Contact Mechanics and Elements of Tribology Lecture 7. Lubrication and Sealing

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Outline

- 1 Lubrication
 - Regimes of lubrication
 - Derivation of the Reynolds equation
 - Analytical solution for hydrostatic lubrication in bearings
 - Elasto-hydrodynamic lubrication
- 2 Sealing
 - Metal-to-metal face seal for nuclear power plant applications
 - Fluid-structure coupling
 - Results of FE numerical simulation

Acknowledgment:

Course "Scientific Computing with Applications in Tribology" A. Almqvist, F. Pérez-Ràfols

Luleå University of Technology, Sweden, 7-9 February 2017

Lubrication: what is it?

- Lubrication: technique to reduce friction and wear between relatively moving surfaces by adding a solid/liquid/gas lubricant
- Studied in Tribology (Greek: tribo "to rub", logy "study of")
 'The Jost Report' (1966): cost of friction, wear and corrosion to UK economy P. Jost (1966)
- Applications:

- gears - seals
- bearings - cams
- piston heads - metal forming
- human joints - HDD ...

 Recent report (2017):
 23% of total world energy losses come from tribological contacts (20% friction, 3% wear)

K. Holmberg, A. Erdemir, Friction (2017)



Lubricant over gears www.iselinc.com



Lubricating a bike chain www.madegood.org

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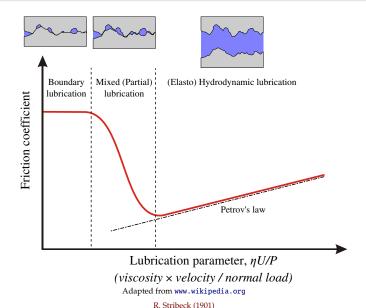


Lubricant over gears www.efficientplantmag.com



Lubricated roller bearing www.bearingtips.com

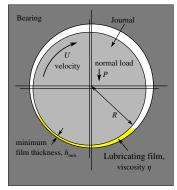
Lubrication regimes: Stribeck curve



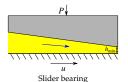
Hydrodynamic lubrication (HL)

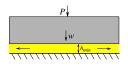
- Conforming surfaces
- No elastic effect
- Normal load fully supported by thin fluid film
- $h_{\min} = f(P, U, \eta, R)$
- $p \le 5$ MPa, $h_{\min} > 1 \mu m$

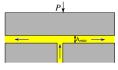
Mechanism of pressure development in fluid film:



Concentric journal bearing Adapted from www.wikipedia.org







Squeeze film bearing

Externally pressurized bearing

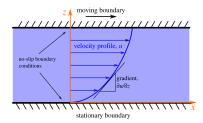
Newtonian fluid

Viscous stresses in flowing fluid are linearly proportional to the strain rate - the gradient of the velocity:

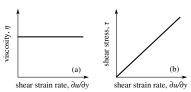
$$\tau = \eta \frac{\partial u}{\partial z}$$

- τ is the shear stress in the fluid
- \blacksquare η is the viscosity of the fluid
- $\frac{\partial u}{\partial y}$ is the shear strain rate
- In general 3D case for arbitrary coordinate system:

$$\tau_{ij} = \eta \left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_i} \right)$$



Laminar shear of fluid between two rigid plates



Properties of Newtonian fluid: (a) viscosity vs shear strain rate (b) shear stress vs shear strain rate

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Petrov's equation

Friction coefficient

Shear stress:

$$\tau = \eta \frac{\partial u}{\partial z} = \eta \frac{U}{h}$$

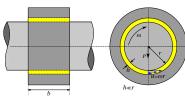
Frictional reaction:

$$T = A\tau = (2\pi rb)\,\eta \frac{U}{h}$$

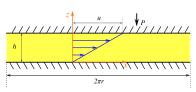
The coefficient of friction:

$$\mu = rac{T}{P} = rac{2\pi r b}{h} rac{\eta U}{P}$$
(Basto) Hydrodynamic habrication

Lubrication parameter, ηU/P
Petrov's law
N.P. Petrov (1883)



