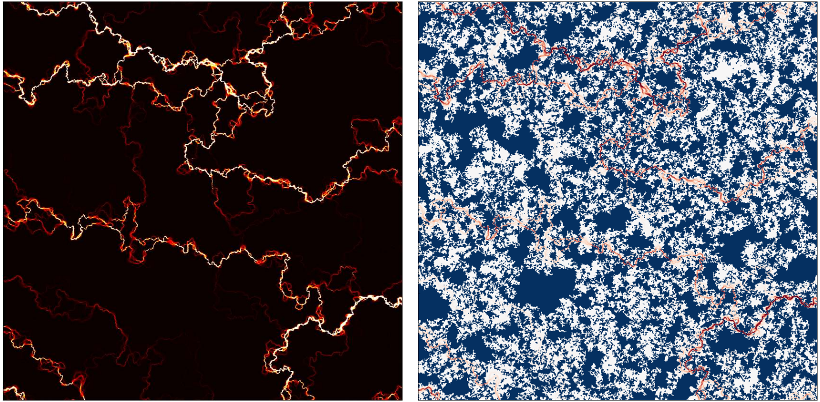

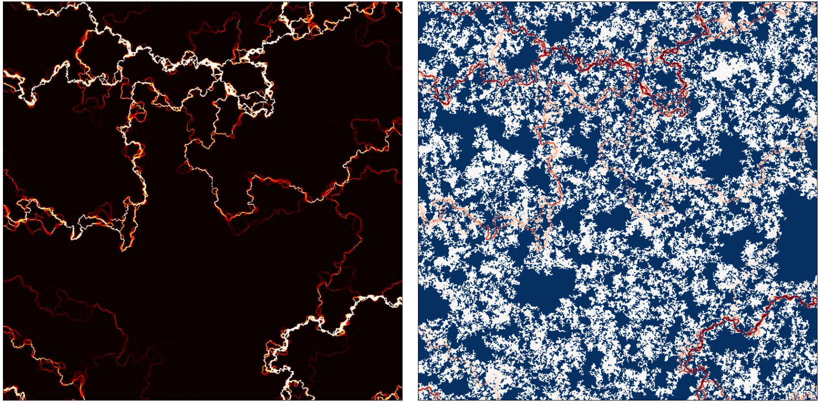

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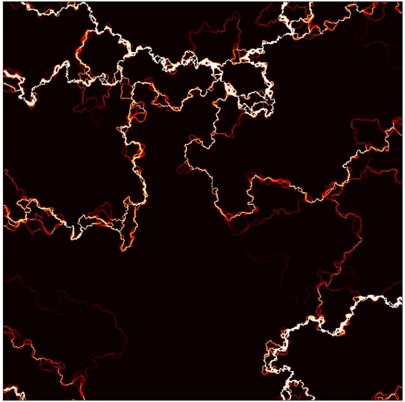

