

Contact Mechanics and Elements of Tribology

Lecture 7. *Friction*

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@ Centre des Matériaux (& virtually)
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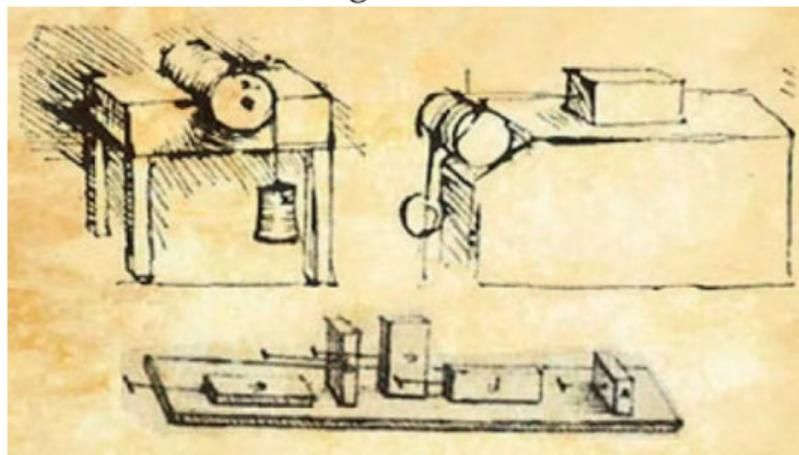
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Outline

- 1 Friction models
- 2 Elastodynamics friction

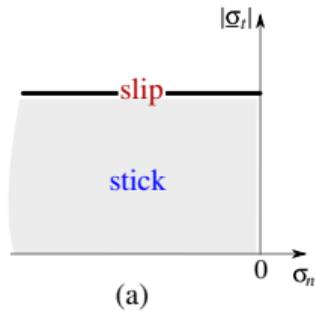
Leonardo da Vinci's experiments

First ever frictional experiments:
notebook drawings of Leonardo da Vinci

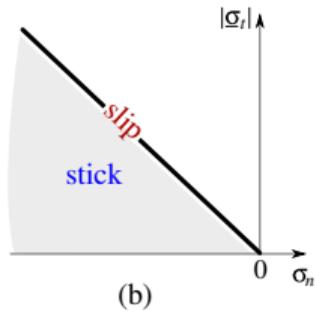


- proportionality between weight and frictional force
- friction is independent on the contact area

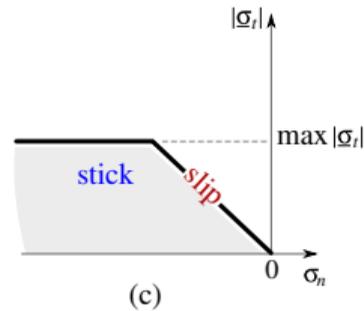
- Static criteria



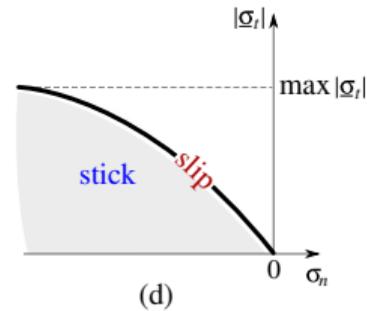
(a) Tresca



(b) Amontons-Coulomb

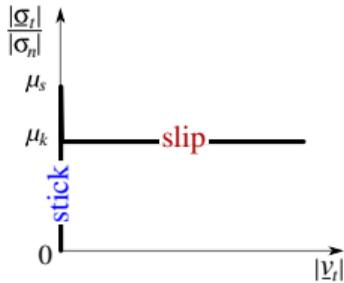


(c) Coulomb-Orowan

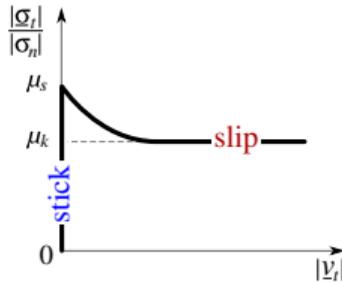


(d) Shaw

• Kinetic criteria

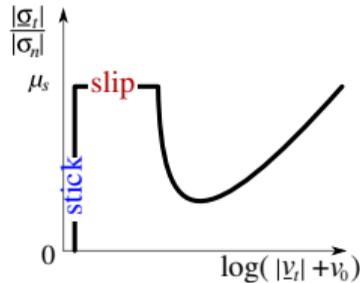


(a)



