

Contact Mechanics and Elements of Tribology

Lecture 6. *Lubrication and Sealing*

Andrei G. Shvarts^a & Vladislav A. Yastrebov^b

^a*University of Glasgow, Glasgow, United Kingdom*

^b*Mines Paris - PSL, CNRS
Centre des Matériaux, Versailles, France*



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1 Lubrication

- Regimes of lubrication
- Derivation of the Reynolds equation
- Analytical solution for hydrostatic lubrication in bearings
- Elasto-hydrodynamic lubrication

2 Sealing

- Metal-to-metal face seal for nuclear power plant applications
- Fluid-structure coupling
- Results of FE numerical simulation

Acknowledgment:

Course "Scientific Computing with Applications in Tribology"

A. Almqvist, F. Pérez-Ràfols

Luleå University of Technology, Sweden, 7-9 February 2017

Lubrication: what is it?

- **Lubrication:** technique to reduce friction and wear between relatively moving surfaces by adding a solid/liquid/gas lubricant
- Studied in **Tribology** (Greek: *tribo* - "to rub", *logy* - "study of")
'The Jost Report' (1966):
cost of friction, wear and corrosion to UK economy P. Jost (1966)
- Applications:
 - gears
 - bearings
 - piston heads
 - human joints
 - seals
 - cams
 - metal forming
 - HDD ...
- Recent report (2017):
23% of total world energy losses come from tribological contacts
(20% friction, 3% wear)
K. Holmberg, A. Erdemir, *Friction* (2017)



Lubricant over gears
www.iselinc.com



Lubricating a bike chain
www.madegood.org

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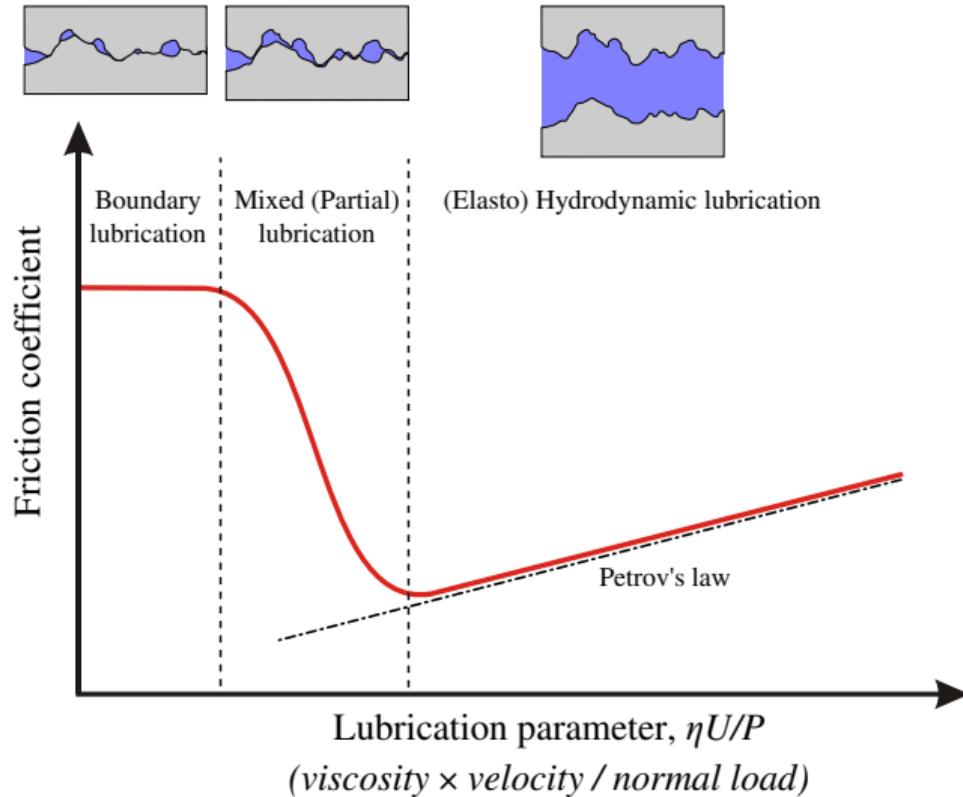


Lubricant over gears
www.efficientplantmag.com



Lubricated roller bearing
www.bearingtips.com

Lubrication regimes: Stribeck curve

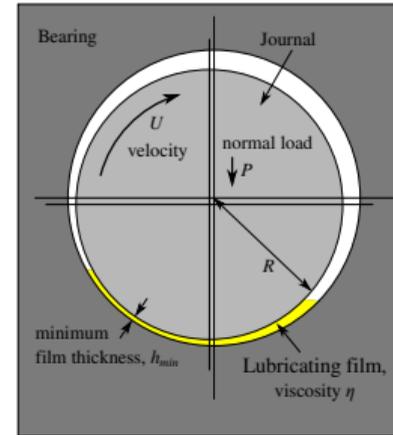


Adapted from www.wikipedia.org

R. Stribeck (1901)

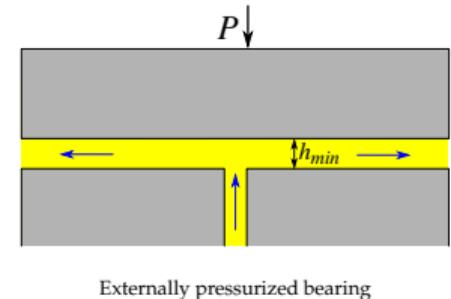
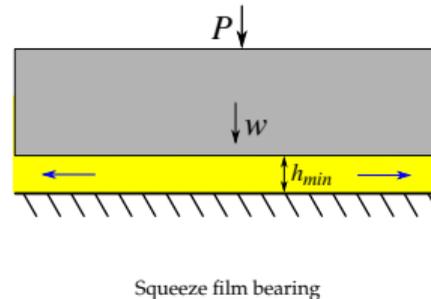
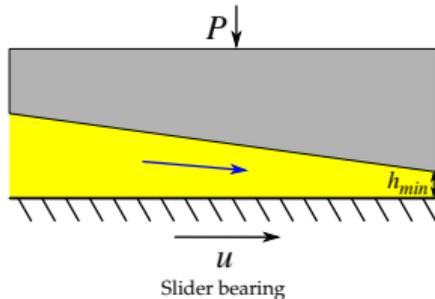
Hydrodynamic lubrication (HL)

- Conforming surfaces
- No elastic effect
- Normal load fully supported by thin fluid film
- $h_{\min} = f(P, U, \eta, R)$
- $p \leq 5 \text{ MPa}, h_{\min} > 1 \mu\text{m}$



Concentric journal bearing
Adapted from www.wikipedia.org

- Mechanism of pressure development in fluid film:



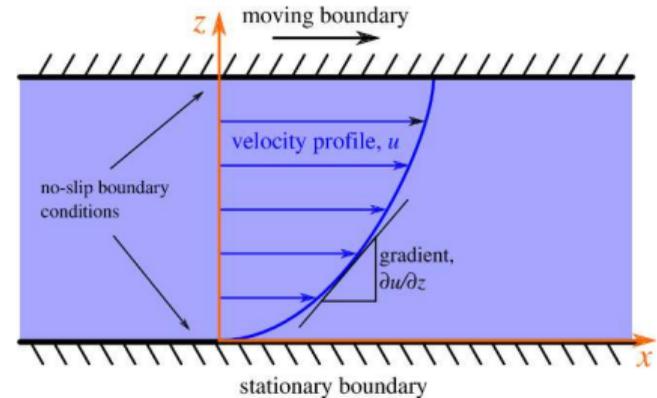
Newtonian fluid

- Viscous stresses in flowing fluid are linearly proportional to the strain rate - the gradient of the velocity:

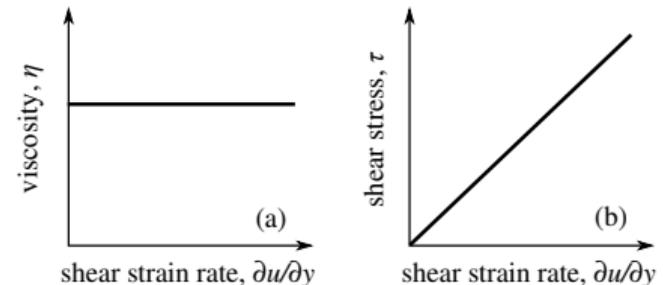
$$\tau = \eta \frac{\partial u}{\partial z}$$

- τ is the shear stress in the fluid
- η is the viscosity of the fluid
- $\frac{\partial u}{\partial y}$ is the shear strain rate
- In general 3D case for arbitrary coordinate system:

$$\tau_{ij} = \eta \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$



Laminar shear of fluid between two rigid plates



Properties of Newtonian fluid:
(a) viscosity vs shear strain rate
(b) shear stress vs shear strain rate

Petrov's equation

- Shear stress:

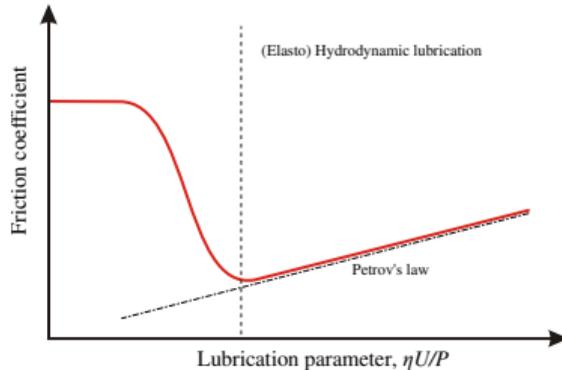
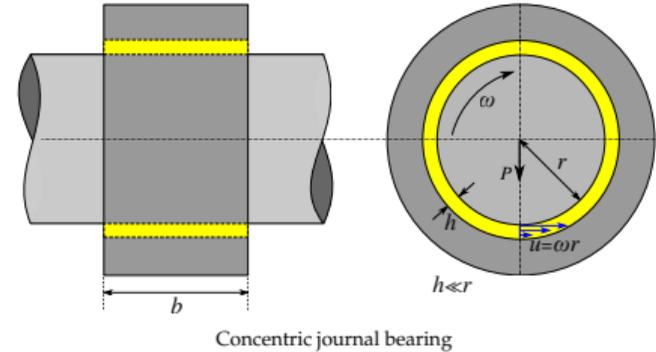
$$\tau = \eta \frac{\partial u}{\partial z} = \eta \frac{U}{h}$$

- Frictional reaction:

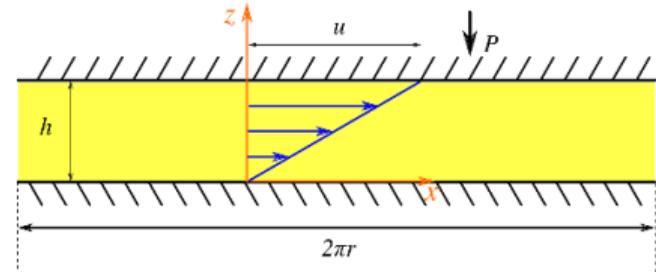
$$T = A\tau = (2\pi r b) \eta \frac{U}{h}$$

- The coefficient of friction:

$$\mu = \frac{T}{P} = \frac{2\pi r b}{h} \frac{\eta U}{P}$$



Petrov's law
N.P. Petrov (1883)



Developed journal and bearing surfaces

r radius of journal

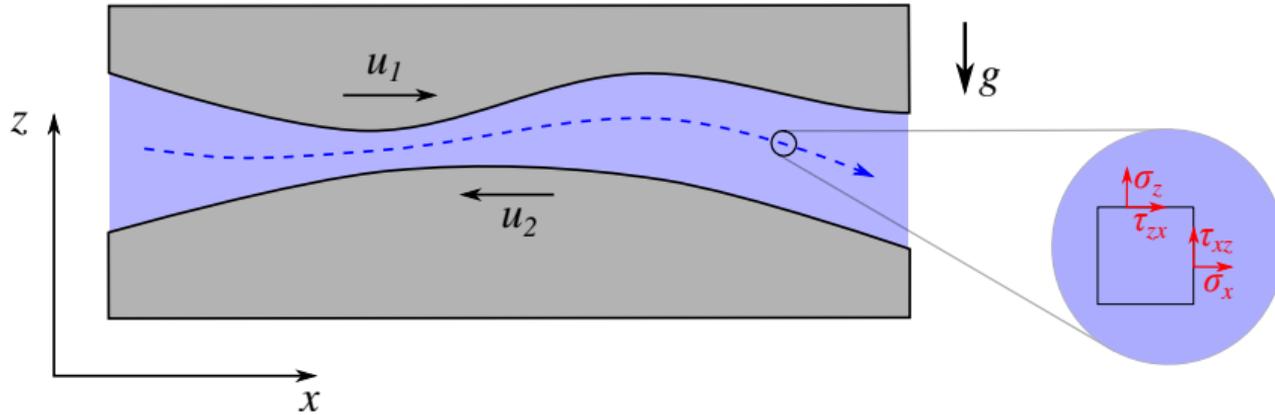
b width of journal

U linear velocity

P normal load

h radial clearance

Stresses on the surface of a fluid element



- Stresses on the surface of a fluid element:

$$\tau_{ij} = \eta \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad \tau_{ij} = \tau_{ji}$$

$$\sigma_i = -p - \frac{2}{3} \eta \nabla \cdot \underline{u} + 2\eta \frac{\partial u_i}{\partial x_i}$$

- η absolute viscosity
- p hydrostatic pressure
- x_i coordinates
- u_i velocity componets
- g acceleration of gravity

Navier-Stokes equations

- Newton's second law of motion for a fluid element:

$$\underbrace{\rho \frac{D\mathbf{u}}{Dt}}_{\text{inertia forces}} = \underbrace{\rho \mathbf{g}}_{\text{body forces}} + \underbrace{\nabla \cdot \underline{\underline{\boldsymbol{\sigma}}}}_{\text{surface forces}}$$

- Material derivative:

$$\underbrace{\frac{Du}{Dt}}_{\text{total derivative}} = \underbrace{\frac{\partial u}{\partial t}}_{\text{local derivative}} + \underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}}_{\text{convective derivative}}$$

- Navier-Stokes equations:

$$\rho \frac{D\mathbf{u}}{Dt} = \rho \mathbf{g} - \nabla p - \frac{2}{3} \nabla (\eta \nabla \cdot \mathbf{u}) + 2 (\nabla \cdot (\eta \nabla)) \mathbf{u} + \nabla \times (\eta (\nabla \times \mathbf{u}))$$

- 4 unknowns: u, v, w, p ; 3 equations + the continuity equation

Continuity equation

- Flux is a mass of fluid flowing per unit time through a unit area:

$$\underline{q} = \rho \underline{u}$$

- Conservation of mass: outflow of mass from a volume equals to decrease of mass within the volume (integral form):

$$\frac{dm}{dt} + \oint_S (\underline{q} \cdot \underline{n}) dS = 0$$

- The divergence theorem:

$$\oint_S (\underline{q} \cdot \underline{n}) dS = \iiint_V (\nabla \cdot \underline{q}) dV$$

- Differential form of continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0$$

- If density ρ is constant: $\nabla \cdot \underline{u} = 0$

Towards the Reynolds equation

- Introducing dimensionless variables:

$$X = \frac{x}{l_0}, \quad Y = \frac{y}{b_0}, \quad Z = \frac{z}{h_0}, \quad T = \frac{t}{t_0}, \quad \bar{u} = \frac{u}{u_0}$$
$$\bar{v} = \frac{v}{v_0}, \quad \bar{w} = \frac{w}{w_0}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{\eta} = \frac{\eta}{\eta_0}, \quad P = \frac{h_0^2 p}{\eta_0 u_0 l_0}$$

- Reynolds number:

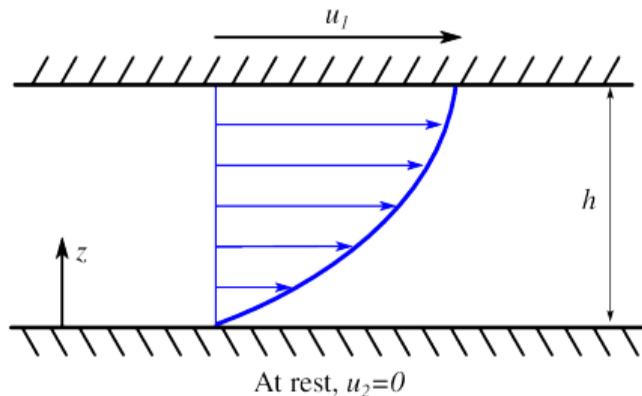
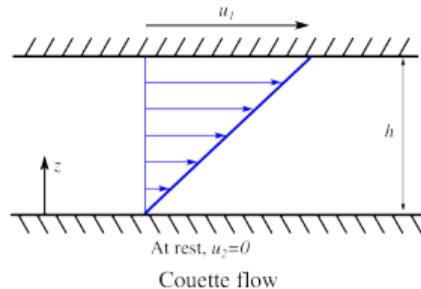
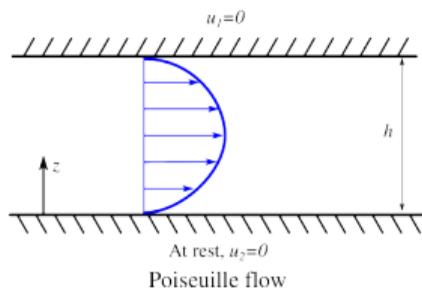
$$\mathcal{R} = \frac{\text{Inertia}}{\text{Viscous}} = \frac{\rho_0 u_0 l_0}{\eta_0}$$

- Thin fluid film: $h_0 \ll l_0, \quad \mathcal{R} \ll 1$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z} \left(\eta \frac{\partial u}{\partial z} \right)$$
$$\frac{\partial p}{\partial y} = \frac{\partial}{\partial z} \left(\eta \frac{\partial v}{\partial z} \right)$$
$$\frac{\partial p}{\partial z} = 0 \rightarrow p = p(x, y)$$

Velocity profile

$$u = \underbrace{\frac{\partial p}{\partial x} \frac{z^2 - zh}{2\eta}}_{\text{Poiseuille flow}} + \underbrace{u_1 \frac{z}{h} + u_2 \left(1 - \frac{z}{h}\right)}_{\text{Couette flow}}$$



Calculating flow rates

$$u = \frac{\partial p}{\partial x} \frac{z^2 - zh}{2\eta} + u_1 \frac{z}{h} + u_2 \left(1 - \frac{z}{h}\right)$$
$$v = \frac{\partial p}{\partial y} \frac{z^2 - zh}{2\eta} + v_1 \frac{z}{h} + v_2 \left(1 - \frac{z}{h}\right)$$

Flow rate per unit width in x and y directions:

$$q'_x = \int_0^h u dz = -\frac{h^3}{12\eta} \frac{\partial p}{\partial x} + \frac{u_1 + u_2}{2} h$$

$$q'_y = \int_0^h v dz = -\frac{h^3}{12\eta} \frac{\partial p}{\partial y} + \frac{v_1 + v_2}{2} h$$

Integrating continuity equation across film thickness:

$$\int_0^h \left[\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{u} \right] dz = 0$$

Reynolds equation

- General form in 3D:

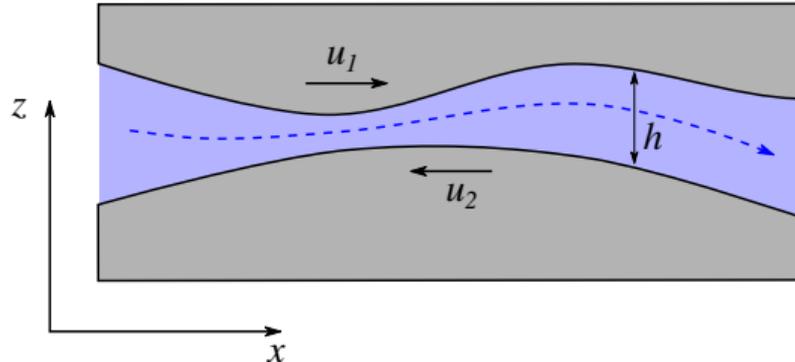
$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial y} \right) = \frac{(u_1 + u_2)}{2} \frac{\partial(\rho h)}{\partial x} + \frac{(v_1 + v_2)}{2} \frac{\partial(\rho h)}{\partial y} + h \frac{\partial \rho}{\partial t}$$

- If ρ, η are constant:

$$\frac{1}{12\eta} \frac{\partial}{\partial x} \left(h^3 \frac{\partial p}{\partial x} \right) + \frac{1}{12\eta} \frac{\partial}{\partial y} \left(h^3 \frac{\partial p}{\partial y} \right) = \frac{(u_1 + u_2)}{2} \frac{\partial h}{\partial x} + \frac{(v_1 + v_2)}{2} \frac{\partial h}{\partial y}$$

- In 2D:

$$\frac{\partial}{\partial x} \left(h^3 \frac{\partial p}{\partial x} \right) = 6\eta(u_1 + u_2) \frac{\partial h}{\partial x}$$



Physically relevant models

- Non-Newtonian fluid:

$$\eta = f\left(\frac{du}{dz}\right)$$

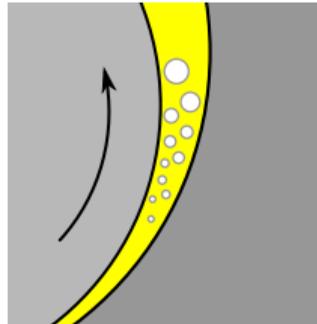
- Viscosity-pressure dependency: Barus law

$$\eta(p) = \eta_0 e^{\alpha p}$$

- Fluid compressibility:

$$K = -V \frac{dp}{dV} = \rho \frac{dp}{d\rho}$$
$$\rho = \rho_0 e^{(p-p_0)/K}$$

- Cavitation: process of bubble generation due to local pressure decline below saturated vapor pressure and subsequent bubble collapse when the pressure is increased.



Step slider bearing

- Reynolds equation
($\rho = \text{const}, \eta = \text{const}$):

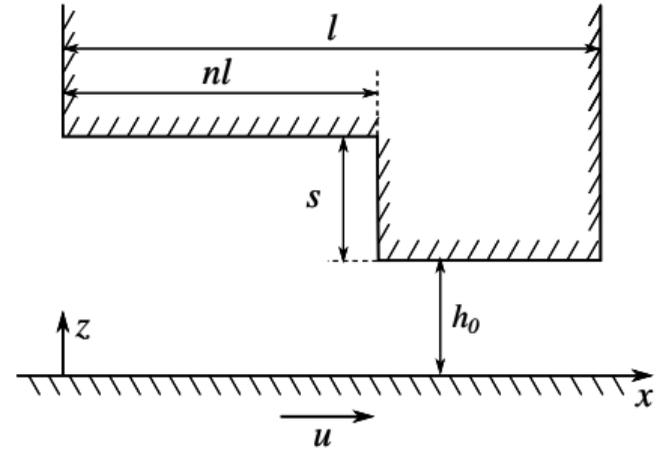
$$\frac{d}{dx} \left(h^3 \frac{dp}{dx} \right) = 6u\eta \frac{dh}{dx}$$

- Constant film thickness in both sections:

$$h(x) = \begin{cases} h_0 + s & 0 < x < nl \\ h_0 & nl < x < l \end{cases}$$

$$\frac{d^2p}{dx^2} = 0, \quad x \in (0; nl) \cup (nl; l)$$

$$\frac{dp}{dx} = \text{const}, \quad x \in (0; nl) \cup (nl; l)$$



Step slider bearing

- Continuity of pressure:

$$p|_{x=nl-0} = p|_{x=nl+0} = p_m$$

$$nl \left(\frac{dp}{dx} \right)_i = -(1-n)l \left(\frac{dp}{dx} \right)_o$$

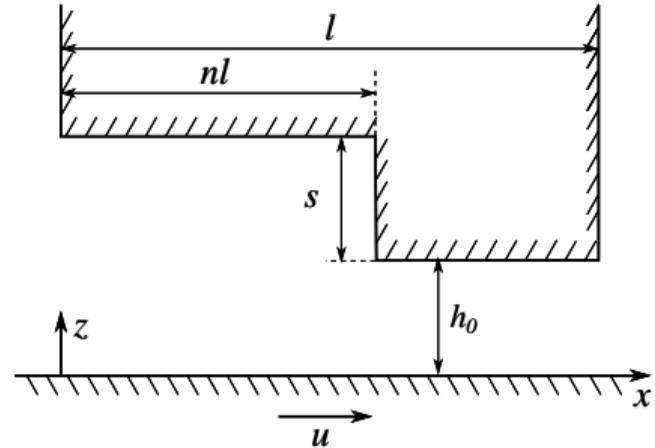
- Continuity of flow rate:

$$q'_{x,i} = q'_{x,o}$$

$$-\frac{(h_0 + s)^3}{12\eta} \left(\frac{dp}{dx} \right)_i + \frac{u(h_0 + s)}{2} = -\frac{h_0^3}{12\eta} \left(\frac{dp}{dx} \right)_o + \frac{uh_0}{2}$$

- Maximum pressure:

$$p_m = 6\eta ul \left[\frac{n(1-n)s}{(1-n)(h_0 + s)^3 + nh_0^3} \right]$$



Step slider bearing

- Pressure distribution:

$$p(x) = \begin{cases} p_m \frac{x}{nl} & 0 < x < nl \\ p_m \frac{l-x}{(1-n)l} & nl < x < l \end{cases}$$

$$p_m = 6\eta ul \left[\frac{n(1-n)s}{(1-n)(h_0+s)^3 + nh_0^3} \right]$$

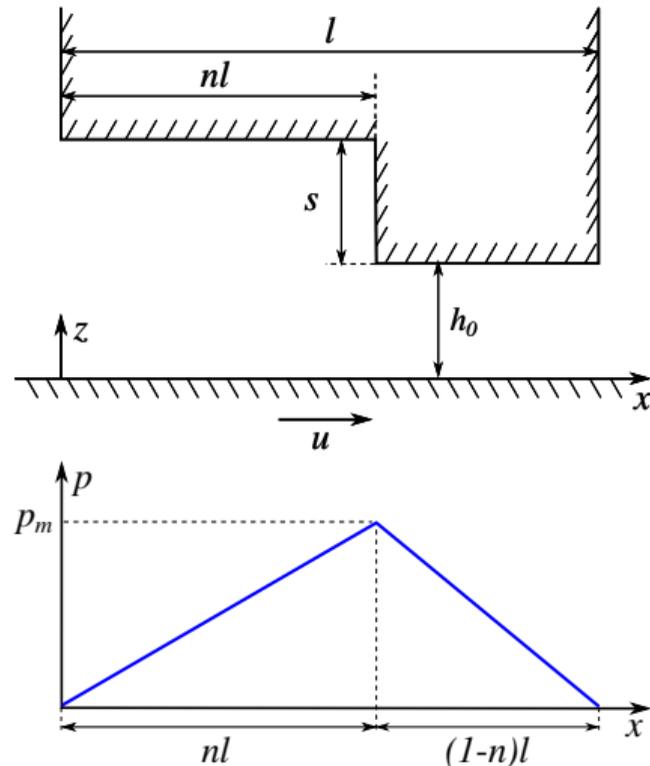
- Optimal bearing configuration to produce the largest p_m :

$$\frac{\partial p_m}{\partial n} = 0 \quad \text{and} \quad \frac{\partial p_m}{\partial s} = 0$$

$$\begin{cases} (1-n)^2(h_0+s)^3 - n^2h_0^3 = 0 \\ (1-n)(h_0+s)^2(h_0-2s) + nh_0^3 = 0 \end{cases}$$

- Optimal values:

$$\frac{h_0}{s} = 1.155, \quad n = 0.7182$$



Elastohydrodynamic lubrication (EHL)

- Non-conforming surfaces
- Elastic deflection of solid walls
- Viscosity-pressure dependence: $\eta(p) = \eta_0 \exp \xi p$

- Hard EHL (metal parts):

$$0.5 \text{ GPa} \leq p \leq 3 \text{ GPa}$$

$$0.1 \text{ } \mu\text{m} \leq h_{\min} \leq 1 \text{ } \mu\text{m}$$

- gears
- rolling bearings
- cams

- Soft EHL (polymer):

$$p \approx 1 \text{ MPa}$$

$$h_{\min} \approx 1 \text{ } \mu\text{m}$$

- seals
- human joints
- tires



Needle roller bearing
www.farazbearing.com



Bevel gear www.linngear.com

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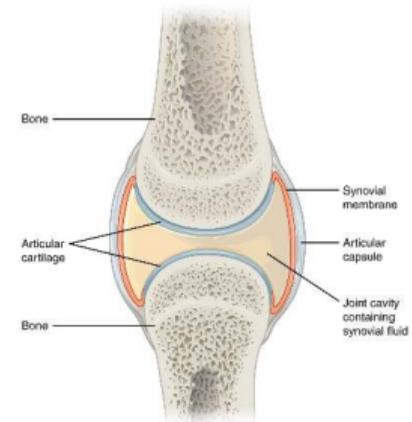
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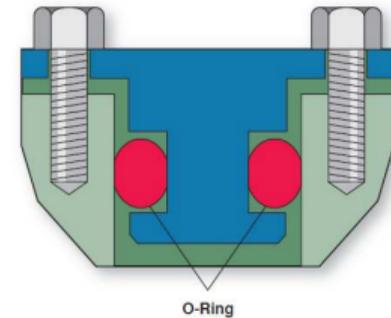
$$p \approx 1 \text{ MPa}$$

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- seals
- human joints
- tires



Nontrivial joint
www.wikipedia.org



O-ring seal
www.ecosealthailand.com

Elastohydrodynamic lubrication (EHL)

- Reynolds equation:

$$\frac{d}{dx} \left(\frac{h^3}{\eta} \frac{dp}{dx} \right) = 6u \frac{dh}{dx}$$

- Viscosity-pressure dependence (Barus law):

$$\eta(p) = \eta_0 e^{\xi p}$$

- Film shape:

$$h(x) = h_0 + S(x) + \delta(x)$$

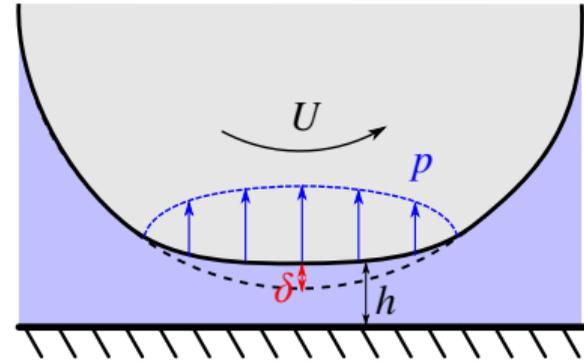
h_0 constant

$$S(x) = \frac{x^2}{2R} \quad \text{undeformed geometry}$$

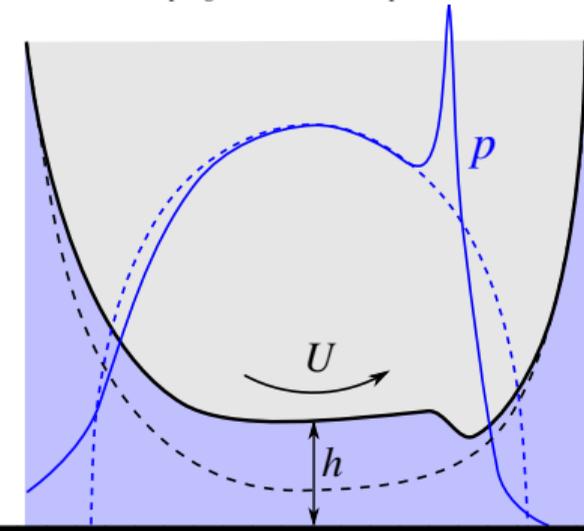
$\delta(x)$ elastic deformation

- Contact constraints:

$$\begin{cases} h_0 + S(x) + \delta(x) = 0, & p > 0 \\ h_0 + S(x) + \delta(x) > 0, & p = 0 \end{cases}$$