Concentric journal bearing



Developed journal and bearing surfaces

r radius of journal

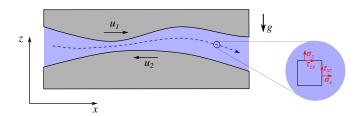
b width of journal

U linear velocity

P normal load

h radial clearance

Stresses on the surface of a fluid element



Stresses on the surface of a fluid element:

$$\tau_{ij} = \eta \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad \tau_{ij} = \tau_{ji}$$

$$\sigma_i = -p - \frac{2}{3}\eta\nabla\cdot\underline{\boldsymbol{u}} + 2\eta\frac{\partial u_i}{\partial x_i}$$

η absolute viscosity

p hydrostatic pressure

 x_i coordinates

 u_i velocity componets

g acceleration of gravity

Navier-Stokes equations

Newton's second law of motion for a fluid element:

$$\rho \frac{D\underline{u}}{Dt} = \rho \underline{\underline{g}} + \nabla \cdot \underline{\underline{g}}$$
inertia forces body forces surface forces

Material derivative:

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$
total derivative local derivative

■ Navier-Stokes equations:

$$\rho \frac{D\underline{u}}{Dt} = \rho \underline{g} - \nabla p - \frac{2}{3} \nabla \left(\eta \nabla \cdot \underline{u} \right) + 2 \left(\nabla \cdot (\eta \nabla) \right) \underline{u} + \nabla \times \left(\eta (\nabla \times \underline{u}) \right)$$

• 4 unknowns: u, v, w, p; 3 equations + the continuity equation

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Continuity equation

■ Flux is a mass of fluid flowing per unit time through a unit area:

$$\underline{q} = \rho \underline{u}$$

Conservation of mass: outflow of mass from a volume equals to decrease of mass within the volume (integral form):

$$\frac{dm}{dt} + \iint_{S} (\underline{q} \cdot \underline{n}) \, dS = 0$$

■ The divergence theorem:

$$\iint_{S} (\underline{q} \cdot \underline{n}) \, dS = \iiint_{V} (\nabla \cdot \underline{q}) \, dV$$

Differential form of continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{\boldsymbol{u}}) = 0$$

10/33

■ If density ρ is constant: $\nabla \cdot \mathbf{u} = 0$

Towards the Reynolds equation

■ Introducing dimensionless variables:

$$X = \frac{x}{l_0}, \quad Y = \frac{y}{b_0}, \quad Z = \frac{z}{h_0}, \quad T = \frac{t}{t_0}, \quad \bar{u} = \frac{u}{u_0}$$

$$\bar{v} = \frac{v}{v_0}, \quad \bar{w} = \frac{w}{w_0}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{\eta} = \frac{\eta}{\eta_0}, \quad P = \frac{h_0^2 p}{\eta_0 u_0 l_0}$$

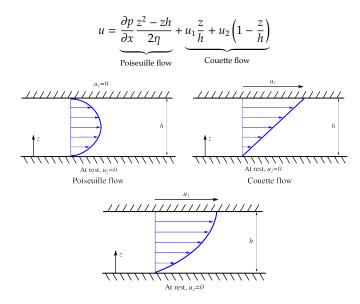
■ Reynolds number:

$$\mathcal{R} = \frac{\text{Inertia}}{\text{Viscous}} = \frac{\rho_0 u_0 l_0}{\eta_0}$$

■ Thin fluid film: $h_0 \ll l_0$, $\mathcal{R} \ll 1$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z} \left(\eta \frac{\partial u}{\partial z} \right)$$
$$\frac{\partial p}{\partial y} = \frac{\partial}{\partial z} \left(\eta \frac{\partial v}{\partial z} \right)$$
$$\frac{\partial p}{\partial z} = 0 \to p = p(x, y)$$

Velocity profile



Calculating flow rates

$$u = \frac{\partial p}{\partial x} \frac{z^2 - zh}{2\eta} + u_1 \frac{z}{h} + u_2 \left(1 - \frac{z}{h} \right)$$
$$v = \frac{\partial p}{\partial y} \frac{z^2 - zh}{2\eta} + v_1 \frac{z}{h} + v_2 \left(1 - \frac{z}{h} \right)$$

Flow rate per unit width in x and y directions:

$$q_x' = \int_0^h u dz = -\frac{h^3}{12\eta} \frac{\partial p}{\partial x} + \frac{u_1 + u_2}{2} h$$
$$q_y' = \int_0^h v dz = -\frac{h^3}{12\eta} \frac{\partial p}{\partial y} + \frac{v_1 + v_2}{2} h$$