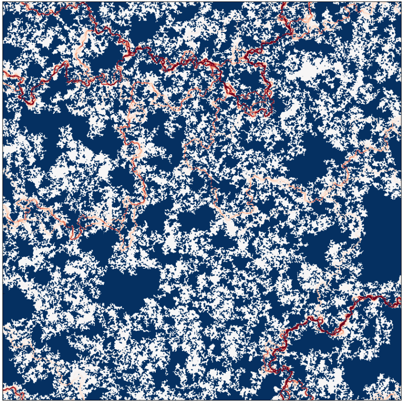

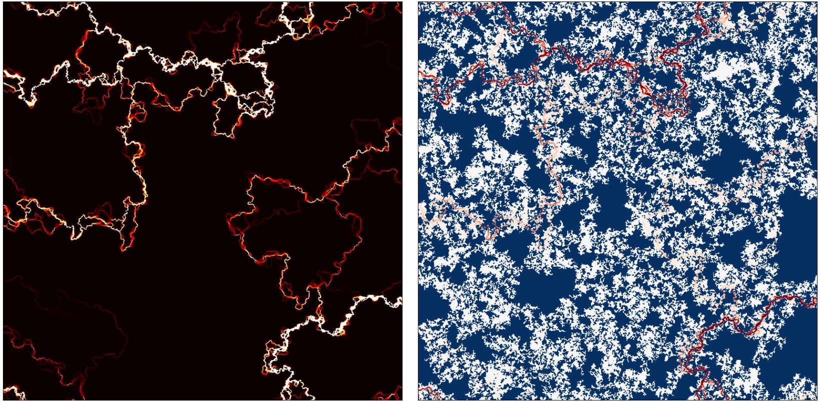

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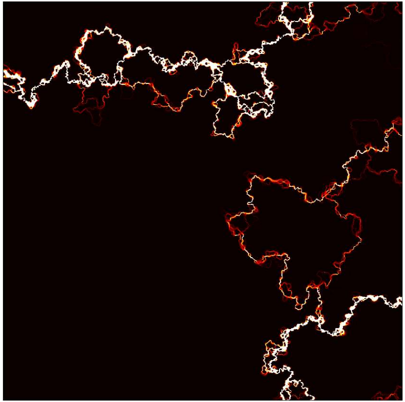

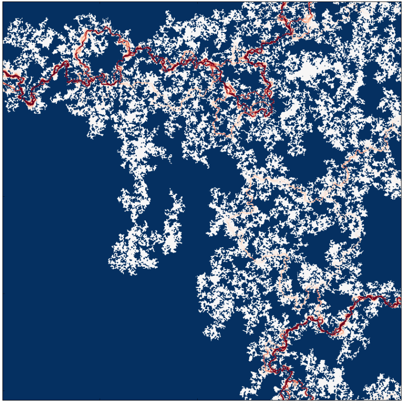

Contact area & trapped fluid

- Contact area does not conduct flow

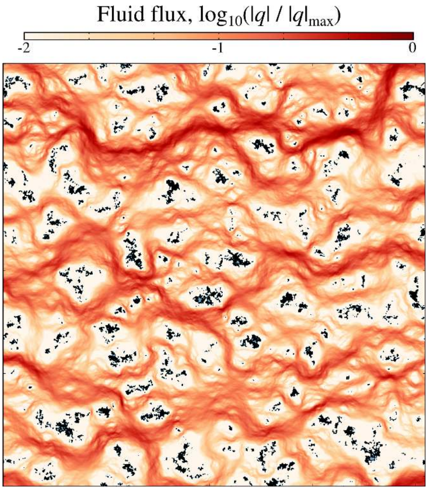

Fig. Fluid flux

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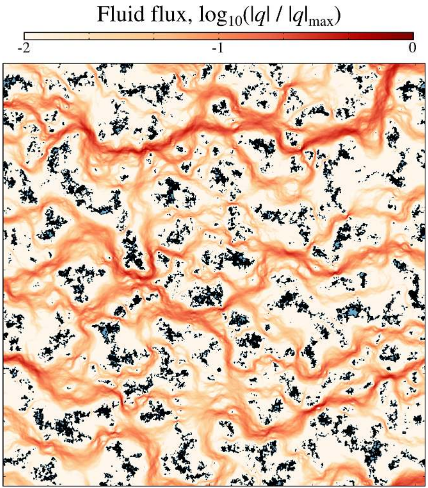

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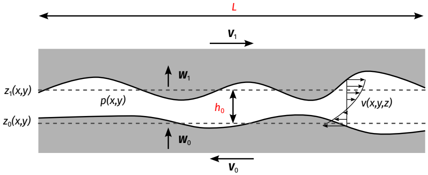

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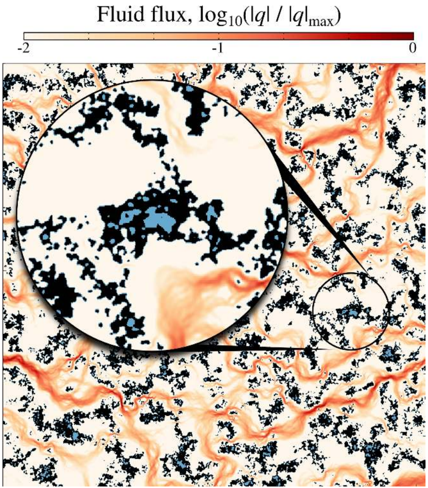