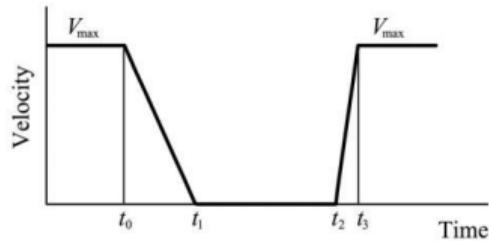


Coupling of fluid and elastic problems



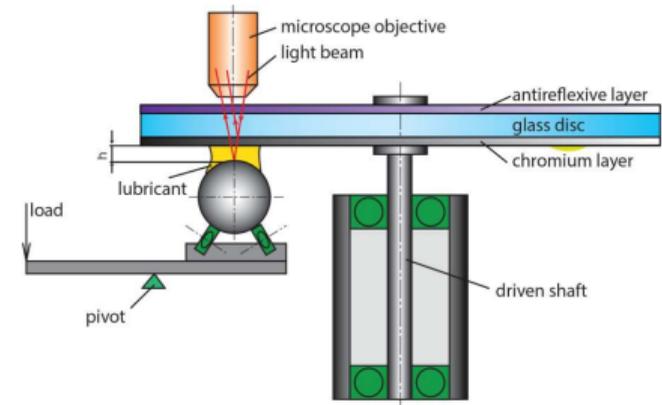
EHL film thickness: experimental observation

- Ball-on-disk optical tribometer
- Measurement based on light interference principle
- Unidirectional start-stop-start motion
- Important for study of rolling contact fatigue and wear



Unidirectional start-stop-start motion^[1]

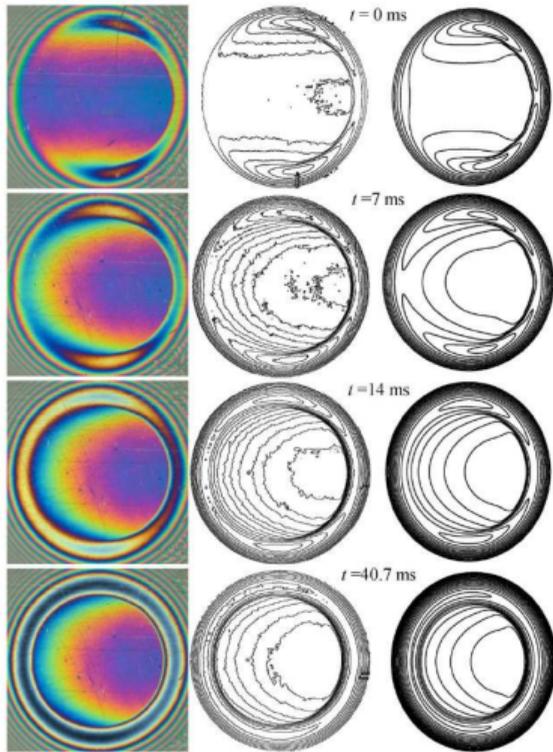
[1] P. Sperka et al, *Journal of Tribology* (2014)



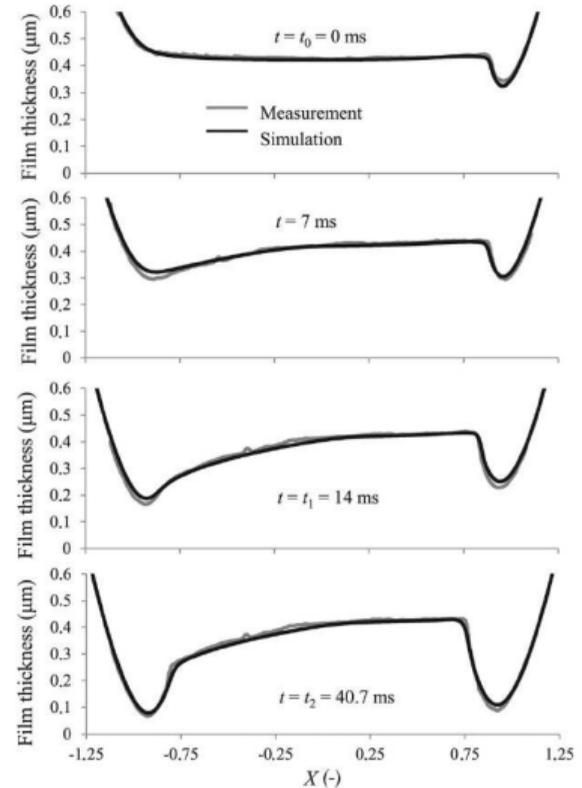
Ball-on-disk apparatus with interferometry^[2]

[2] D. Kostal et al, *Journal of Tribology* (2017)

EHL film thickness: experimental observation

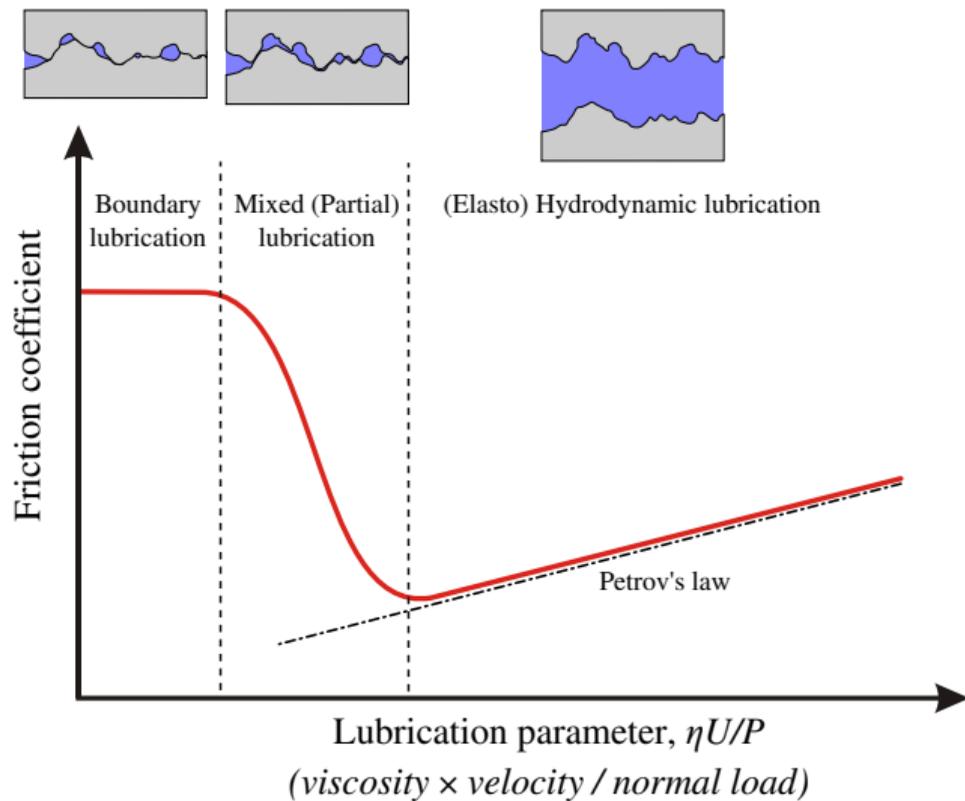


Left to right:
snapshots of interferograms, film thickness contour maps, results of
numerical simulation^[1]



Midplane film thickness profiles along rolling direction^[1]

Lubrication regimes: Stribeck curve



Adapted from www.wikipedia.org

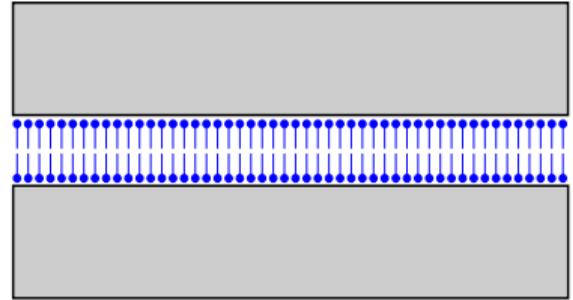
Boundary lubrication

- Asperities come in contact
- $1 \text{ nm} \leq h_{\min} \leq 10 \text{ nm}$
- Bulk lubricant properties (i.e. viscosity) are not important
- Physical and chemical properties of the surface and of the fluid film are important
- Lubricant film of molecular size^[1]
- Breakdown of lubricant film at localized regions^[2], frictional force:

$$F = A(\alpha s_m + (1 - \alpha)s_l)$$

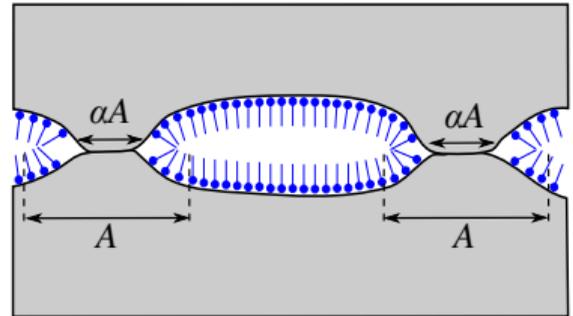
- A the area that supports the load
 α fraction of breakdown area
 s_m shear stress in solid contact
 s_l shear stress in the lubricating film

Therefore, if α - const, then $F \propto A$.



The frictional resistance is due to interaction between the outer surfaces of the adsorbed monolayers without any solid contact occurring^[1]

[1] W.B. Hardy, (1936)



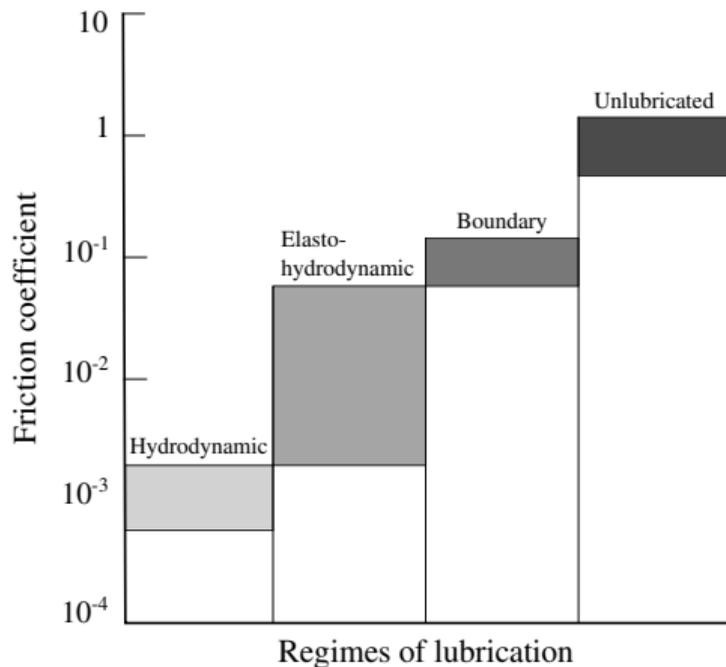
Mechanism involving breakdown of the lubricant film at small localized regions^[2]

[2] F.P. Bowden and D. Tabor, "The friction and Lubrication of Solids" (1950)

Mixed (partial) lubrication

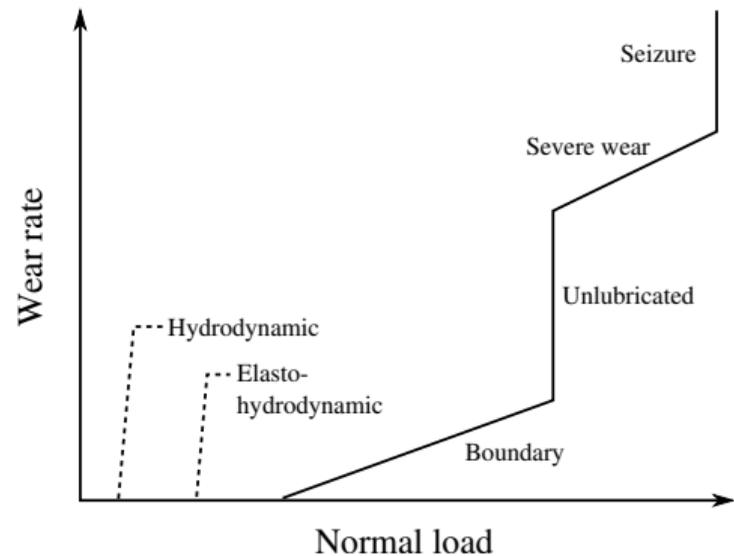
- Combination of boundary and fluid film effects
- Some asperity contact
- Film layer of one or more molecular layers
- Smooth transition
- $0.01 \mu\text{m} \leq h_{\min} \leq 1 \mu\text{m}$

Lubrication: friction coefficient and wear



Bar diagram of friction coefficient for various lubrication conditions

B.J. Hamrock, "Fundamental of fluid film lubrication" (1991)



Wear rate for various lubrication regimes

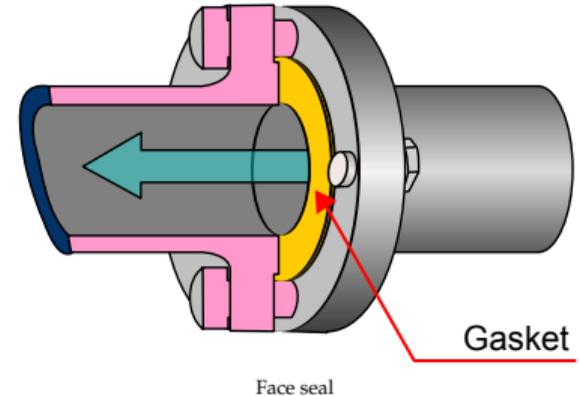
Beerbower (1972)

Lubrication: take home messages

- Lubrication: reduction of wear and friction between relatively moving surfaces by adding lubricant
- Hydrodynamic: full separation of solids, thin fluid film lubrication (Reynolds equation), dynamic viscosity is important
- EHL: solids deform elastically (hard - metals, soft - polymers), affected by viscosity-pressure dependence
- Boundary: contact of asperities, but still thin molecular level of lubricant, chemical properties important, breakdown of fluid film
- Mixed (partial): smooth transition
- Recommended literature:
 - 1 B.J. Hamrock et al, "Fundamentals of fluid film lubrication" (2004)
 - 2 F.P. Bowden and D. Tabor, "The friction and Lubrication of Solids (1950)
 - 3 D. Dowson, "Elastohydrodynamic and micro-elastohydrodynamic lubrication", *Wear*, 190 (1995)

Sealing: what is it?