Integrating continuity equation across film thickness:

$$\int_{0}^{h} \left[\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{\boldsymbol{u}} \right] dz = 0$$

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Reynolds equation

General form in 3D:

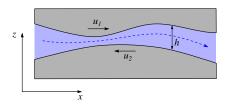
$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial y} \right) = \frac{(u_1 + u_2)}{2} \frac{\partial (\rho h)}{\partial x} + \frac{(v_1 + v_2)}{2} \frac{\partial (\rho h)}{\partial y} + h \frac{\partial \rho}{\partial t}$$

• If ρ , η are constant:

$$\frac{1}{12\eta}\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{1}{12\eta}\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right) = \frac{(u_1 + u_2)}{2}\frac{\partial h}{\partial x} + \frac{(v_1 + v_2)}{2}\frac{\partial h}{\partial y}$$

■ In 2D:

$$\frac{\partial}{\partial x} \left(h^3 \frac{\partial p}{\partial x} \right) = 6\eta (u_1 + u_2) \frac{\partial h}{\partial x}$$



Physically relevant models

■ Non-Newtonian fluid:

$$\eta = f\left(\frac{du}{dz}\right)$$

Viscosity-pressure dependency: Barus law

$$\eta(p) = \eta_0 e^{\alpha p}$$

■ Fluid compressibility:

$$K = -V \frac{dp}{dV} = \rho \frac{dp}{d\rho}$$
$$\rho = \rho_0 e^{(p-p_0)/K}$$

 Cavitation: process of bubble generation due to local pressure decline below saturated vapor pressure and subsequent bubble collapse when the pressure is increased.



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Step slider bearing

• Reynolds equation $(\rho = \text{const}, \eta = \text{const})$:

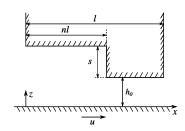
$$\frac{d}{dx}\left(h^3\frac{dp}{dx}\right) = 6u\eta\frac{dh}{dx}$$

Constant film thickness in both sections:

$$h(x) = \begin{cases} h_0 + s & 0 < x < nl \\ h_0 & nl < x < l \end{cases}$$

$$\frac{d^2p}{dx^2} = 0, \quad x \in (0; nl) \cup (nl; l)$$

$$\frac{dp}{dx} = const, \quad x \in (0; nl) \cup (nl; l)$$



Step slider bearing

Continuity of pressure:

$$p|_{x=nl-0} = p|_{x=nl+0} = p_m$$

$$nl\left(\frac{dp}{dx}\right)_i = -(1-n)l\left(\frac{dp}{dx}\right)_o$$

Continuity of flow rate:

$$q'_{x,i} = q'_{x,o}$$

 $u(h_0 + s)$ h_0^3 (1)

$$-\frac{(h_0+s)^3}{12\eta} \left(\frac{dp}{dx}\right)_i + \frac{u(h_0+s)}{2} = -\frac{h_0^3}{12\eta} \left(\frac{dp}{dx}\right)_o + \frac{uh_0}{2}$$

Maximum pressure:

$$p_m = 6\eta u l \left[\frac{n(1-n)s}{(1-n)(h_0+s)^3 + nh_0^3} \right]$$

Step slider bearing

Pressure distribution:

$$p(x) = \begin{cases} p_m \frac{x}{nl} & 0 < x < nl \\ p_m \frac{l-x}{(1-n)l} & nl < x < l \end{cases}$$

$$p_m = 6\eta u l \left[\frac{n(1-n)s}{(1-n)(h_0+s)^3 + nh_0^3} \right]$$

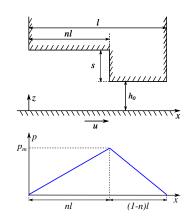
• Optimal bearing configuration to produce the largest p_m :

$$\frac{\partial p_m}{\partial n} = 0 \quad \text{and} \quad \frac{\partial p_m}{\partial s} = 0$$

$$\begin{cases} (1 - n)^2 (h_0 + s)^3 - n^2 h_0^3 = 0\\ (1 - n)(h_0 + s)^2 (h_0 - 2s) + nh_0^3 = 0 \end{cases}$$