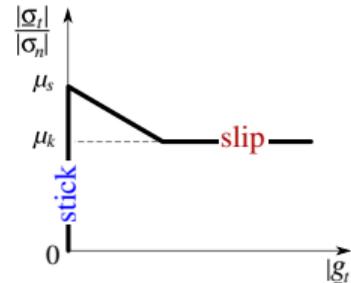
(b)

(a,b) velocity weakening



(c)

(c) velocity
weakening-strengthening

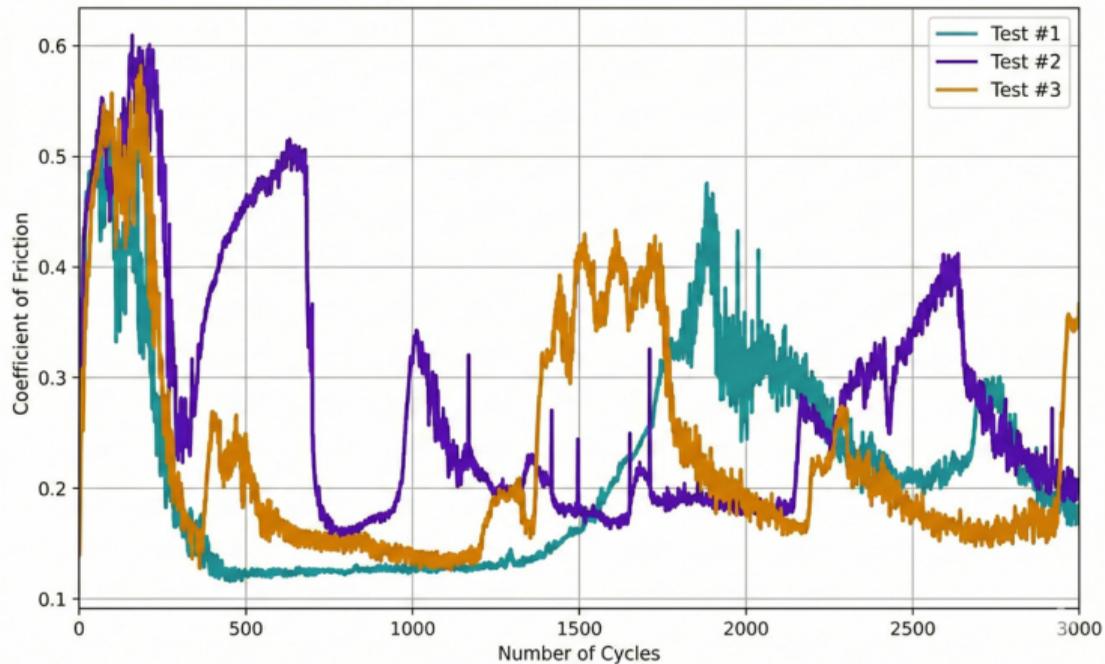


(d)

(d) linear *slip* weakening

μ_s static and μ_k kinetic coefficients of friction.

Models and reality



Dry friction between a DLC coated-steel and a steel

Rate and state friction and regularization

- **Rate and state friction law**

- Rate $v_t = |\underline{v}_t|$ – relative slip velocity

- State θ – \approx internal time

- Dieterich–Ruina–Perrin (1979, 83, 95)

Frictional resistance

$$\sigma_t^c = |\sigma_n| [\mu_s + b\theta + a \ln(v_t/v_0)]$$

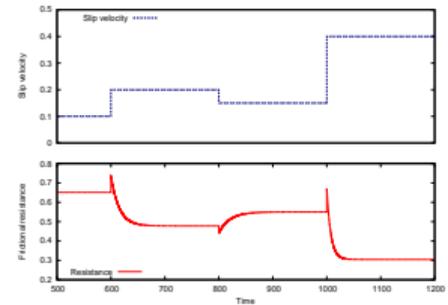
Evolution of the state variable

$$\dot{\theta} = -\frac{v_t}{L} \left[\theta + \ln\left(\frac{v_t}{v_0}\right) \right]$$

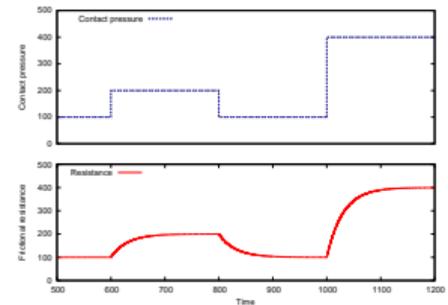
- **Prakash-Clifton friction law (1992,2000)**