Fig. Fluid flux (zoom)

Contact area & trapped fluid

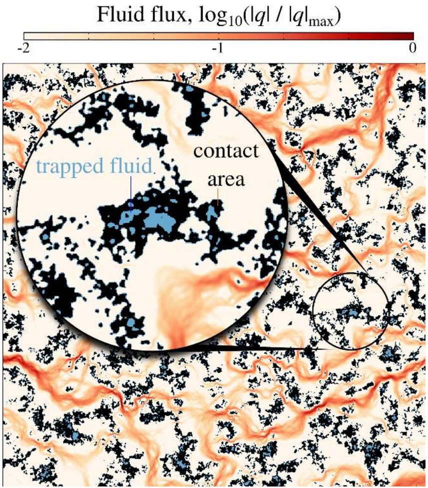
- Contact area does not conduct flow
- Islands of trapped fluid ≡
  non-simply connected contact spots do not contribute to conduction
- Thus the effective transmissivity depends on the
  effective contact area:

$$A'_{\text{eff}} = A' + A'_t$$

A' is the contact area fraction
A'_t is the area of trapped fluid

- Effective medium transmissivity:

$$(1 - A' \ ) \int_0^\infty \frac{g^3 P(g)}{g^3 + K'_{\text{eff}} m_0^{3/2}}\, dg = \frac{1}{2}$$



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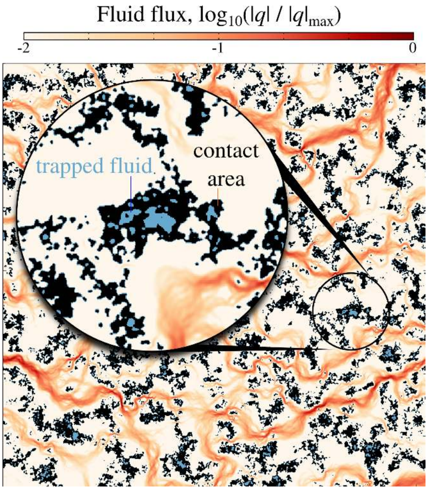

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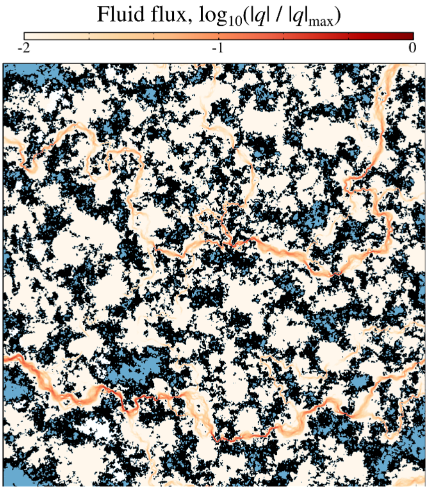

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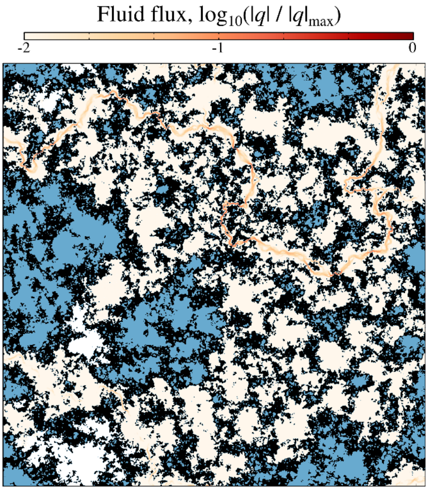

