Contact Mechanics and Elements of Tribology

Practical work on planar contact Integration of Flamant's solution

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January 22, 2024

Introduction

Let's try to find the stress fields in the vicinity of contact, and displacements on the surface, induced by three specific contact-pressure distributions defined on the interval $x \in [-a, a]$:

1. Uniform pressure (Fig. 1(a)): $p = p_0$
2. Singular at edges pressure (Fig. 1(b)): $p = \dfrac{p_0}{\sqrt{1 - x^2/a^2}}$
3. "Elliptic" pressure (Fig. 1(c)): $p = p_0\sqrt{1 - x^2/a^2}$

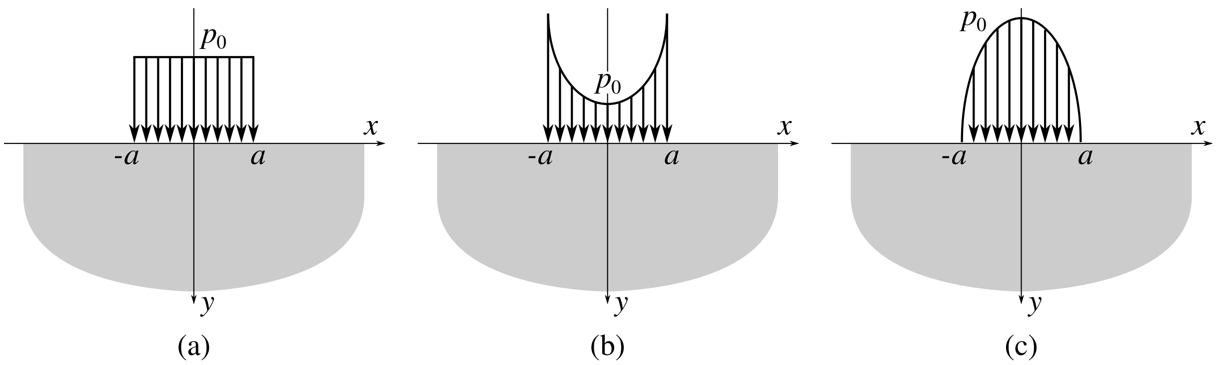

Figure 1: Pressure distribution on the half-space: (a) uniform, (b) singular at edges, (c) "elliptic" or Hertzian pressure.

To find the stress field and the displacement, we will use the following equations (use numerical integration):

$$\sigma_x(x,y) = -\frac{2y}{\pi}\int\limits_{-b}^{a} \frac{p(s)(x-s)^2\,ds}{((x-s)^2+y^2)^2}$$

$$\sigma_y(x,y) = -\frac{2y^3}{\pi}\int\limits_{-b}^{a} \frac{p(s)\,ds}{((x-s)^2+y^2)^2}$$

$$\sigma_{xy}(x,y) = -\frac{2y^2}{\pi}\int\limits_{-b}^{a} \frac{p(s)(x-s)\,ds}{((x-s)^2+y^2)^2}$$

$$u_x(x,0) = -\mathrm{sign}(x)\frac{(1-2\nu)(1+\nu)}{2E}\left[\int\limits_{-b}^{x} p(s)\,ds - \int\limits_{x}^{a} p(s)\,ds\right] + C_1$$

$$u_y(x,0) = -\frac{2(1-\nu^2)}{\pi E}\int_{-b}^{a} p(s)\ln|x-s|\,ds + C_2$$

Code

You are provided with two simple Python scripts for evaluation of above-mentioned integrals, however, you'll need to substitute correct expression for the integrands.

- To evaluate stresses
  `compute_stresses_virgin.py`
- To evaluate displacements
  `compute_displacements_virgin.py`

Objectives

1. Find the stress field in a region which extends to $x \in [-2a : 2a]$ and $y \in [-4a : 0]$ for all three pressure distributions mentioned above.
2. Using Hertz's contact formulae, analyze 2D contact between a flat steel substrate ($E = 210$ GPa, $\nu = 0.3$) and a diamond "cylindrical" indenter of radius $R = 10$ mm. Determine the point of the onset of plasticity and the corresponding force F_c using von Mises yield criterion for the yield stress $R = 350$ MPa. The von Mises stress is given by

   $$\sigma_{vM} = \sqrt{\frac{3}{2}\underline{\underline{s}} : \underline{\underline{s}}}$$

   where the deviator of the stress tensor is given by

   $$\underline{\underline{s}} = \underline{\underline{\sigma}} - \frac{1}{3}\mathrm{tr}(\underline{\underline{\sigma}})\underline{\underline{I}}.$$

   Alternatively, the von Mises stress could be found as:

   $$\sigma_{vM} = \sqrt{\frac{1}{2}\left[(\sigma_x-\sigma_y)^2 + (\sigma_x-\sigma_z)^2 + (\sigma_y-\sigma_z)^2 + 6\left(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2\right)\right]}$$

   Hint: don't forget about σ_{zz}.
3. Following a similar procedure, find displacements on the top surface $u_x(x,0)$, $u_y(x,0)$ for all three pressure distributions mentioned above.