- **Sealing:** technique to prevent or reduce leakage of fluid from one chamber to another using seals
- Different types:
 - face seals
 - O-ring seals
 - labyrinth seals
- Dynamic/static
- Material: polymer/metallic
- Operate in EHL/mixed/boundary regimes



O-ring

Application: metal-to-metal static face seal

- Metal-to-metal static face seals used in fluid system of nuclear power plants
- Coating of the seal is made of material Norem^[1]: elasto-plastic
Elastic moduli:

$$E = 175 \text{ GPa}, \nu = 0.3$$

Yield stress:

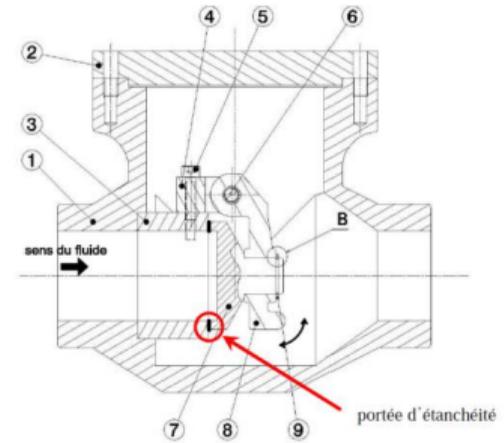
$$\sigma_Y = R_0 + Q(1 - e^{-bp})$$

$$R_0 = 442.7 \text{ MPa}$$

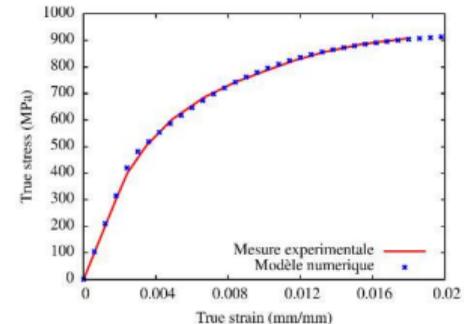
$$Q = 493.5 \text{ MPa}$$

$$b = 242.2$$

J. Durand, PhD thesis (2012)

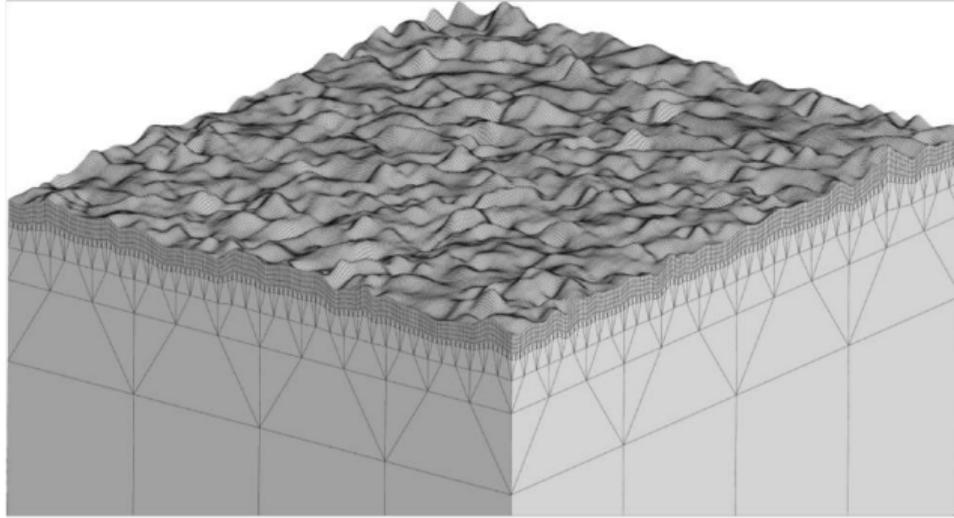


Sketch of a valve^[1]

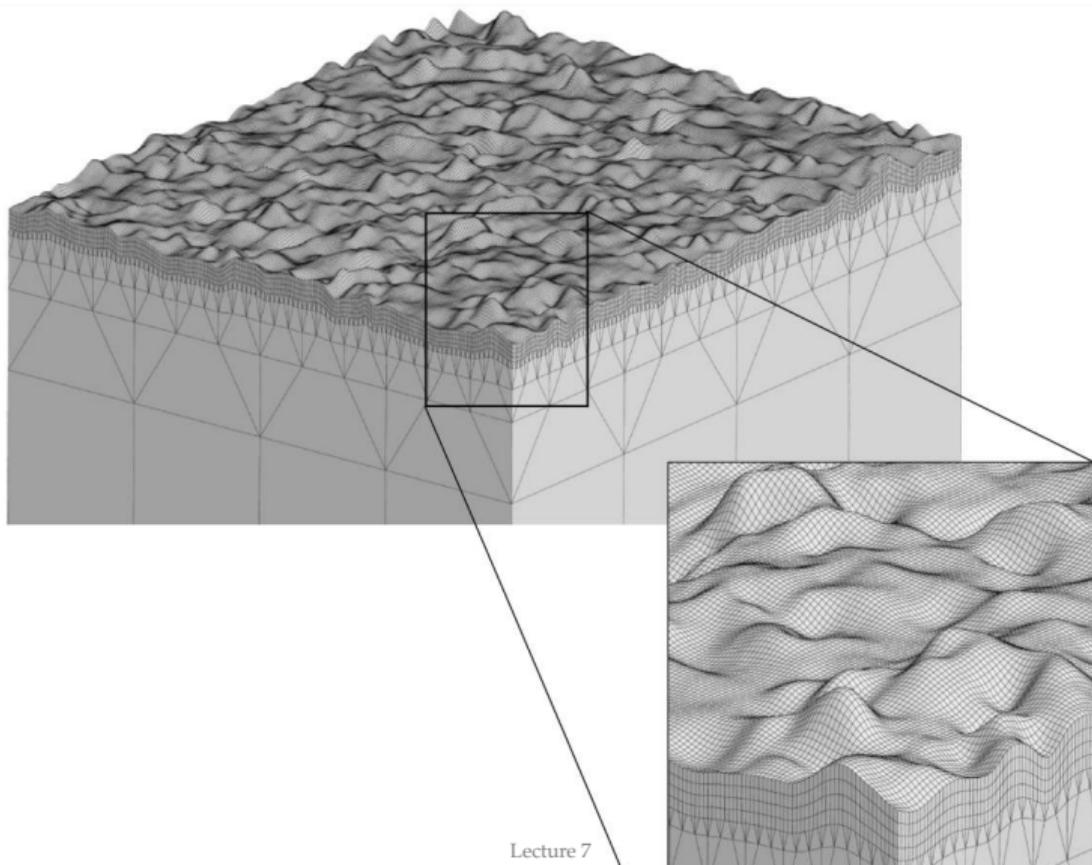


Material behavior of Norem^[1]

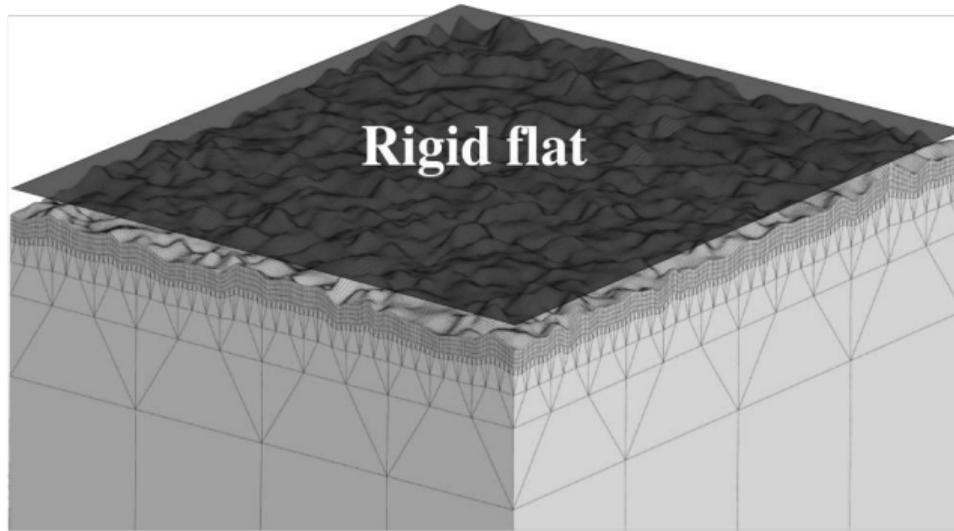
Problem statement



Problem statement

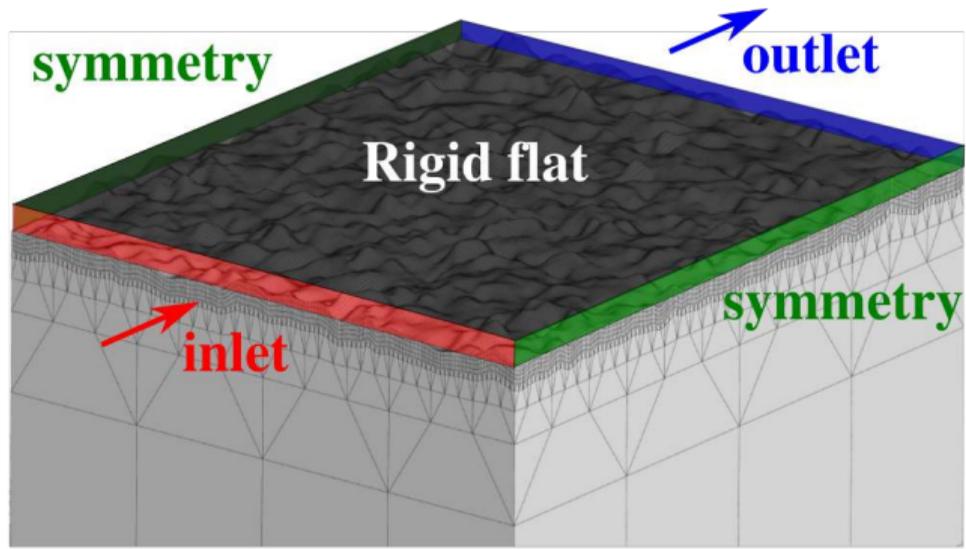


Problem statement



external load

Problem statement



Problem statement

- Mechanical contact (unilateral):

$$\begin{cases} \nabla \cdot \underline{\underline{\sigma}}(\underline{\mathbf{u}}) = 0 & \text{in } \Omega_s, \\ g(\underline{\mathbf{u}}) \geq 0, \sigma_n(\underline{\mathbf{u}}) \leq 0, g(\underline{\mathbf{u}}) \sigma_n(\underline{\mathbf{u}}) = 0 & \text{at } \Gamma_c, \\ u_x|_{x=0, \lambda/2} = 0, \quad u_y|_{y=0, L} = 0, \end{cases}$$

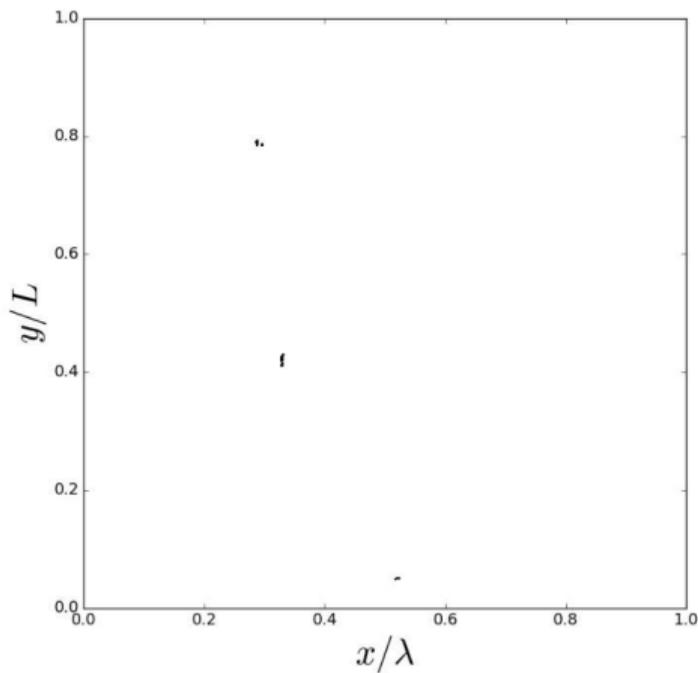
- Thin fluid flow with immobile walls (Reynolds equation):

$$\begin{cases} \nabla \cdot [g(\underline{\mathbf{u}})^3 \nabla p_f] = 0 & \text{in } \Gamma_f \\ p_f|_{y=0} = p_i, \quad p_f|_{y=L} = p_o \\ [\nabla p_f \cdot \underline{\mathbf{e}}_x]|_{x=0, \lambda/2} = 0, \end{cases}$$

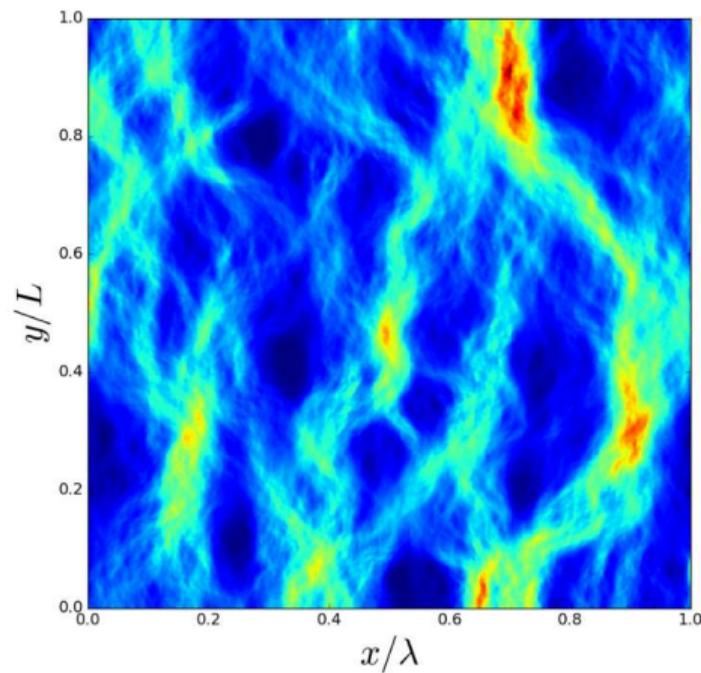
- Fluid/structure interface:

$$\sigma_n(\underline{\mathbf{u}}) = -p_f \quad \text{at } \Gamma_f$$

Results of the numerical simulation

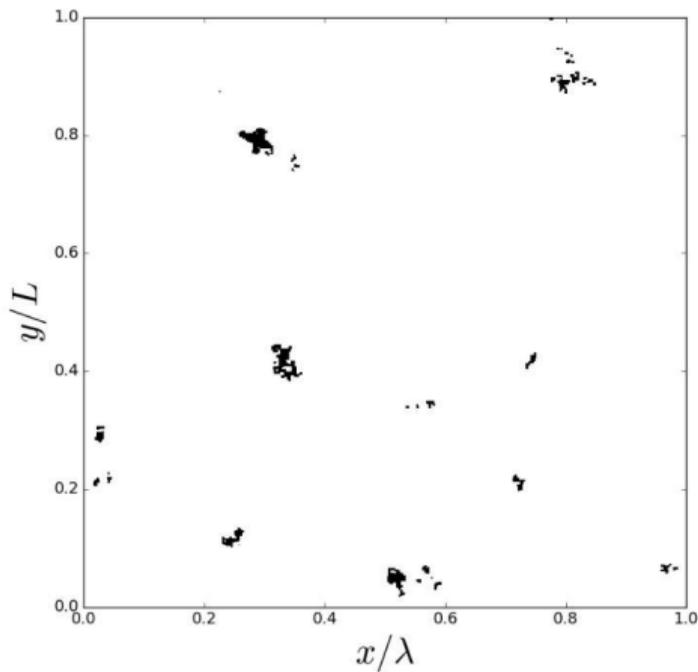


Morphology of the contact interface

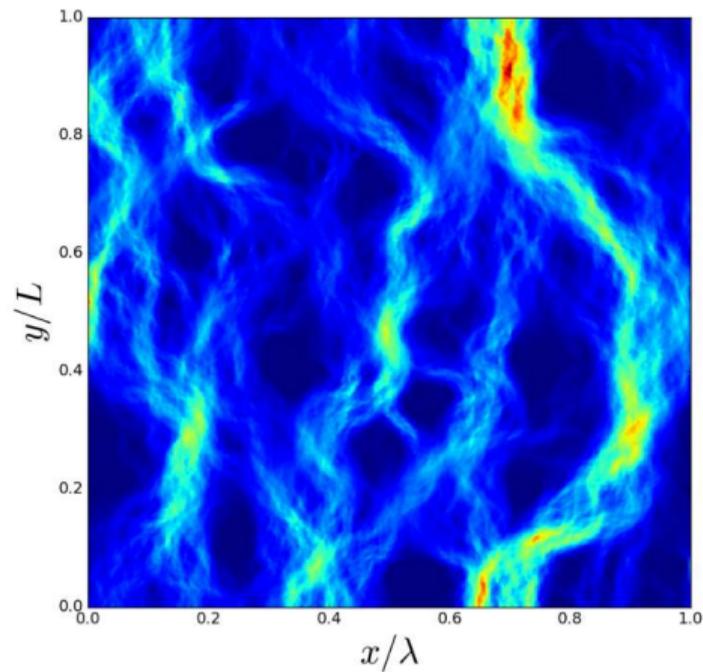


Intensity of the fluid flux

Results of the numerical simulation

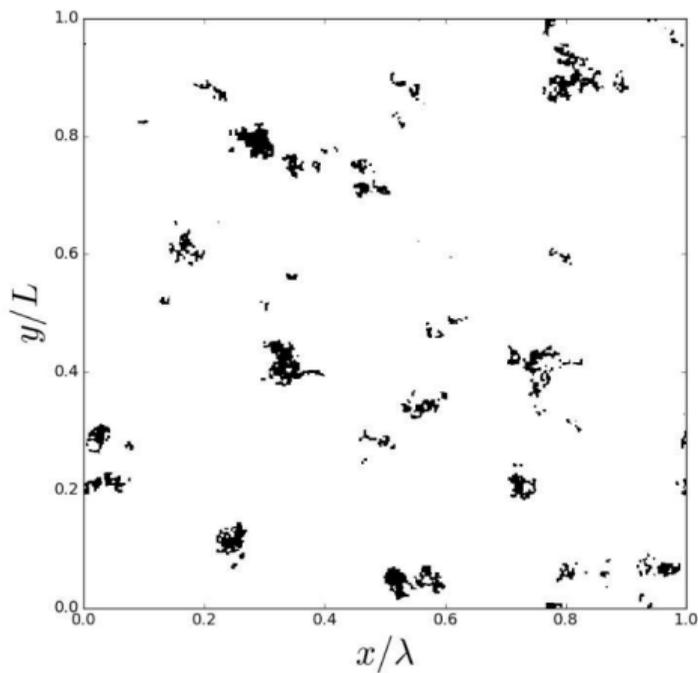


Morphology of the contact interface

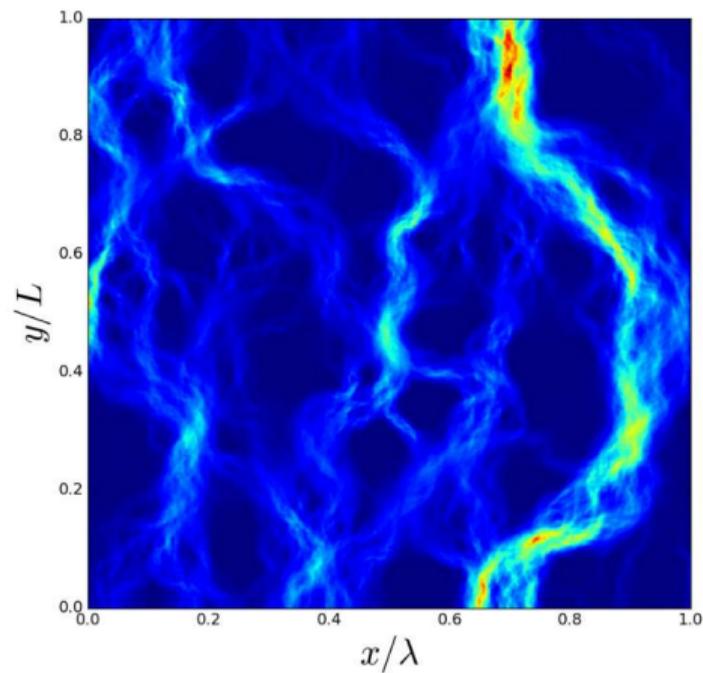


Intensity of the fluid flux

Results of the numerical simulation

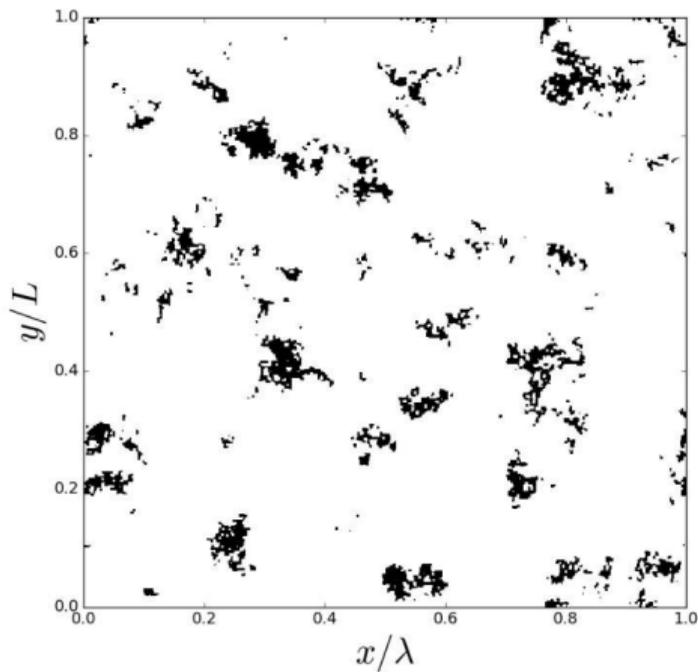


Morphology of the contact interface

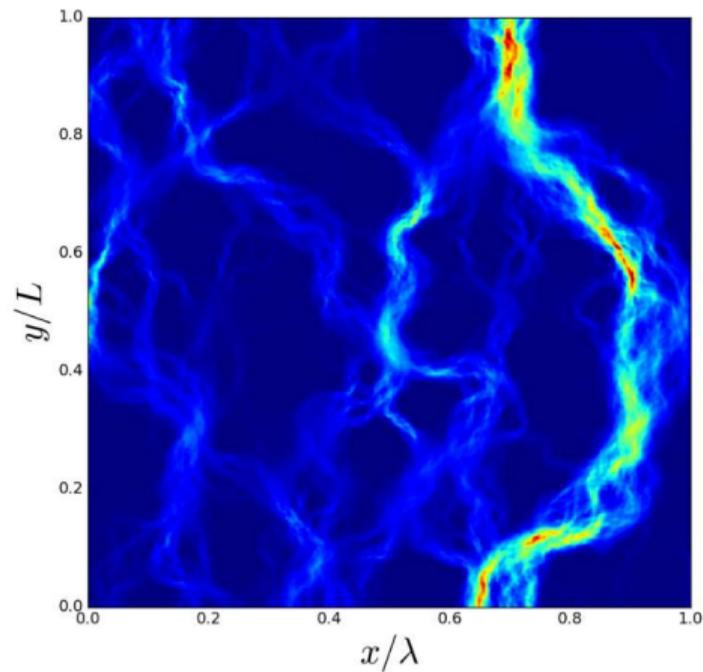


Intensity of the fluid flux

Results of the numerical simulation

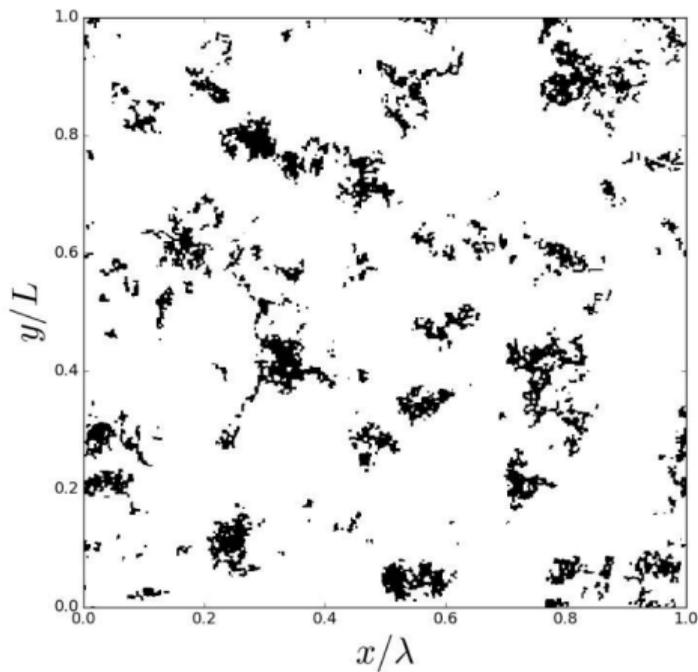


Morphology of the contact interface

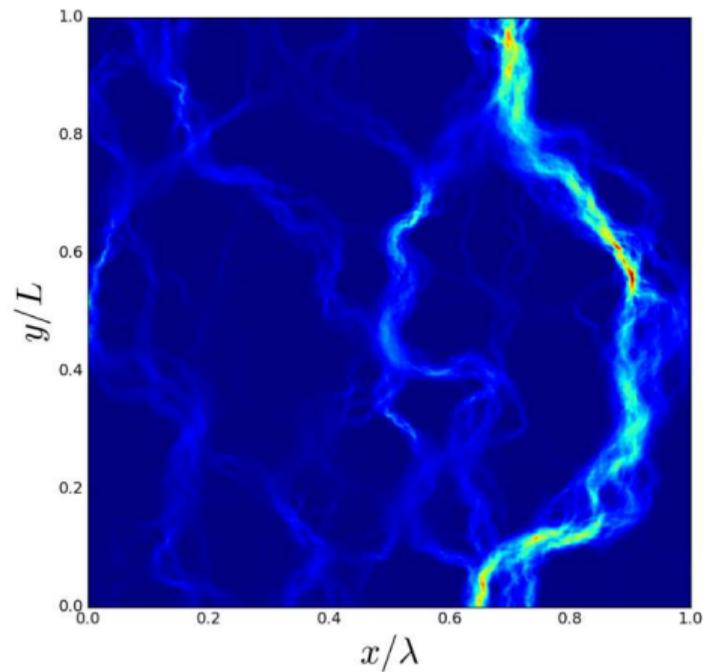


Intensity of the fluid flux

Results of the numerical simulation

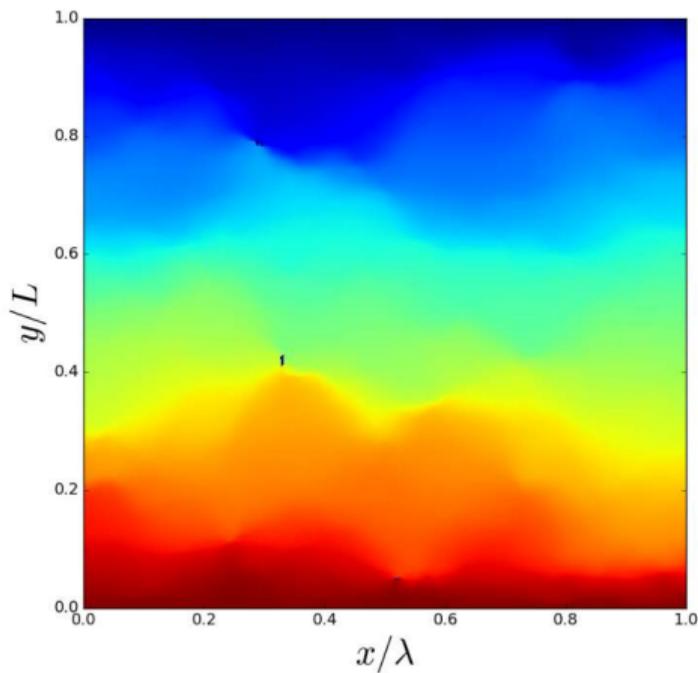


Morphology of the contact interface

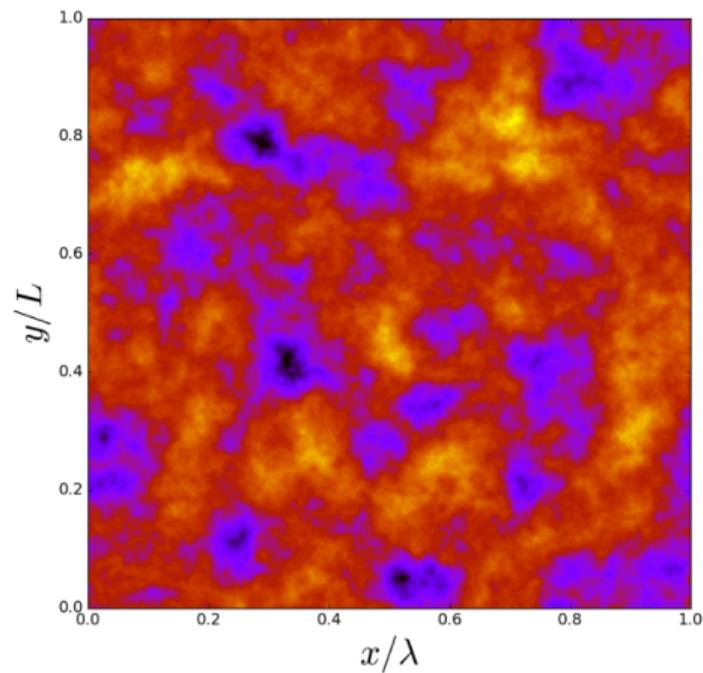


Intensity of the fluid flux

Results of the numerical simulation

