Practical Work: Fretting Fatigue

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1. Introduction

Fretting fatigue is on of the central problems in tribology in numerous applications. Everywhere where two bodies are in contact and subjected to cyclic loading, fretting fatigue could a problem. In this practical work, we will learn how to simulate fretting fatigue using the fundamental solution of the elastic half-space.

2. Problem Set-up

Consider a slider of a shape described by the following equation:

$$y = A(1 - \cos(2\pi x/\lambda)), \quad x \in [-\lambda/2, \lambda/2]$$

which is pressed by a force P into a long elastic sheet of height H and width W (see Fig. 1). A symmetric set-up is considered, i.e. the same slider is pressed on the opposite side of the sheet. The sheet is subjected to a cyclic loading Q with a constant amplitude Q_0 and a frequency ω.

$$Q = Q_0 \sin(\omega t).$$

The material of the slider and the sheet is assumed to be similar Ti-6Al-4V alloy with Young's modulus $E = 115$ GPa and Poisson ratio $\nu = 0.33$. The coefficient of friction between the slider and the sheet is $\mu = 0.4$. Let us assume a plane strain problem, i.e. $\varepsilon_{zz} = 0$. Then the problem is reduced to a 2D problem in the x, y plane.

3. Analytical Solution

We will use Cattaneo-Mindlin analytical solution to simulate the problem. The solution for the stick zone half-width c is given by:

$$\frac{c}{a} = \sqrt{1 - \frac{Q_0}{\mu P}},$$

where a is the contact half-width which can be determined from Hertz solution for line contact (see lecture notes). The slip zone is located at the edges of the

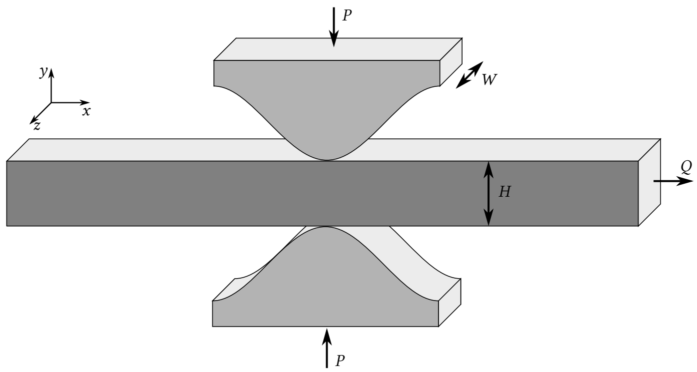

Figure 1: Problem set-up.

contact zone, and its width is given by: $b = a - c$. The contact pressure is given by:

$$p(x) = \begin{cases} p_0\sqrt{1 - \dfrac{x^2}{a^2}}, & \text{if } |x| \leq a; \\ 0, & \text{elsewhere,} \end{cases}$$

where $p_0 = 2P/(\pi a)$. The shear tractions in the interface are

$$q(x) = \begin{cases} \mu p_0 \left(\sqrt{1 - x^2/a^2} - \sqrt{c^2/a^2 - x^2/a^2}\right) & \text{, if } |x| \leq c; \\ \mu p_0 \left(\sqrt{1 - x^2/a^2}\right) & \text{, if } c < |x| \leq a; \\ 0 & \text{, elsewhere.} \end{cases} \tag{1}$$

Surface tractions are shown in Fig. 2.

4. Numerical Solution

We will use the fundamental solutions of the elastic half-space to simulate the problem. The expressions are provided in the lecture notes. Before looking for the stress field, we need to solve Hertz problem for our material and loading conditions. We need to compute the contact half-width a and the contact pressure p_0 we also need to compute the stick zone c. Note that such a solution would be valid for the first cycle only, since the history of sliding for past cycles is not taken into account.

When the contact problem is solved, we can compute the stress field in the bulk and close to surface.

As soon as it is done, we can find the orientation of planes of maximum shear stress in the whole specimen and the corresponding shear stress values. We will find as well how these planes' orientation change with the loading. Consider

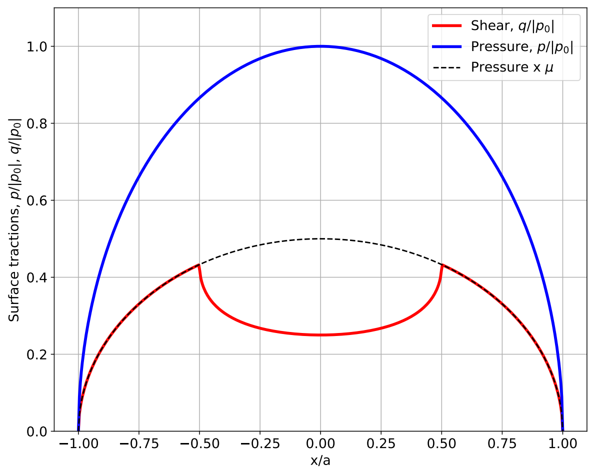

Figure 2: Surface tractions.

the first loading quarter-cycle, i.e. $t \in [0, \pi/\omega/2]$. Report the orientation of the planes of maximum shear stress and the maximum shear stress at time $t = \pi/\omega/4$ and $t = \pi/\omega/2$.