Optimal values:

$$\frac{h_0}{s} = 1.155, \quad n = 0.7182$$



Elastohydrodynamic lubrication (EHL)

- Non-conforming surfaces
- Elastic deflection of solid walls
- Viscosity-pressure dependence: $\eta(p) = \eta_0 \exp \xi p$
- Hard EHL (metal parts):

$$0.5 \text{ GPa} \le p \le 3 \text{ GPa}$$

 $0.1 \mu\text{m} \le h_{\text{min}} \le 1 \mu\text{m}$

- gears
- rolling bearings
- cams
- Soft EHL (polymer):

$$p \approx 1 \text{ MPa}$$

 $h_{\min} \approx 1 \mu \text{m}$

- seals
- human joints
- tires



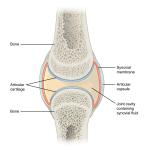
Needle roller bearing www.farazbearing.com



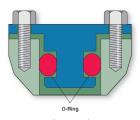
Bevel gear www.linngear.com

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Nontrivial joint www.wikipedia.org



O-ring seal www.ecosealthailand.com

Elastohydrodynamic lubrication (EHL)

Reynolds equation:

$$\frac{d}{dx}\left(\frac{h^3}{\eta}\frac{dp}{dx}\right) = 6u\frac{dh}{dx}$$

Viscosity-pressure dependence (Barus law):

$$\eta(p)=\eta_0\,e^{\xi p}$$

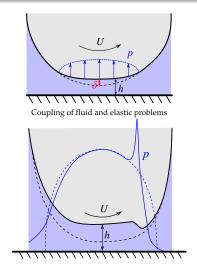
Film shape:

$$h(x) = h_0 + S(x) + \delta(x)$$

 h_0 constant h_0 constant

Contact constraints:

$$\begin{cases} h_0 + S(x) + \delta(x) = 0, & p > 0 \\ h_0 + S(x) + \delta(x) > 0, & p = 0 \end{cases}$$



Results of numerical simulations^[1,2]

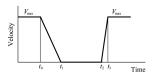
[1] D. Dowson, Wear (1995)

[2] B.J. Hamrock, "Fundamental of fluid film lubrica-

tion"" (1991)

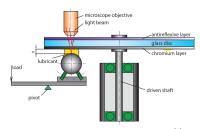
EHL film thickness: experimental observation

- Ball-on-disk optical tribometer
- Measurement based on light interference principle
- Unidirectional start-stop-start motion
- Important for study of rolling contact fatigue and wear



Unidirectional start-stop-start motion^[1]

[1] P. Sperka et al, Journal of Tribology (2014)

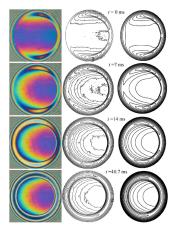


Ball-on-disk apparatus with interferometry^[2]

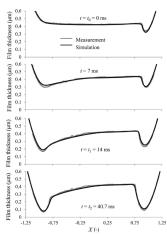
[2] D. Kostal et al, Journal of Tribology (2017)

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EHL film thickness: experimental observation



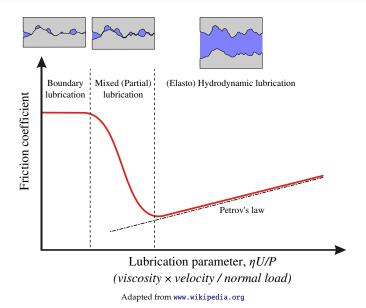
[1] P. Sperka et al, Journal of Tribology (2014)



Midplane film thickness profiles along rolling direction^[1]

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Lubrication regimes: Stribeck curve



Boundary lubrication

- Asperities come in contact
- $1 \text{ nm} \le h_{\min} \le 10 \text{ nm}$
- Bulk lubricant properties (i.e. viscosity) are not important
- Physical and chemical properties of the surface and of the fluid film are important
- Lubricant film of molecular size^[1]
- Breakdown of lubricant film at localized regions^[2], frictional force:

$$F = A \left(\alpha s_m + (1 - \alpha) s_l\right)$$

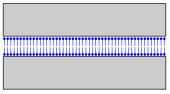
A the area that supports the load

 α fraction of breakdown area

 s_m shear stress in solid contact

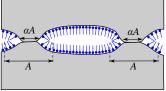
 s_l shear stress in the lubricating film

Therefore, if α - const, then $F \propto A$.