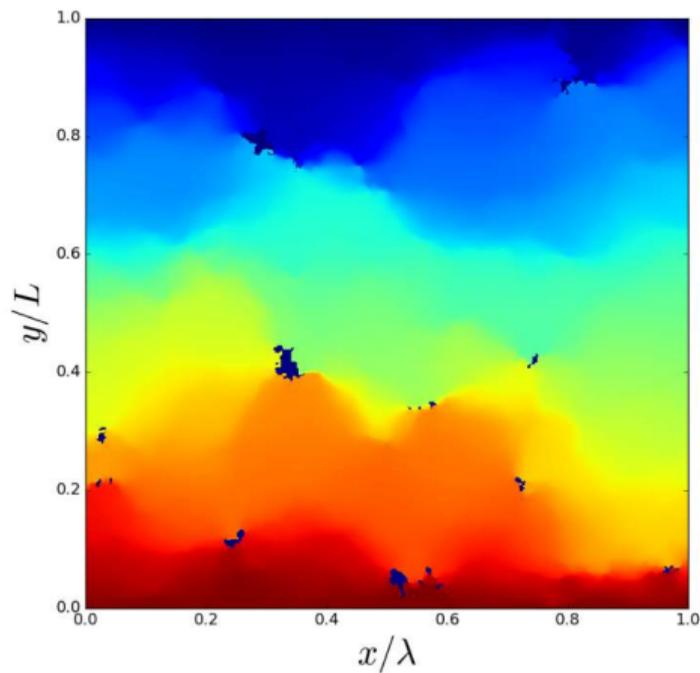


Distribution of the fluid pressure

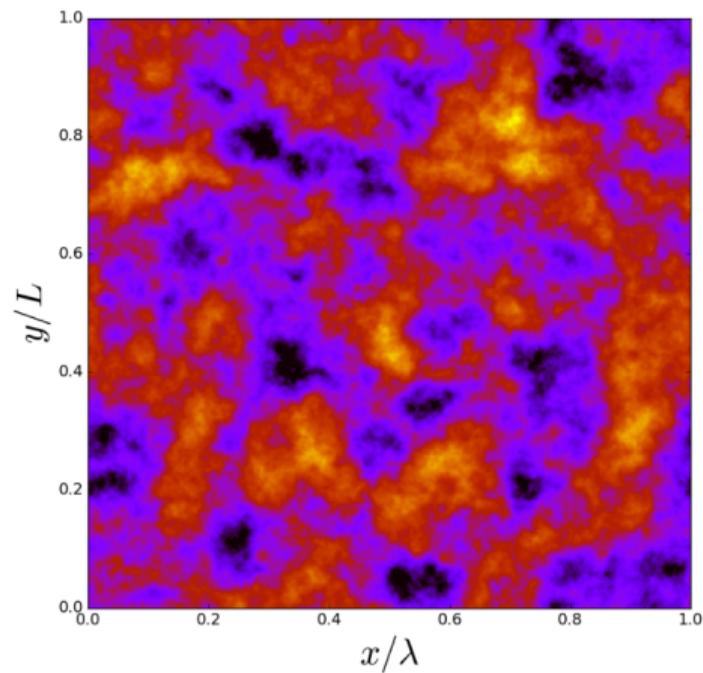


Distribution of the free volume

Results of the numerical simulation

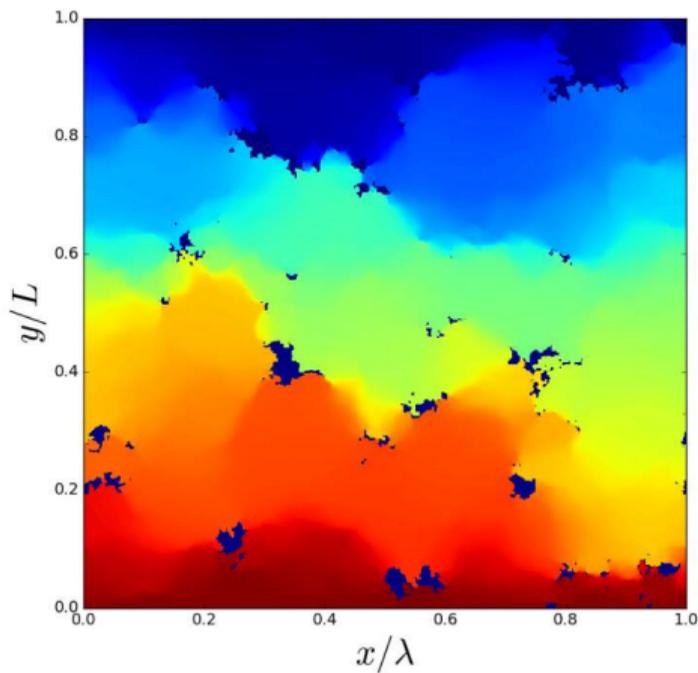


Distribution of the fluid pressure

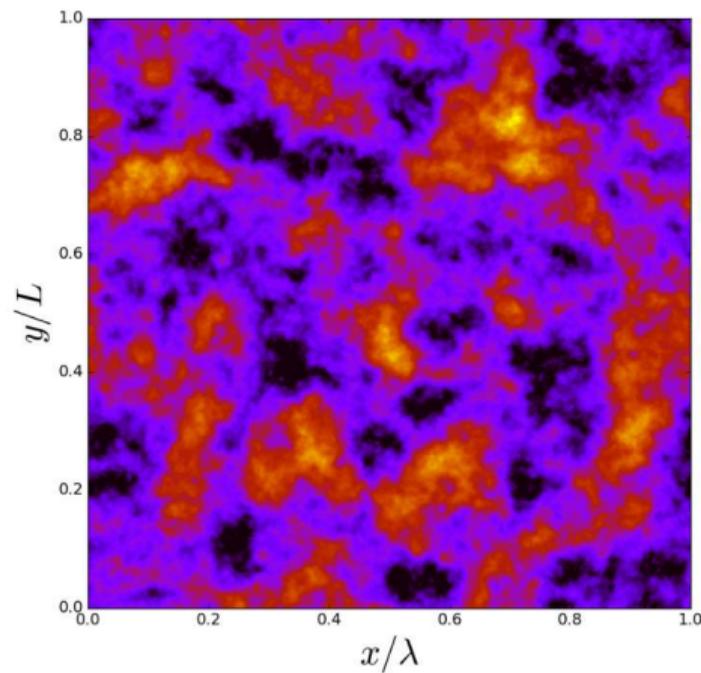


Distribution of the free volume

Results of the numerical simulation

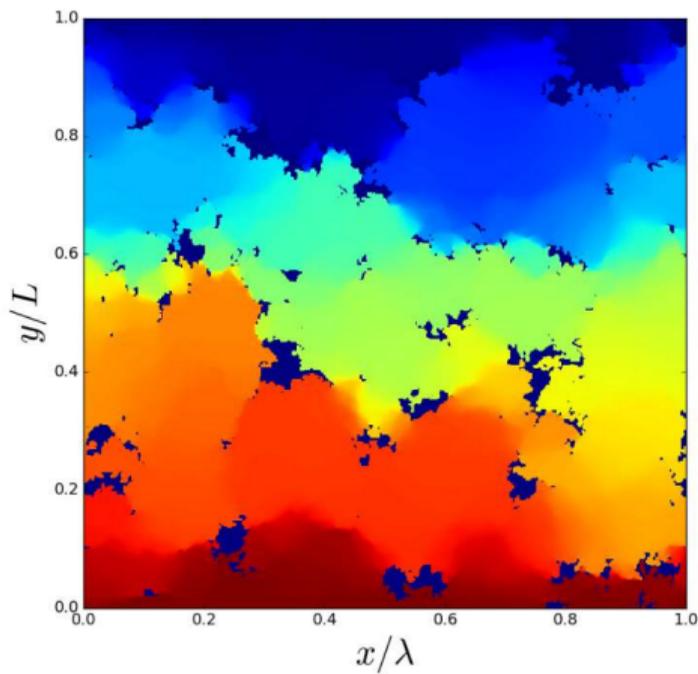


Distribution of the fluid pressure

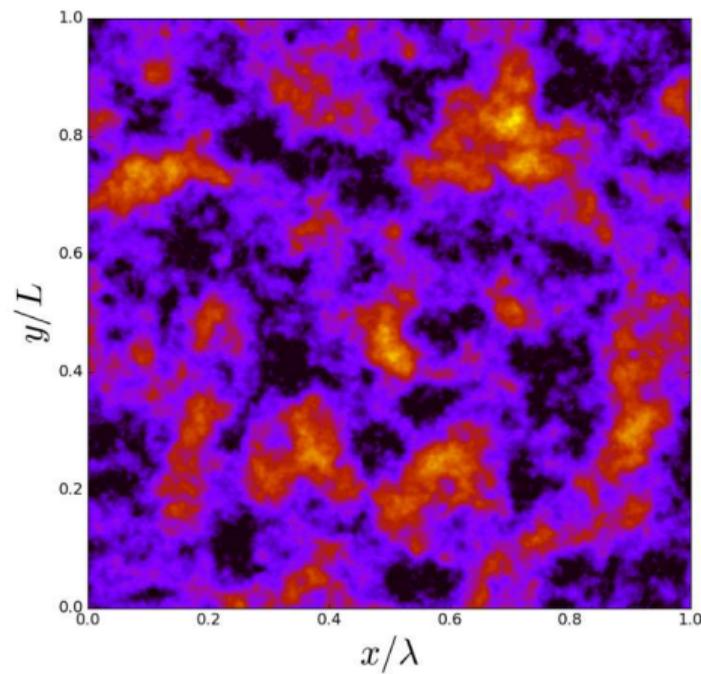


Distribution of the free volume

Results of the numerical simulation

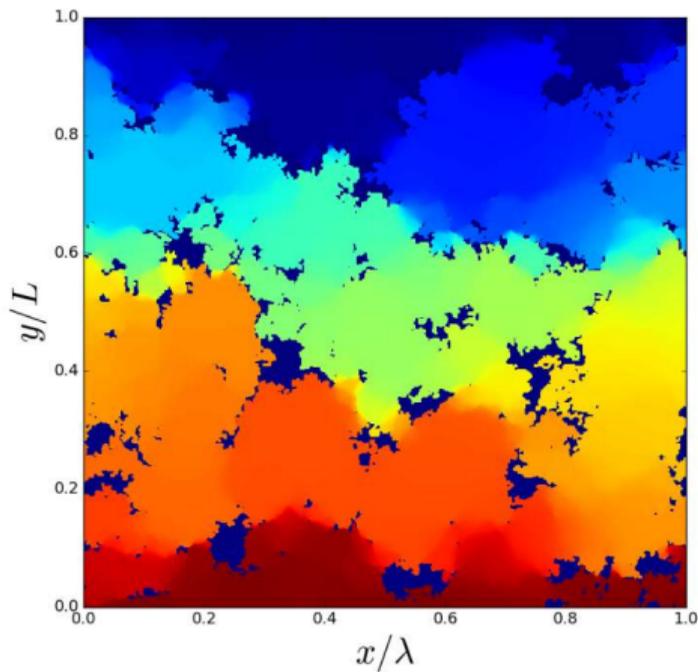


Distribution of the fluid pressure

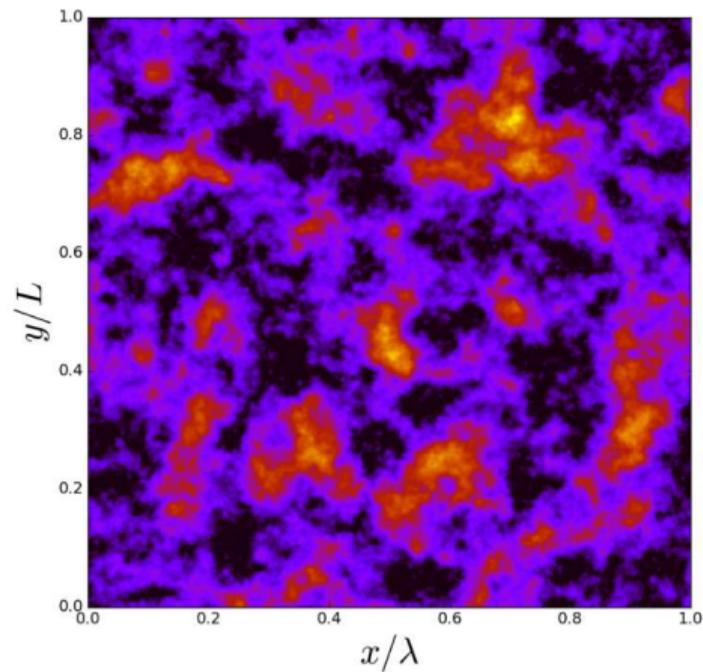


Distribution of the free volume

Results of the numerical simulation

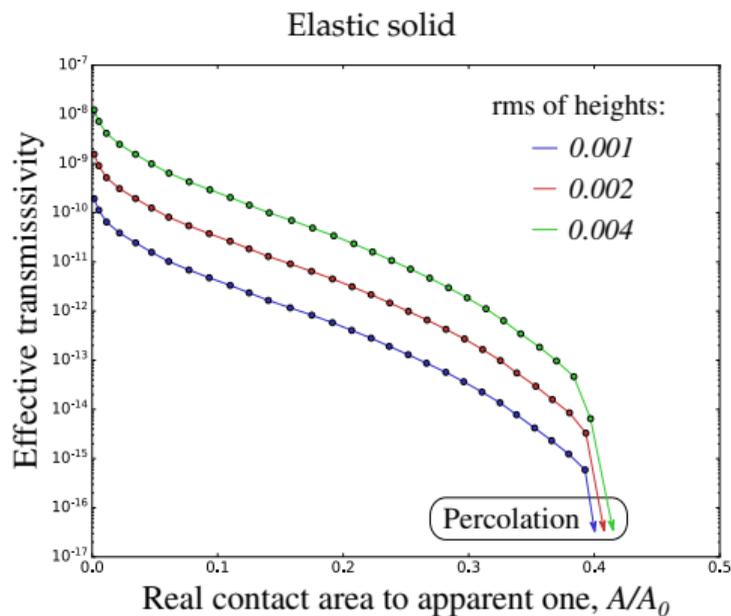


Distribution of the fluid pressure



Distribution of the free volume

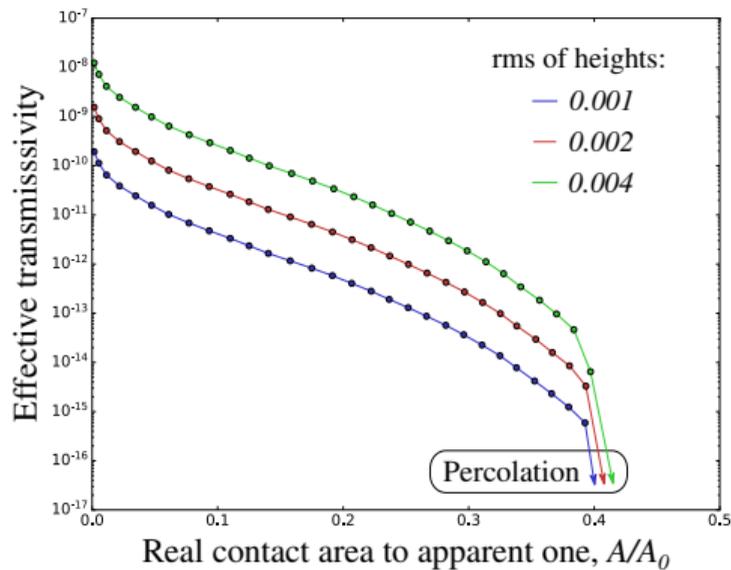
Transmissivity of the interface



Effective transmissivity of the interface
in case of elastic material
(loading until percolation)