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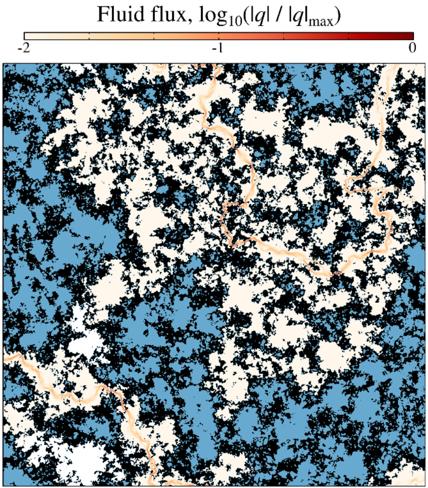

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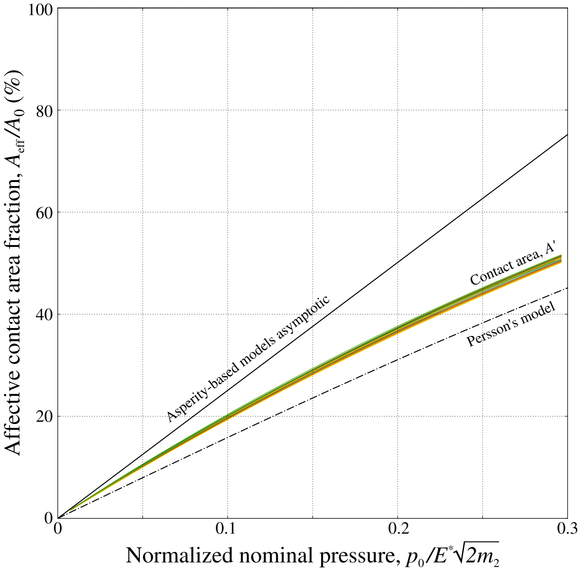
Effective contact area



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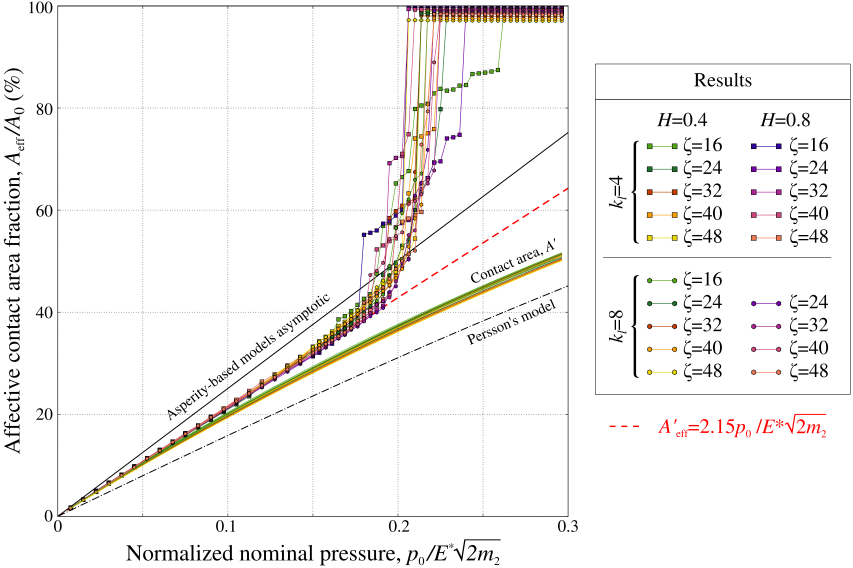
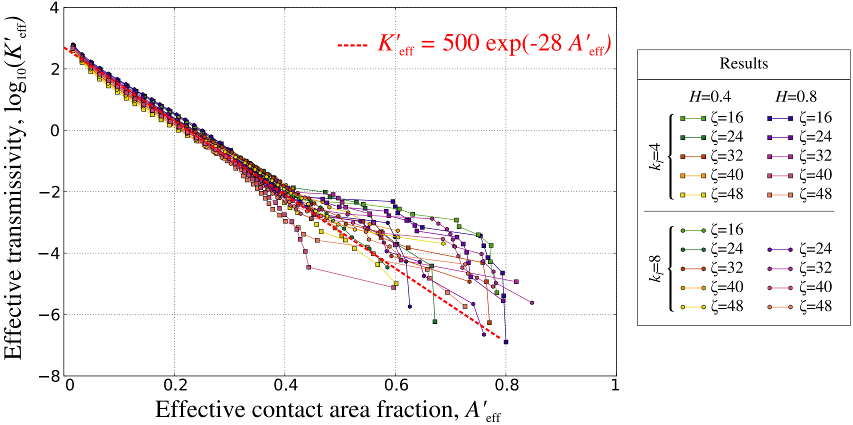