The frictional resistance is due to interaction between the outer surfaces of the adsorbed monolayers without any solid contact occurring $^{[1]}$



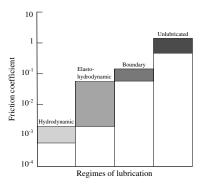


Mechanism involving breakdown of the lubricant film at small localized regions^[2]
[2] F.P. Bowden and D. Tabor, "The friction and Lubrication of Solids" (1950)

Mixed (partial) lubrication

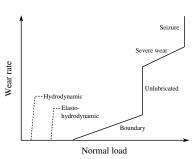
- Combination of boundary and fluid film effects
- Some asperity contact
- Film layer of one or more molecular layers
- Smooth transition
- $0.01 \ \mu \text{m} \le h_{\text{min}} \le 1 \ \mu \text{m}$

Lubrication: friction coefficient and wear



Bar diagram of friction coefficient for various lubrication conditions

B.J. Hamrock, "Fundamental of fluid film lubrication" (1991)



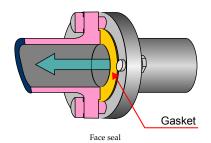
Wear rate for various lubrication regimes
Beerbower (1972)

Lubrication: take home messages

- Lubrication: reduction of wear and friction between relatively moving surfaces by adding lubricant
- Hydrodynamic: full separation of solids, thin fluid film lubrication (Reynolds equation), dynamic viscosity is important
- EHL: solids deform elastically (hard metals, soft polymers), affected by viscosity-pressure dependence
- Boundary: contact of asperities, but still thin molecular level of lubricant, chemical properties important, breakdown of fluid film
- Mixed (partial): smooth transition
- Recommended literature:
 - 1 B.J. Hamrock et al, "Fundamentals of fluid film lubrication" (2004)
 - 2 F.P. Bowden and D. Tabor, "The friction and Lubrication of Solids (1950)
 - 3 D. Dowson, "Elastohydrodynamic and micro-elastohydrodynamic lubrication", Wear, 190 (1995)

Sealing: what is it?

- Sealing: technique to prevent or reduce leakage of fluid from one chamber to another using seals
- Different types:
 - face seals
 - O-ring seals
 - labyrinth seals
- Dynamic/static
- Material: polymer/metallic
- Operate in EHL/mixed/boundary regimes





O-ring

Application: metal-to-metal static face seal

- Metal-to-metal static face seals used in fluid system of nuclear power plants
- Coting of the seal is made of material Norem^[1]: elasto-plastic Elastic moduli:

$$E = 175 \,\text{GPa}, \nu = 0.3$$

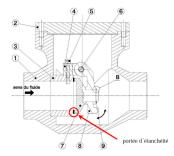
Yield stress:

$$\sigma_Y = R_0 + Q(1 - e^{-bp})$$

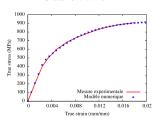
$$R_0 = 442.7 \text{ MPa}$$

 $Q = 493.5 \text{ MPa}$
 $b = 242.2$

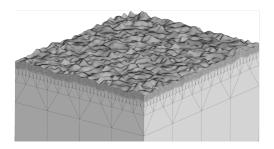
J. Durand, PhD thesis (2012)

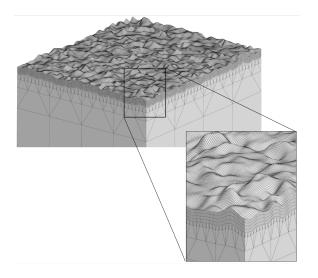


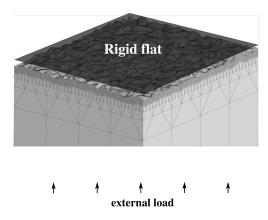


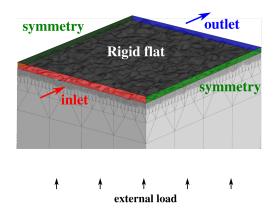


Material behavior of Norem^[1]









	Surface discretization	Total DOFs	RAM	Cores	Time
	256 × 256	1.4M	30 Gb	8	2-4 days
Ì	512 × 512	5.7M	140 Gb	16	4-8 days