- Viscous type evolution of frictional resistance σ_t

- $\dot{\sigma}_t = -\frac{v_t}{L} (\sigma_t + \mu\sigma_n)$



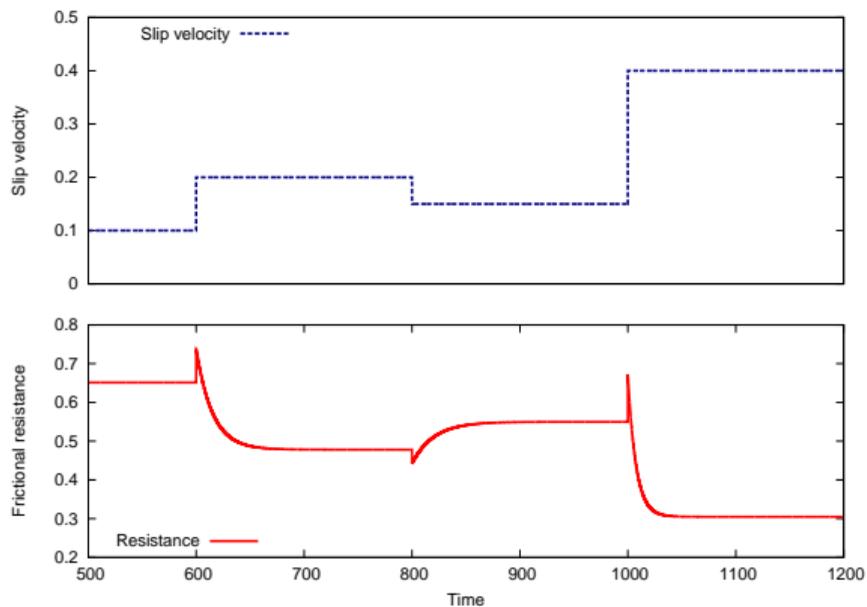
Rate and state friction law



Prakash-Clifton regularization

Rate and state friction and regularization

- Rate and state friction law



Rate and state friction and regularization

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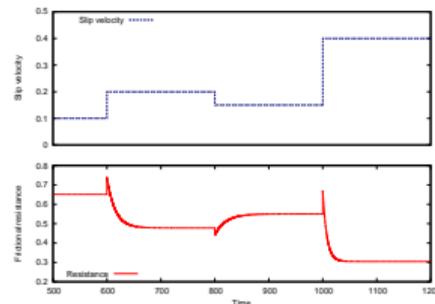
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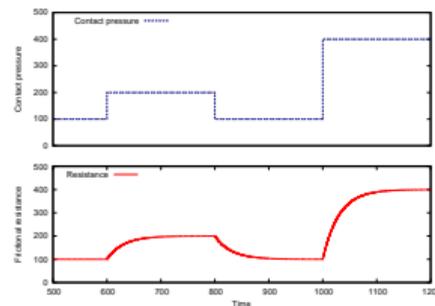
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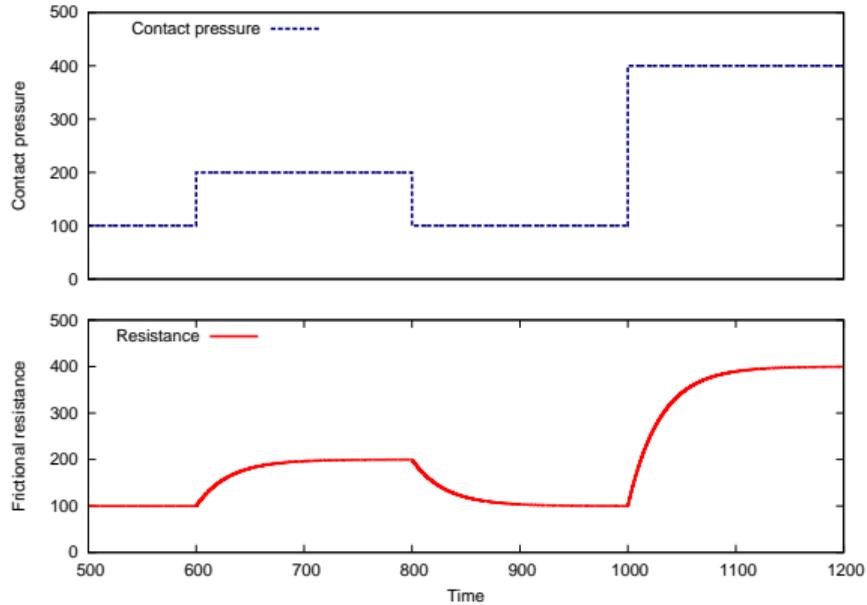
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