$A'_{\text{eff}}=2.15 p_0 / E^*\sqrt{2m_2}$

Normalized effective transmissivity



~ normalized nominal pressure p'

Fig. Evolution of the effective transmissivity

Effective transmissivity

- Effective area wrt load:

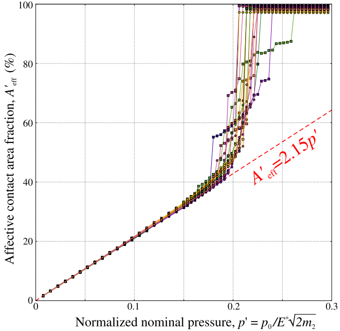
  $$A'_{\text{eff}} \approx 2.15p'$$

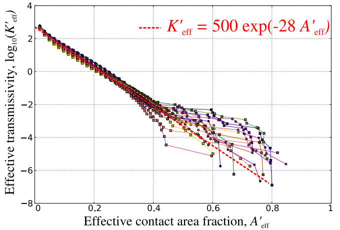
- Normalized load:

  $$p' = p_0/E^* \sqrt{2m_2}$$

- Normalized effective transmissivity wrt effective area:

  $$K'_{\text{eff}} \approx 500\exp(-28A'_{\text{eff}})$$





Effective transmissivity

- Effective area wrt load:

$$A'_{\text{eff}} \approx 2.15p'$$

- Normalized load:

$$p' = p_0/E^* \sqrt{2m_2}$$

- Normalized effective transmissivity wrt effective area:

$$K'_{\text{eff}} \approx 500 \exp(-28A'_{\text{eff}})$$

- Recall:

$$K'_{\text{eff}} = -\frac{12\mu\langle\, q_x \,\rangle L}{m_0^{3/2}\Delta P_f}$$

- Express the mean flow:

$$\langle\, q_x \,\rangle = -\frac{K'_{\text{eff}} m_0^{3/2}\Delta P_f}{12\mu L}$$

- Finally:

$$\boxed{\langle\, q_x \,\rangle \approx -\frac{41.7 \exp(-42.57 p_0/E^* \sqrt{m_2}) m_0^{3/2}\Delta P_f}{\mu L}}$$

Conclusion & current work

Main result:

Mean flow (far from the percolation) through contact of nominal area $L \times L$:

$$\langle q_x \rangle \approx -\frac{41.7 m_0^{3/2} \Delta P_f}{\mu L} \cdot \exp\left(-42.57 \frac{p_0}{E^* \sqrt{m_2}}\right)$$

μ is dynamic viscosity,
ΔP_f is the pressure drop between the inlet and the outlet,
p_0 is the nominal applied pressure,
E^* is the effective elastic modulus.
Roughness parameters:
m_0 is the variance of roughness,
$2m_2$ is the variance of roughness gradient.

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Beyond the one-way coupling:

- Monolithic two-way FEM[1] framework coupling solid and fluid equations (thin flow, Reynolds equation) with contacts including islands of non-linear compressible fluid

[1] A.G. Shvarts, J. Vignollet, V.A. Yastrebov. "Computational framework for monolithic coupling for thin fluid flow in contact interfaces". Computer Methods in Applied Mechanics and Engineering, 379:113738 (2021).

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- Add some physics:
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- But, at Å-scales, continuum mechanics and especially continuum contact[1] do not work.
- Search for relevant physics that could justify $\lambda_s \gg$ Å.
- Candidates: plasticity (scale dependent), surface energy and adhesion, interaction potential.

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☺ Thank you for your attention!