Mechanical contact (unilateral):

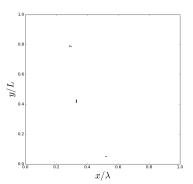
$$\begin{cases} \nabla \cdot \underline{\underline{\sigma}}(\underline{u}) = 0 & \text{in } \Omega_s, \\ g(\underline{u}) \ge 0, \ \sigma_n(\underline{u}) \le 0, \ g(\underline{u}) \ \sigma_n(\underline{u}) = 0 & \text{at } \Gamma_c, \\ u_x|_{x=0,\lambda/2} = 0, \quad u_y|_{y=0,L} = 0, \end{cases}$$

■ Thin fluid flow with immobile walls (Reynolds equation):

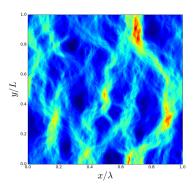
$$\begin{cases} \nabla \cdot \left[g(\underline{\boldsymbol{u}})^{3} \nabla p_{f} \right] = 0 & \text{in } \Gamma_{f} \\ p_{f} \big|_{y=0} = p_{i}, & p_{f} \big|_{y=L} = p_{o} \\ \left[\nabla p_{f} \cdot \underline{\boldsymbol{e}}_{x} \right] \big|_{x=0,\lambda/2} = 0, \end{cases}$$

Fluid/structure interface:

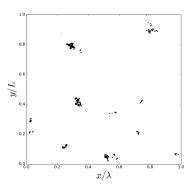
$$\sigma_n(\mathbf{u}) = -\mathbf{p}_f$$
 at Γ_f