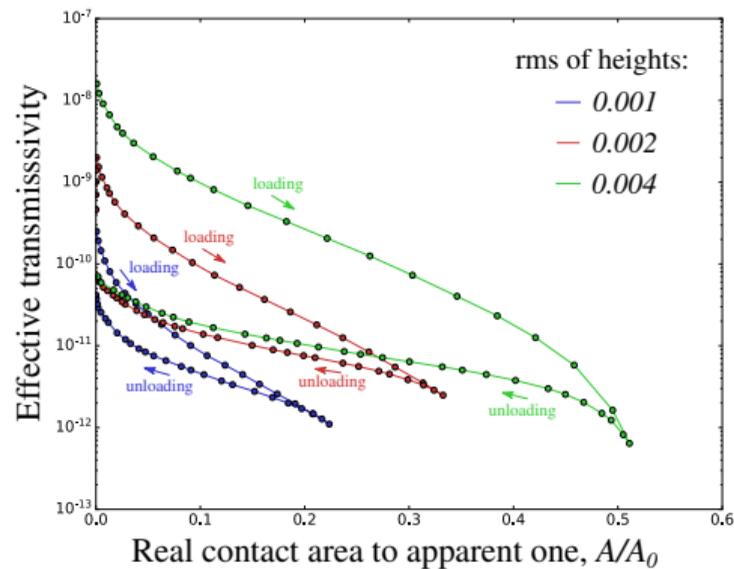
Transmissivity of the interface

Elastic solid

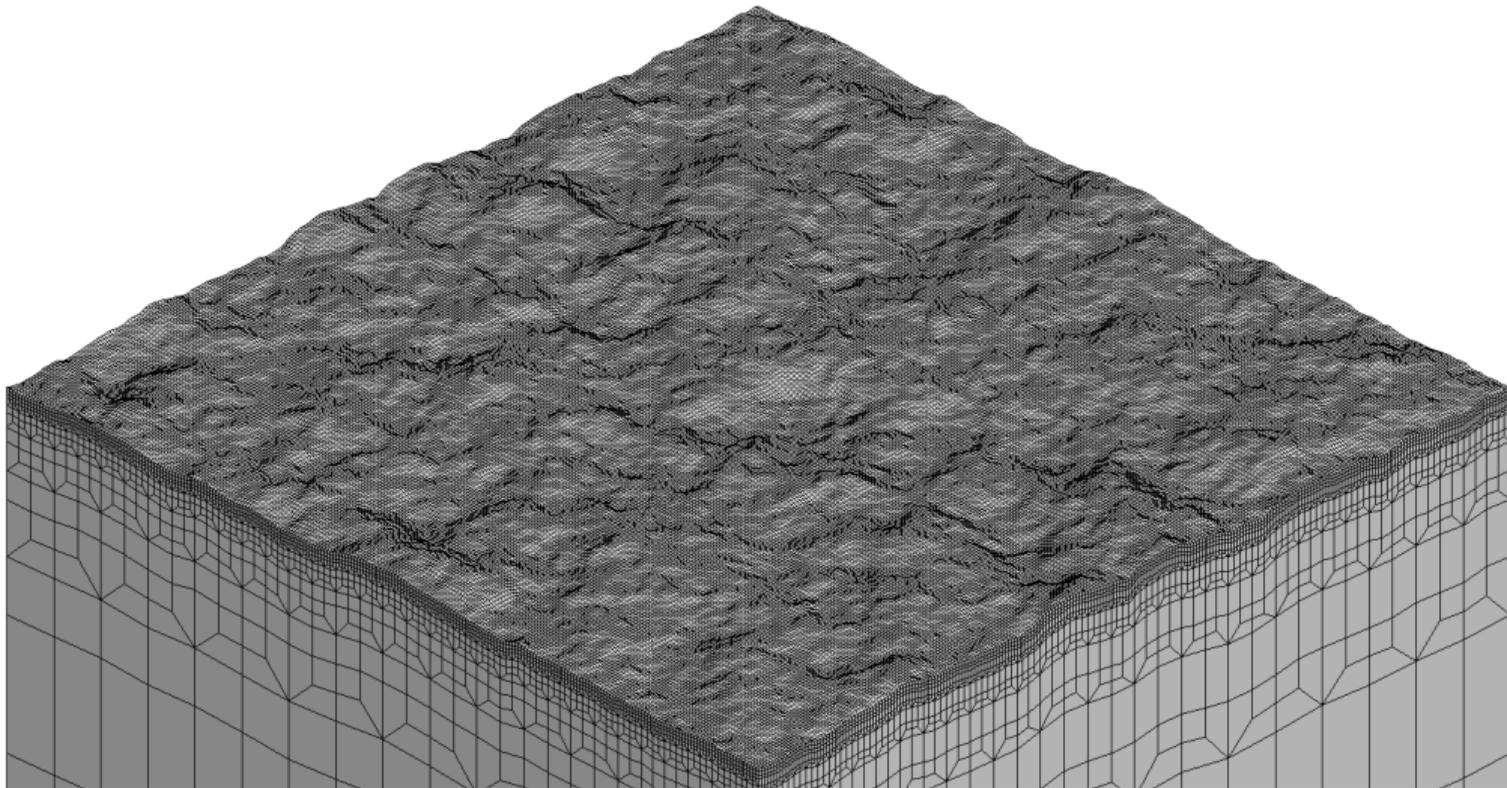


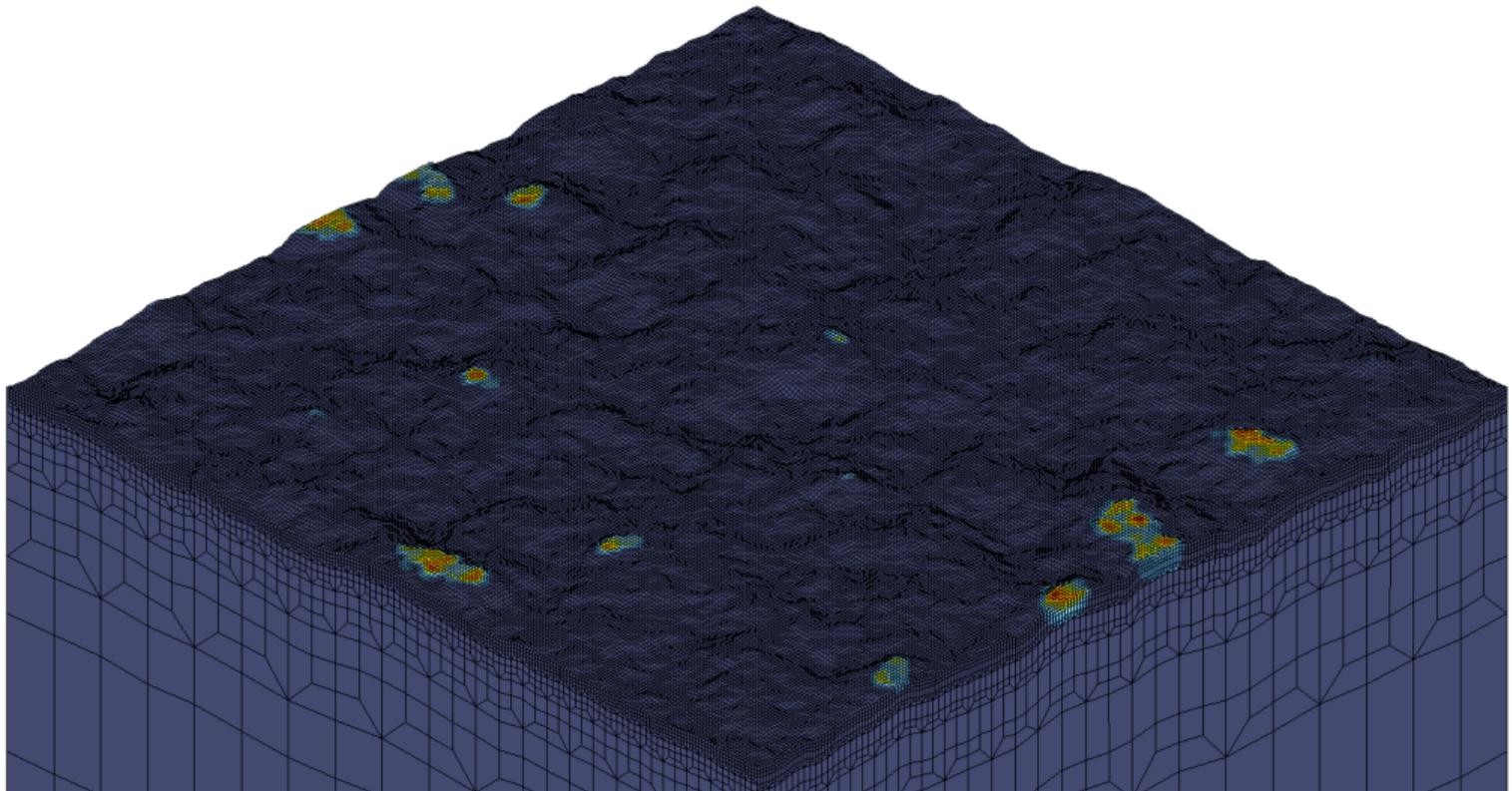
Effective transmissivity of the interface
in case of elastic material
(loading until percolation)

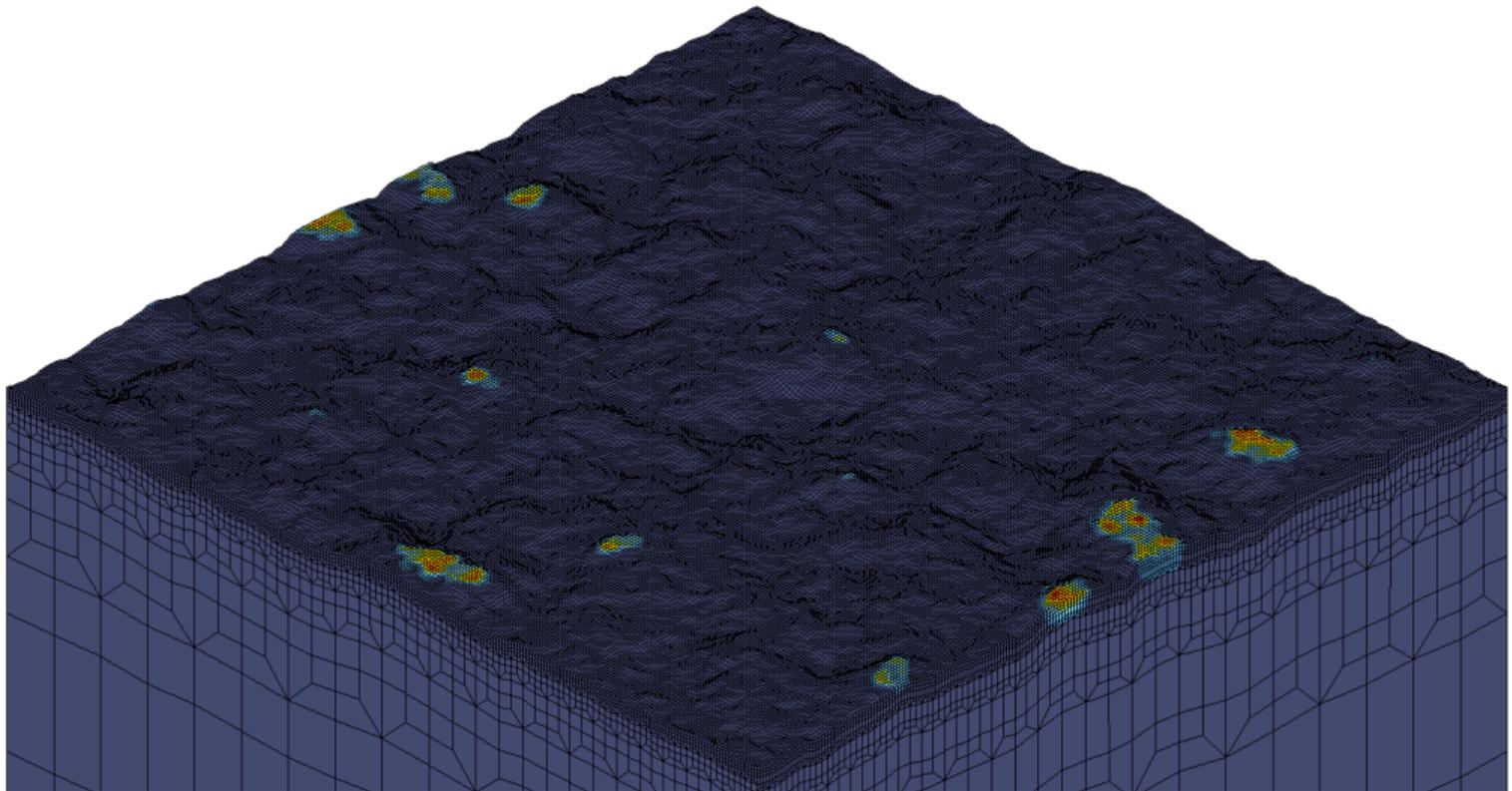
Elasto-plastic solid

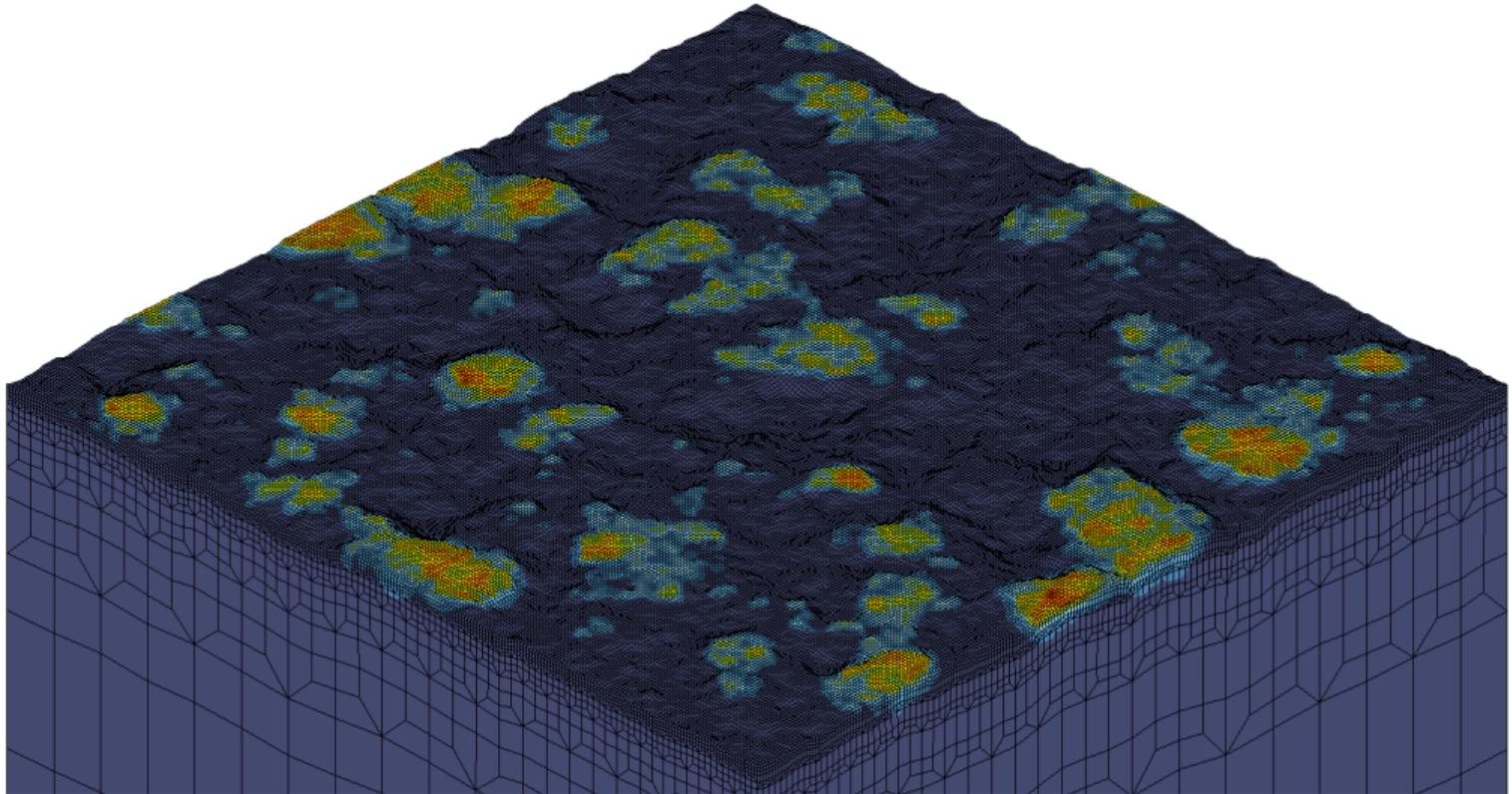


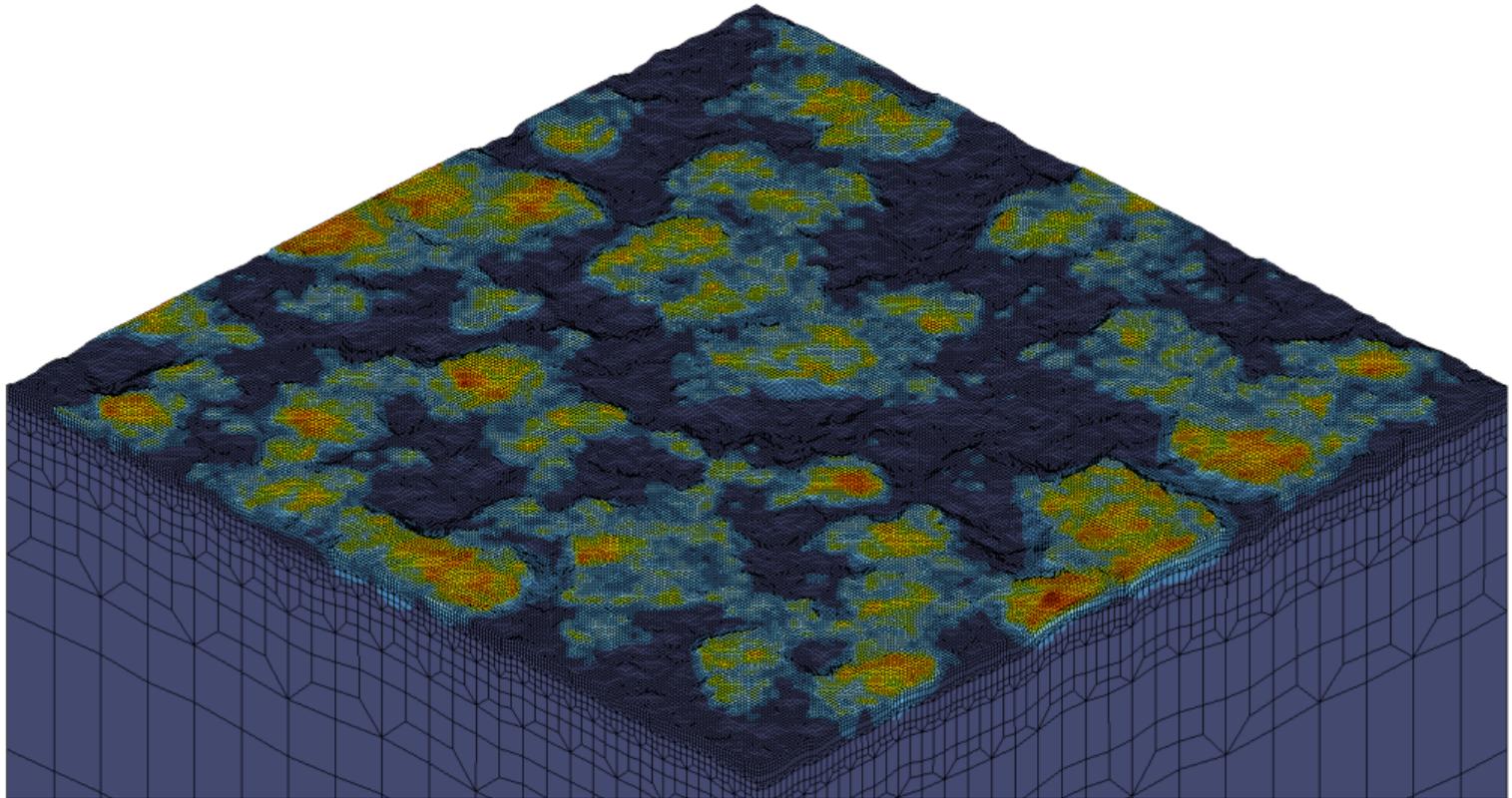
Effective transmissivity of the interface
in case of elasto-plastic material
(loading-unloading cycle)