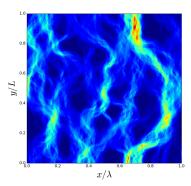
Morphology of the contact interface



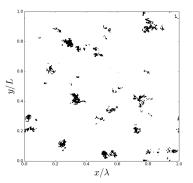
Intensity of the fluid flux



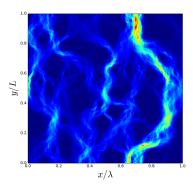
Morphology of the contact interface



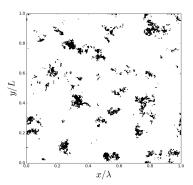
Intensity of the fluid flux



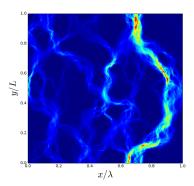
Morphology of the contact interface



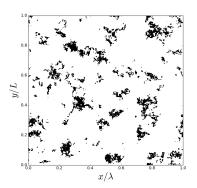
Intensity of the fluid flux



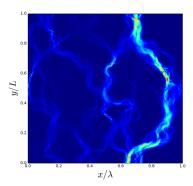
Morphology of the contact interface



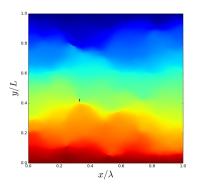
Intensity of the fluid flux



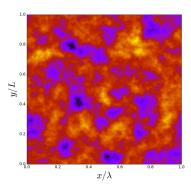
Morphology of the contact interface



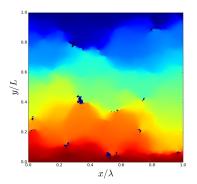
Intensity of the fluid flux



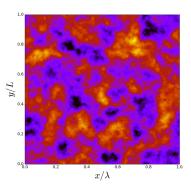
Distribution of the fluid pressure



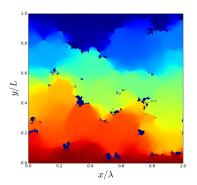
Distribution of the free volume



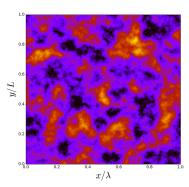
Distribution of the fluid pressure



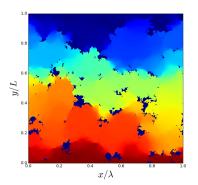
Distribution of the free volume



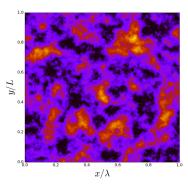
Distribution of the fluid pressure



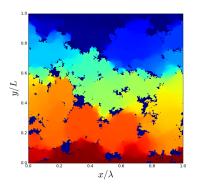
Distribution of the free volume



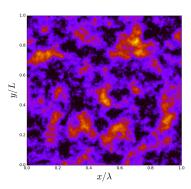
Distribution of the fluid pressure



Distribution of the free volume

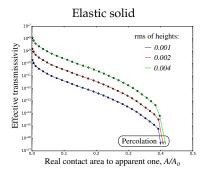


Distribution of the fluid pressure



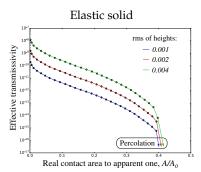
Distribution of the free volume

Transmissivity of the interface



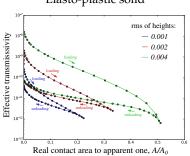
Effective transmissivity of the interface in case of elastic material (loading until percolation)

Transmissivity of the interface

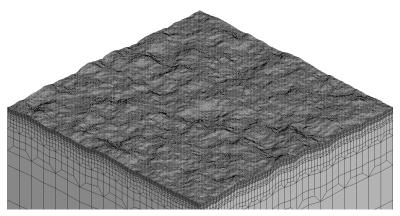


Effective transmissivity of the interface in case of elastic material (loading until percolation)

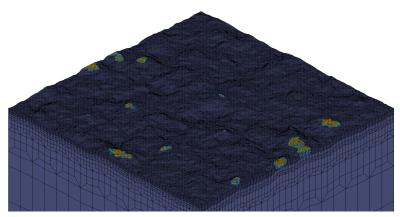
Elasto-plastic solid



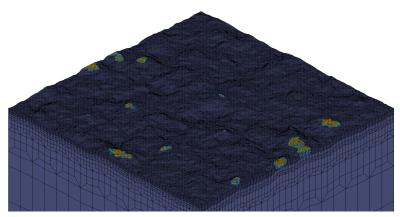
Effective transmissivity of the interface in case of elasto-plastic material (loading-unloading cycle)



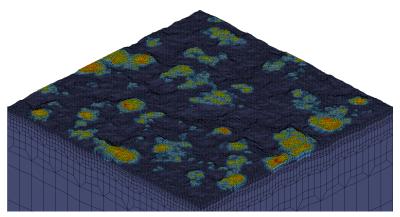
FE mesh



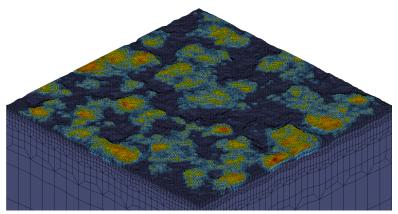
Accumulated plastic strain during loading



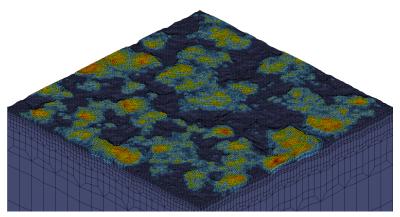
Accumulated plastic strain during loading



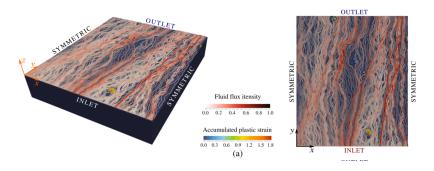
Accumulated plastic strain during loading

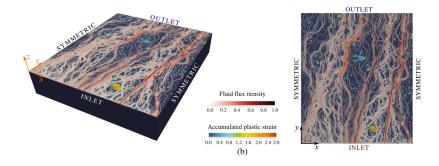


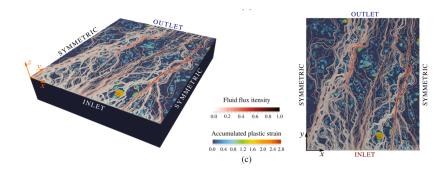
Accumulated plastic strain during loading

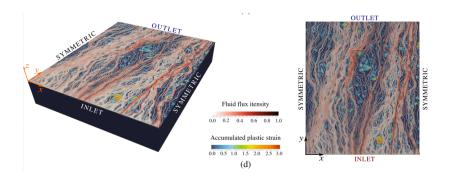


Accumulated plastic strain during unloading









Thank you for your attention!