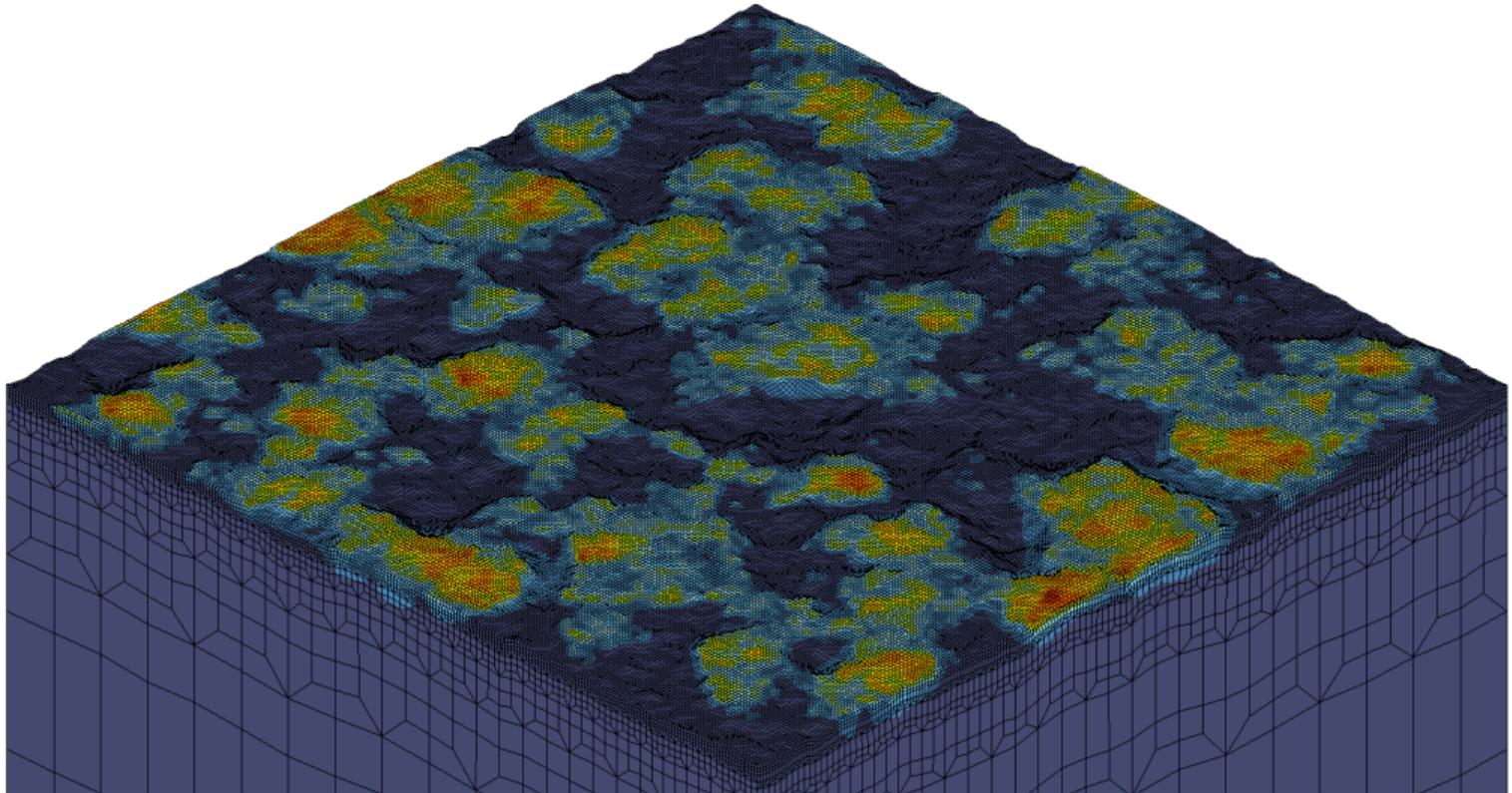




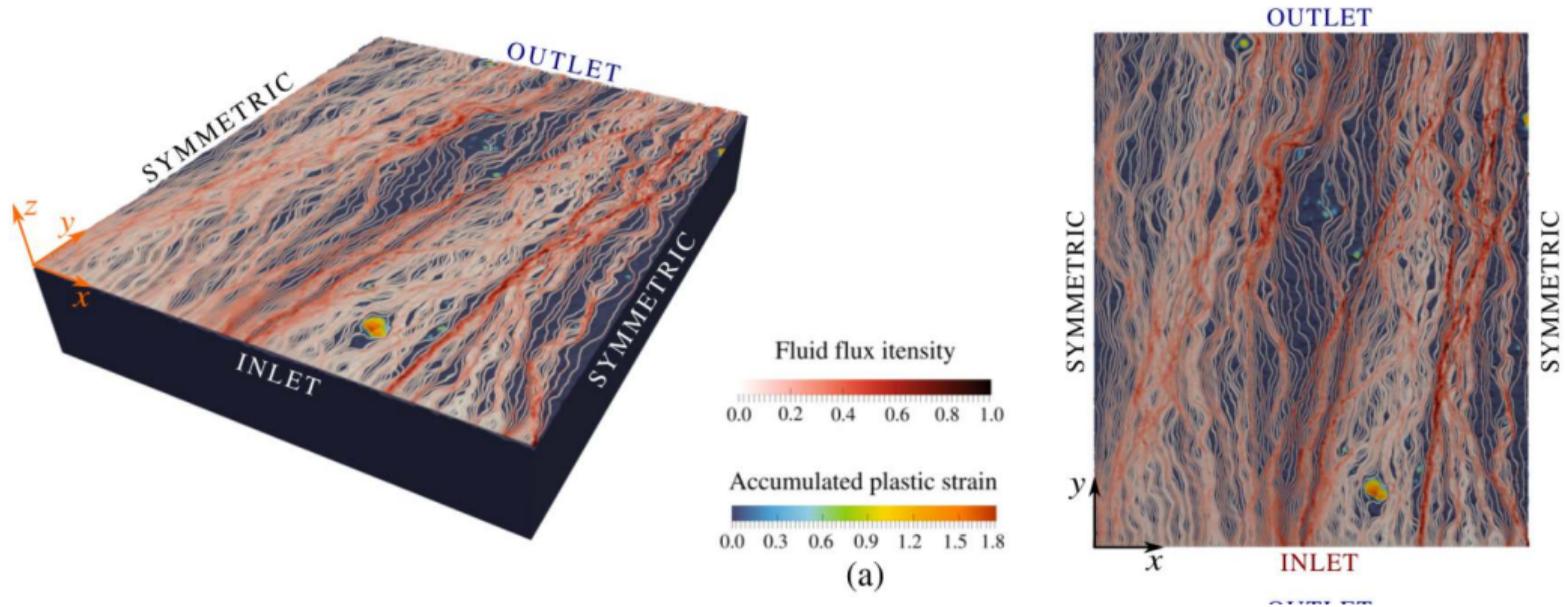




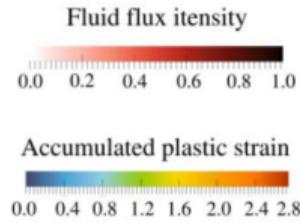
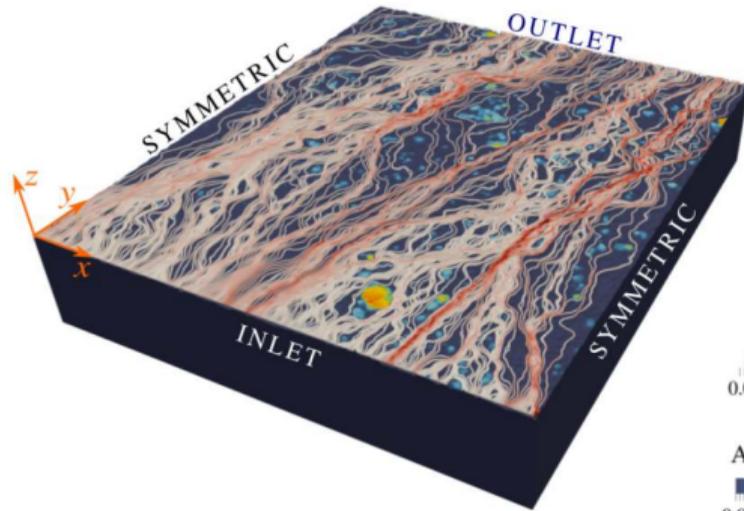




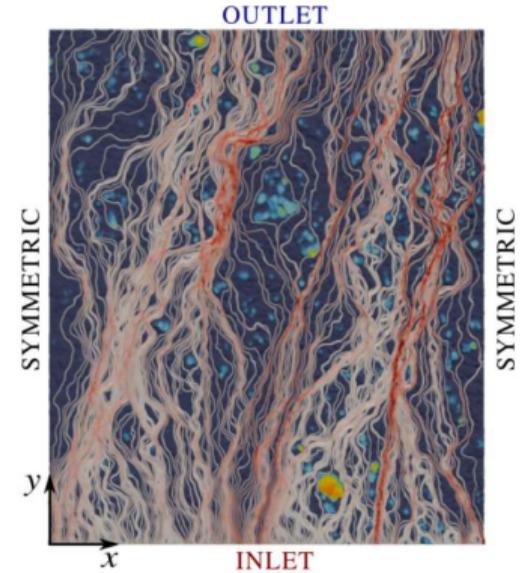
Fluid+solid FE Results



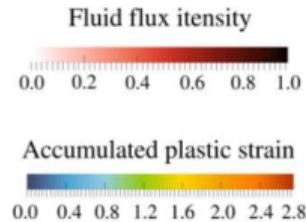
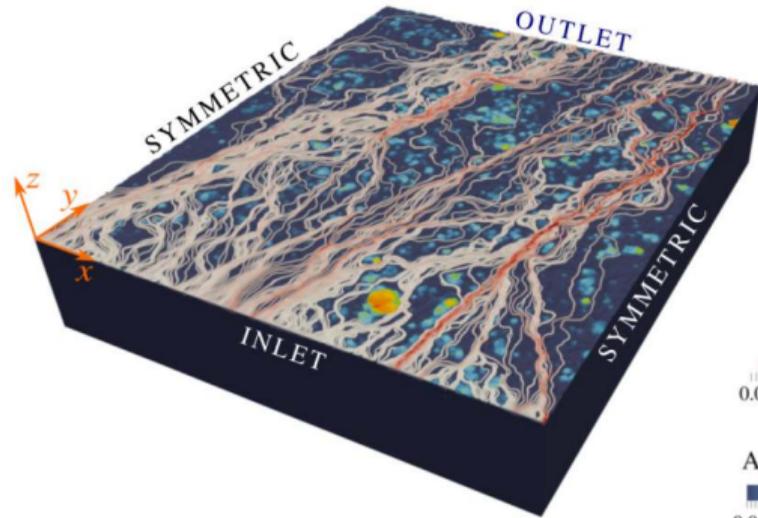
Fluid+solid FE Results



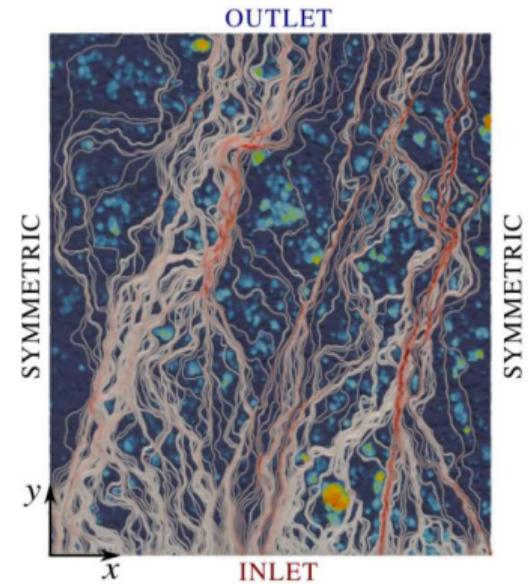
(b)



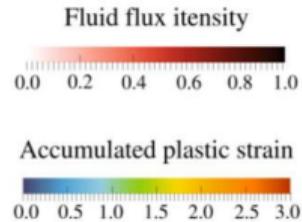
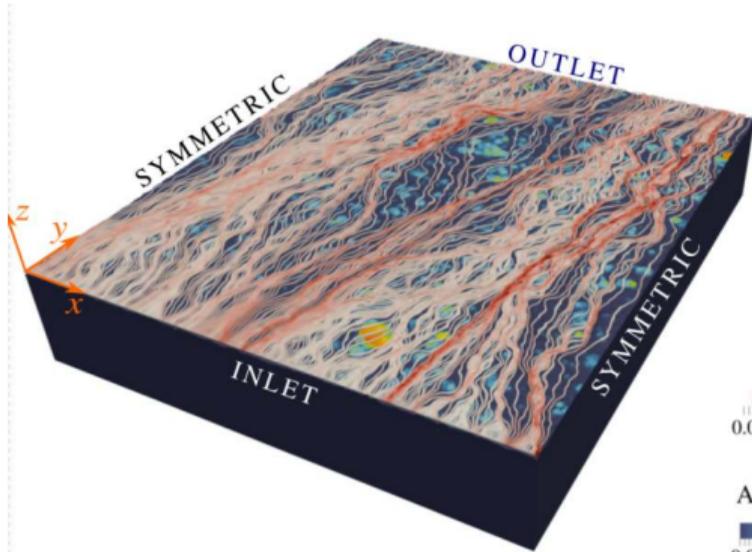
Fluid+solid FE Results



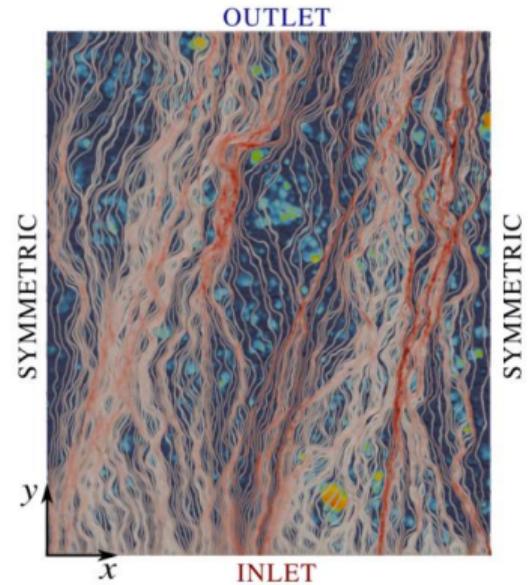
(c)



Fluid+solid FE Results



(d)





Thank you for your attention!
